



A Note on Constant-Gain Digital Resonators

Author(s): Ken Steiglitz

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cal language (algorithms, fractals) preferred by the audience in a time of economic problems, growing population, growing pollution, and political conservatism? Or is a traditionally valued music with clearly defined timbre ideals (presets), performance traditions, and player limitations of more interest? Making electroacoustic music more interesting just by including instrumental players in a composition or by creating new instruments for a better imitation of acoustic instruments is not a solution for this problem. It is more likely helpful to define new instrument-player interactions instead of reproducing the traditional ones. New ways of interaction might create a new aesthetic, but does the listener want this?

In the context of human interaction between composition and performance, we can identify fundamental differences between electroacoustic music and instrumental music that cause the problem of why "less-highly-valued examples of electroacoustic music can indeed be significantly 'worse' than even very 'bad' traditional music." When performed by a good performer, a bad composition is still an interesting experience because the performer modifies the information of the score by applying interpretive habits and the timbre of a good instrument. The performer would shape the results in electroacoustic music as well by his or her personal view, applying an interpretive language to the formal structure of the piece. This helps the listener perceive a complex musical structure or enriches a poor structure. A personal interpretation adds a common grammar to the piece and eases the process of understanding or adds some good qualities to the otherwise bad piece. Because electroacoustic music does not go through

this process of modification, a bad piece will be played as poorly as it is, and even a good piece has more problems in the process of communication because no common interpretational grammar helps in the communicative process. Live interaction may supply a solution for this dilemma in electroacoustic music, as mentioned above, but do we practitioners deliver something equivalent to the positive effect of the "stereotypical" interpretive habits or timbres if we are not reproducing instrumental performance behavior?

I think composers should place more emphasis on the development of expressive grammatical elements in their compositions, and I believe that radio and television stations should broadcast more electroacoustic music and should support the discussion of technical and aesthetic concepts in the program as a part of music perception. Composers of electroacoustic music should include more of the left out visual sphere into their compositions—light, movement, space (not only in the acoustical sense), visual art, and others. Otherwise, the continuation of a conservative aesthetic will lead electroacoustic music to become even more isolated as an art form.

Ludger Brümmer  
Essen, Germany

### **Electronic Resources for Computer Music**

I recently picked up a copy of *Computer Music Journal* and found that the MAX user's mailing list was missing from your compilation of net resources. The MAX list moved to McGill University's Faculty of Music early in 1994 and was subsequently transferred to a list server

machine at McGill's Computing Centre. This mailing list averages a membership of more than 200 and discusses everything from simple user questions to the development of custom objects. To subscribe, readers should send an electronic letter with the contents SUBSCRIBE MAX {first and last name} to [listserv@vm1.mcgill.ca](mailto:listserv@vm1.mcgill.ca). They will then receive a help file generated by the list server utility informing them of their request status. Once subscribed, readers should send correspondence intended for list subscribers to [max@vm1.mcgill.ca](mailto:max@vm1.mcgill.ca). Note that because of the local site change, the previous address at McGill ([max@music.mcgill.ca](mailto:max@music.mcgill.ca)) is no longer valid. If any problems or difficulties arise, the human list owner can be reached by sending mail to [max-owner@vm1.mcgill.ca](mailto:max-owner@vm1.mcgill.ca).

Jason D. Vantomme  
Evanston, Illinois, USA

In the on-line editor's note concerning available on-line resources, the list server for American University is listed as being at [auvm.auvm.edu](http://auvm.auvm.edu). This was changed to [auvm.american.edu](http://auvm.american.edu) some time ago.

Joe McMahan  
[xrjdm@farside.gsfc.nasa.gov](mailto:xrjdm@farside.gsfc.nasa.gov)

### **A Note on Constant-Gain Digital Resonators**

The two-pole digital resonator is the simplest band-pass filter and is widely used in computer music as a fast way to shape the spectra of signals and noise. Smith and Angell (1982) proposed improving the two-pole resonator by adding zeros at  $z = \pm 1$  or  $z = \pm\sqrt{R}$ , where  $R$  is the pole radius. This controls the varia-

tion in peak gain as the resonant frequency of the filter is swept, and it introduces notches in the gain curve at zero and the Nyquist frequencies, improving its shape when the resonant frequency is very low or very high. With zeros at  $\pm 1$ , the filter transfer function becomes

$$H(z) = G \frac{1 - z^{-2}}{1 - 2R \cos \theta z^{-1} + R^2 z^{-2}}$$

(RESON\_1)

in place of the simple two-pole resonator with no zeros:

$$H(z) = G \frac{1}{1 - 2R \cos \theta z^{-1} + R^2 z^{-2}}$$

(RESON)

where  $\theta$  is the pole angle. The implementation equation of RESON\_1 is simply

$$y_i = G \cdot (x_i - x_{i-2}) + (2R \cos \theta) y_{i-1} - (R^2) y_{i-2} \quad (1)$$

Thus the computation required by RESON\_1 is just one add operation more per sample than RESON because the numerator coefficient is unity.

The difficulty noted by Smith and Angell (1982) is that the gain of RESON\_1 at  $\theta$ , although a much less sensitive function of  $\theta$  than the peak gain of RESON, is still not exactly constant if  $\theta$  is swept while keeping  $R$  fixed.

Smith and Angell (1982) then suggest a remarkable solution; they move the zeros to  $\pm \sqrt{R}$ , corresponding to the transfer function

$$H(z) = G \frac{1 - Rz^{-2}}{1 - 2R \cos \theta z^{-1} + R^2 z^{-2}}$$

(RESON\_R)

The magnitude response at the pole angle  $\theta$  is now independent of

$\theta$ , being in fact  $1/(1-R)$ , so we use  $G=1-R$  to normalize the gain at  $\theta$  to unity. A disadvantage of RESON\_R with respect to RESON\_1 is the fact that the sharp notches at  $\omega = 0$  and  $\pi$  due to the zeros at  $z = 1$  and  $-1$  are degraded.

### Scaling Gain at True Peak

An interesting property of all three of these RESON filters is that the peak magnitude response does not occur precisely at the pole angle  $\theta$ . This means that for RESON\_R the magnitude response at the actual peak frequency is not really independent of resonant frequency. Throughout this note I will denote the true peak frequency by  $\psi$  and the pole angle by  $\theta$ .

In the case of RESON, the simple resonator with no zeros, the actual peak frequency occurs at (Steiglitz 1974)

$$\cos \psi = \frac{1 + R^2}{2R} \cos \theta. \quad (2)$$

Note that there are some pairs of values of  $\theta$  and  $R$  for which there is no  $\psi$  because the factor

$$\frac{1 + R^2}{2R} \quad (3)$$

is greater than 1 for all  $R > 0$ . This happens when the bandwidth is large compared with the center frequency and the center frequency is small, making  $R$  small and  $\cos \theta$  large. The peak in the magnitude response then occurs at zero frequency, and  $\cos \psi > 1$  in Equation 2.

It turns out, however, that the gain at the true peak  $\psi$  of RESON\_1 is actually independent of resonant frequency, just as the gain at  $\theta$  of RESON\_R is independent of resonant frequency. We can therefore

sweep the pole angle of RESON\_1 keeping  $R$  constant without recomputing the gain constant  $G$ . This makes RESON\_1 faster to implement than RESON\_R and restores the advantage of the zeros at  $\pm 1$ .

The algebra is somewhat tedious but straightforward: differentiate the magnitude response with respect to frequency and set the result to zero; then substitute the result back to find the gain at the peak. I will give the results. The relationship between  $\theta$  and  $\psi$  for RESON\_1 is

$$\cos \psi = \frac{2R}{1 + R^2} \cos \theta. \quad (4)$$

The multiplier of  $\cos \theta$  is the reciprocal of what it is for RESON, which implies that the magnitude response always has a peak at non-zero frequencies. On the other hand, if we choose  $\psi$  and try to find an appropriate  $\theta$ , there may be no solution if  $R$  is too small. The special cases  $\psi = 0$  or  $\pi$  are impossible to achieve with RESON\_1 because of the zeros at those frequencies, but there is always a filter with peak at  $\psi \neq 0$  or  $\pi$  for  $R$  sufficiently close to one (sufficiently narrow bandwidth).

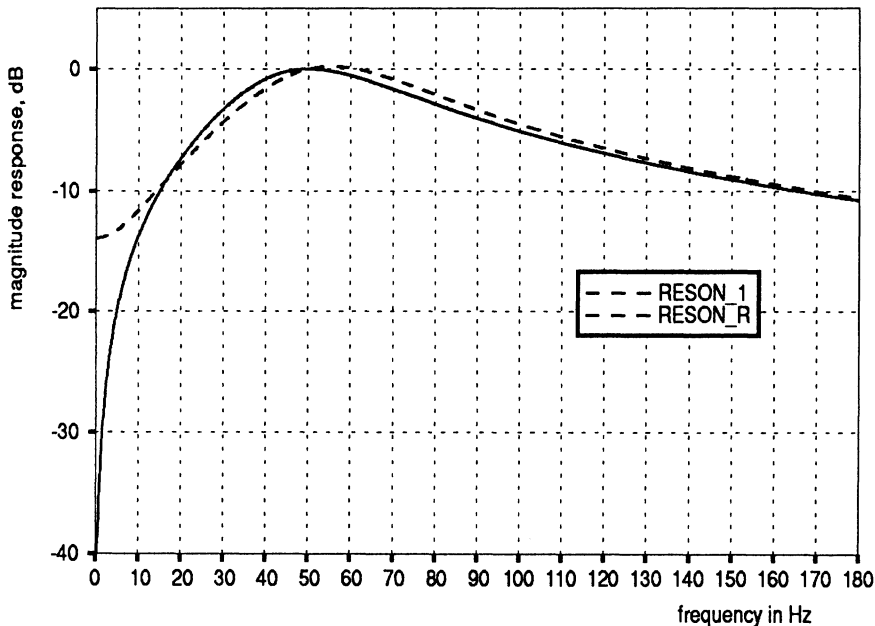
The gain at the peak frequency  $\psi$  of RESON\_1 is

$$2/(1 - R^2) \quad (5)$$

so we normalize with  $G = (1 - R^2)/2$ .

Figure 1 shows a comparison between RESON\_1 and RESON\_R for the case of a peak at  $\psi = 50$  Hz and a half-power bandwidth of 50 Hz, assuming a sampling frequency of 44,100 Hz. The RESON\_1 was designed by calculating  $\theta$  from  $\psi$  using Equation 4, and RESON\_R was designed by choosing  $\theta = \psi$ . RESON\_R is correctly normalized to unity (0 dB) at 50 Hz, but its peak occurs at 55 Hz. The excess gain at 55 Hz is only about 0.2 dB.

Figure 1. Comparison of the magnitude response of RESON\_1 and RESON\_R.



The difference between the pole angle  $\theta$  and the peak  $\psi$  is appreciable only when the bandwidth is large, when it is least noticeable to the ear. In practice, we can therefore specify RESON\_1 by taking  $\theta = \psi$ . The result is a resonator with truly constant peak gain at the price of only one more add per sample than RESON.

### Scaling Power Gain

When a resonator is used to shape the spectrum of white noise, we may want to choose the gain con-

stant  $G$  to scale the output power rather than the peak magnitude response. Call the output power of a filter with unit-variance white-noise input its *power gain*  $P$ . It can be computed in either the time or frequency domain as follows:

$$P = \sum_{t=-\infty}^{\infty} |h(t)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\theta})|^2 d\theta \quad (6)$$

It is a pleasant fact that the power gain of RESON\_1 is simply  $2/(1-R^2)$ , the same as its peak magnitude, and also independent of  $\theta$ . As before, the algebra is a bit messy but straightforward. Thus to scale for unit output power with unit-variance white-

noise input, we should use the gain constant  $G = \left[ (1-R^2)/2 \right]^{1/2}$ .

### Conclusions

The peak gain of the Smith-Angell resonator with zeros at  $\pm 1$  is independent of resonant frequency if we consider the gain at the true peak, rather than at the pole angle. So is the power gain. Thus the resonator with zeros at  $\pm 1$  is preferable to the one with zeros at  $\pm \sqrt{R}$  in three respects: (1) it has truly constant gain as resonant frequency is changed with fixed bandwidth; (2) it has sharper notches in the gain curve at zero and Nyquist frequency; and (3) it requires one fewer multiply per sample.

### References

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Ken Steiglitz  
Princeton, New Jersey 08544 USA  
Ken@Princeton.edu