COS 423: Projects 4

due Wed 05/16/2012

Project 1: Generation of Simple Polygons

Design, analyze and implement an algorithm for contructing simple polygons. You may use a probabilistic algorithm. The sole input to your program is an integer n. The output should be a "complicated" simple polygon with n vertices. The quality of your program will depend on speed, but more important, on how winding, irregular, and cluster-free your polygons are. You should produce a number of printouts to display your polygons for various values of n.

Project 2: Low-Cutting Paths

Implement an algorithm that given n points in the plane connects them in a simple path that no line can cut in more than $O(\sqrt{n})$ points. Give graphical evidence that your program works.

Project 3: Voronoi Diagrams

Learn about and implement Fortune's Voronoi diagram algorithm (see lecture notes). Provide graphical evidence that your code works.

Project 4: Minimum Enclosing Disk

Implement the minimum-enclosing disk algorithm and provide graphical evidence that your code works.

Project 5: Linear Programming

Implement the randomized LP algorithm in fixed, arbitrary dimension, and provide graphical evidence that your code works.

Project 6: Random Independent Sets

Consider an undirected, connected graph $G = (V, E)$ and define the following Markov chain: a state is any independent set in G (ie, any subset of nodes with no two of them adjacent to each other); The chain has an edge from S to S' if they differ in exactly one element. The transition probabilities are inferred by the following process. Given the current state S_t, pick a random node v uniformly in V: (i) if $v \in S_t$, then set $S_{t+1} = S_t \setminus \{v\}$; (ii) if v is neither in S_t nor adjacent to any node in S_t, then set $S_{t+1} = S_t \cup \{v\}$; (iii) in all other cases, set $S_{t+1} = S_t$. Prove that the chain is ergodic (ie, irreducible and aperiodic). Find its stationary distribution.

It is often desired to achieve a given stationary distribution with a reversible chain. Here is the idea. Pick some $\mu > 0$ and suppose that you want to sample a random independent set S with probability proportional to $\mu^{|S|}$. (The case $\mu = 1$ gives the uniform distribution.) We modify the Markov chain as follows: Given the current state S_t, pick a random node v uniformly in V: (i) if $v \in S_t$, then set $S_{t+1} = S_t \setminus \{v\}$ with probability $\min\{1, 1/\lambda\}$; (ii) if v is neither in S_t nor adjacent to any node in S_t, then set $S_{t+1} = S_t \cup \{v\}$ with probability $\min\{1, \lambda\}$; (iii) in all other cases, set $S_{t+1} = S_t$. Prove that the chain is ergodic and time-reversible. Find its stationary distribution.

Pick a random graph G in $\mathbf{G}(n, p)$, ie, with n nodes and each pair of them connected at random, independently with probability p. Implement the first Markov chain above and estimate the average size of a random independent set as a function of p and n. Repeat the experiment with the second chain and study the dependency on μ.