Quantum phase noise reduction in soliton collisions

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***Abstract*— We show that soliton collisions can reduce quantum phase noise. This effect can improve quantum nondemolition measurements, simply by changing the parameter regime in which the measurement is performed. Successful implementation favors short propagation distances, small wavelength separation between solitons, and larger probe than signal solitons.**

In a quantum nondemolition (QND) measurement, a quantity of interest can be measured repeatedly and any noise is introduced only to conjugate observables. The collision of two solitons provides one realization for such a measurement, in which the phase of a probe soliton provides a QND reading of the photon number of a signal soliton [1], [2].

An accurate reading of the probe soliton phase is important. However, experiments encounter several sources of noise. One is guided acoustic-wave Brillouin scattering (GAWBS) [3], which can be suppressed using a closely spaced reference pulse such that both the reference and probe solitons acquire the same GAWBS noise [1], [2]. A second noise source originates from soliton propagation—vacuum-induced amplitude fluctuations couple with the Kerr nonlinearity of the medium to cause an overall phase diffusion through self-phase modulation (SPM). Though several ways to combat this have been proposed, including a specially prepared local oscillator [4], [5], a phase shifting cavity [6], and a near-resonant two-level medium [7], none have been experimentally demonstrated.

In this paper we show that phase noise can be reduced by a soliton collision due to a negative correlation between phase fluctuations induced by SPM and cross-phase modulation (XPM). We then show how this effect can improve QND measurements.

The field is described by the quantum nonlinear Schrödinger equation,

$$i\frac{\partial}{\partial z}\hat{\phi} = -\frac{\partial^2}{\partial x^2}\hat{\phi} + 2\hat{\phi}^\dagger\hat{\phi}\hat{\phi}, \tag{1}$$

with dimensionless propagation distance z normalized to $2T_0^2/|\beta_2|$, and the normalized field operator $\hat{\phi}$ is scaled such that the photon number is $\bar{n} \equiv 2\lambda|\beta_2|/hc\gamma T_0$. Dispersion is given by β_2, $T_0 = T_{\text{FWHM}}/1.763$, wavelength λ, and the nonlinearity parameter $\gamma = 2\pi n_2/\lambda A_{\text{eff}}$, with Kerr nonlinearity n_2 and effective mode area A_{eff}.

We decompose the field into a mean classical field ϕ_0 and quantum noise term $\Delta\hat{\phi}$ as $\hat{\phi}(x,z) = \phi_0(x,z) + \Delta\hat{\phi}(x,z)$ [8]. The mean field ϕ_0 is the classical fundamental soliton solution of Eq. (1) [9]:

$$\begin{aligned}\phi_0 = \frac{A}{2}\exp\left(ip(x-x_0) - ip^2 z + i\theta + i\frac{A^2}{4}z\right)\\ \times \operatorname{sech}\left(\frac{A}{2}(x - x_0 - 2pz)\right).\end{aligned} \tag{2}$$

This solution is described by four parameters: amplitude A, frequency (momentum per photon) p, position x_0, and phase θ. The perturbation $\Delta\hat{\phi}$ is expanded to describe the changes in the four soliton parameters:

$$\Delta\hat{\phi}_{\text{sol}} = \sum_m f_m(x,z)\Delta\hat{m}(z)\exp\left(i\Phi(z)\right), \ \ m \in \{A, x_0, p, \theta\}, \tag{3}$$

where basis vectors $f_m(x,z) \equiv (\partial\phi_0/\partial m)|_{x_0=p=0}$, classical phase shift $\Phi(z) \equiv (A_0^2/4)z$, and $\Delta\hat{m}(z)$ represents the quantum fluctuations of each soliton parameter.

These fluctuation operators evolve according to [8]

$$\begin{aligned}\Delta\hat{A}(z) &= \Delta\hat{A}(0), \quad \Delta\hat{\theta}(z) = \Delta\hat{\theta}(0) + \frac{A}{2}\Delta\hat{A}(0)z,\\ \Delta\hat{p}(z) &= \Delta\hat{p}(0), \quad \Delta\hat{x}_0(z) = \Delta\hat{x}_0(0) + 2\Delta\hat{p}(0)z.\end{aligned} \tag{4}$$

As a result, the soliton experiences phase diffusion and wave packet spreading, while the amplitude and frequency propagate unchanged. The initial variances are found by projecting statistics of the coherent state, in which $\langle\Delta\hat{\phi}(x)\Delta\hat{\phi}^\dagger(x')\rangle = \delta(x-x')$, yielding [8]

$$\begin{aligned}\langle\Delta\hat{A}^2\rangle_0 &= A, \quad \langle\Delta\hat{\theta}^2\rangle_0 = \frac{1}{3}\left(1 + \frac{\pi^2}{12}\right)\frac{1}{A},\\ \langle\Delta\hat{p}^2\rangle_0 &= \frac{A}{12}, \quad \langle\Delta\hat{x}_0^2\rangle_0 = \frac{\pi^2}{3A^3}.\end{aligned} \tag{5}$$

A soliton collision, which leaves the photon numbers invariant, transcribes the signal photon number onto the phase of the probe [1], [2]. After collision, the asymptotic phase shift experienced by the probe is [9]

$$\delta\theta_p = 2\left(\arctan\left(\frac{A_p + A_s}{2|p_p - p_s|}\right) - \arctan\left(\frac{A_p - A_s}{2|p_p - p_s|}\right)\right). \tag{6}$$

The variance of probe phase fluctuations is given by

$$\begin{aligned}\langle\Delta\hat{\theta}_p^2(z)\rangle = \langle\Delta\hat{\theta}_p^2\rangle_0 + \left(\frac{A_p^2}{4}z^2 + A_p\frac{\partial\delta\theta_p}{\partial A_p}z + \left(\frac{\partial\delta\theta_p}{\partial A_p}\right)^2\right)\langle\Delta\hat{A}_p^2\rangle_0\\ + \left(\frac{\partial\delta\theta_p}{\partial A_s}\right)^2\langle\Delta\hat{A}_s^2\rangle_0 + \left(\frac{\partial\delta\theta_p}{\partial p_s}\right)^2\langle\Delta\hat{p}_s^2\rangle_0 + \left(\frac{\partial\delta\theta_p}{\partial p_p}\right)^2\langle\Delta\hat{p}_p^2\rangle_0.\end{aligned} \tag{7}$$

The term in Eq. (7) that increases quadratically with z comes about due to SPM. The term linearly proportional to z originates from a cross-correlation between the SPM- and XPM-induced phase fluctuations. This term depends on the derivative of the collision-induced phase shift with respect to the probe amplitude, given by

$$\frac{\partial\delta\theta_p}{\partial A_p} = \frac{4|p_s|}{4|p_s|^2 + (A_p + A_s)^2} - \frac{4|p_s|}{4|p_s|^2 + (A_p - A_s)^2}, \tag{8}$$

which is *always* negative and accounts for phase noise reduction in certain parameter regimes.

The overall phase fluctuations are measured through $R \equiv \langle\Delta\hat{\theta}_p^2(z)\rangle/\langle\Delta\hat{\theta}_p^2(z)\rangle_{\text{nc}}$, defined as the ratio of the phase noise

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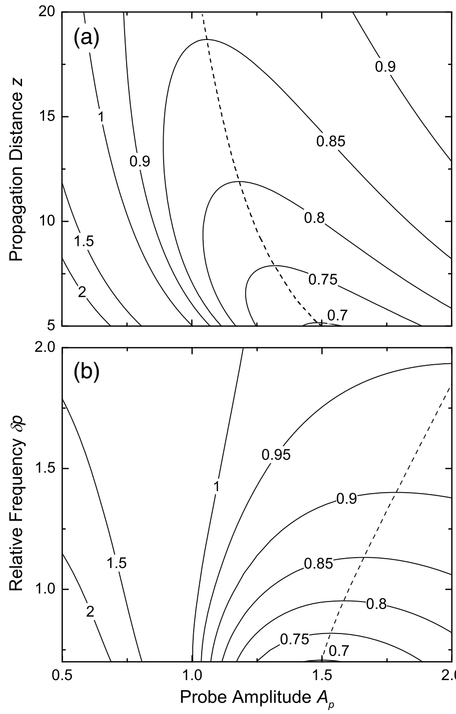

Fig. 1. Contour plots of variance ratio R with respect to (a) propagation distance z with $\delta p = 0.7$ and (b) relative frequency δp with $z = 5$ as a function of probe amplitude A_p and fixed signal amplitude $A_s = 1$. The dashed curves are contours of minimum R.

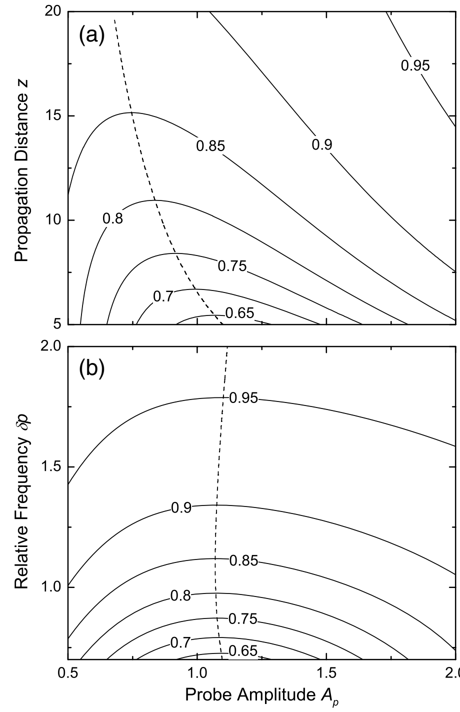

Fig. 2. Contour plots of normalized QND error variance S with respect to (a) propagation distance z with $\delta p = 0.7$ and (b) relative frequency δp with $z = 5$ as a function of probe amplitude A_p and fixed signal amplitude $A_s = 1$. The dashed curves are contours of minimum S.

variance in Eq. (7) with respect to the variance in the absence of collision $\langle \Delta\hat{\theta}_p^2(z)\rangle_{\rm nc} = \langle \Delta\hat{\theta}_p^2\rangle_0 + A_p^2 z^2 \langle \Delta\hat{A}_p^2\rangle_0/4$. In Fig. 1, we plot the dependence of R on z and relative frequency $\delta p = -p_s$ as a function of A_p, keeping the signal amplitude constant, which is arbitrarily fixed at $A_s = 1$. We allow for at least 4 collision lengths, corresponding to $z\delta p > 3.5$, where $\delta p = \pi c T_0 \Delta\lambda/\lambda^2$ and $\Delta\lambda$ is the wavelength separation between probe and signal, collision length $L_{\rm coll} = 2T_{\rm FWHM}/D\Delta\lambda$, and fiber dispersion D. The phase fluctuations can be reduced by over 30%, and three trends are clear—phase noise is reduced at low propagation distance, small relative frequency, and larger probe than signal amplitude.

Next, we calculate the normalized QND error variance with which a measurement of the probe phase will infer the signal amplitude [4],

$$S = 1 - \left[\frac{\langle \Delta\hat{A}_s \Delta\hat{\theta}_p(z)\rangle^2}{\langle \Delta\hat{A}_s^2\rangle\langle \Delta\hat{\theta}_p^2(z)\rangle}\right]^{1/2}, \qquad (9)$$

where $S = 0$ corresponds to the minimum achievable inference error of the signal amplitude. Contours of S are plotted in Fig. 2 in the same way as Fig. 1. Even though a maximum reduction in probe phase noise is not directly connected with an optimization of the QND measurement, some trends are similar. For short propagation distances, a larger probe is optimal, as seen in Fig. 2(a). At longer lengths, however, smaller probes favor a more accurate measurement.

The experimental implementation of the above scheme is feasible with current technology. As an example, we assume FWHM pulse widths of 1 and 1.2 ps for the probe and signal, respectively, $\Delta\lambda = 3$ nm, fiber length of 150 m, and standard fiber parameters of dispersion $D = 17$ ps/km nm and nonlinearity $\gamma = 1$ W^{-1}/km at 1550 nm. For transform-limited solitons, the spectral widths of the signal and probe are 2.1 and 2.5 nm, respectively. These parameters accommodate over 4 collision lengths, with $L_{\rm coll} = 2T_{\rm FWHM}/D\Delta\lambda$, $\delta p \approx 0.75$, and $z = 5$. The photon numbers for the signal and probe solitons are 5×10^8 and 6×10^8, respectively. A QND measurement performed in this experiment will achieve $S \approx 0.65$. It should be noted that both the single and double QND experiments [1], [2] were conducted in a regime with no phase noise reduction and $S > 0.9$.

In this paper, we described a new quantum soliton effect: quantum phase noise reduction in soliton collisions. As an application, we showed improvements in the accuracy of QND measurements. In general, an optimized measurement favors small propagation distances, small wavelength separation between solitons, and a larger probe amplitude relative to the signal.

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