COS 423: Homework Assignment 4

due Wed 04/18/2012

Problem 1

Design a linear-time algorithm, which given n segments parallel to each other, determines if a line can pass through all of them (assume that the order of the segment is available beforehand). How hard is it to compute an implicit description of all the stabbing lines?

Problem 2

Given n points in the plane, how can you construct a simple polygon having each point as a vertex and none other? (Don't use the crossing-removal technique used in the low-stabbing cutting theorem.) Still more interesting is to prove that in the worst case $\Omega(n \log n)$ operations are required. Why is that so?

Problem 3

Given n points in the plane sorted by x-coordinates, find an $O(n)$ time algorithm for computing the two points whose inter-distance is maximum. Establish the correctness of your algorithm.

Problem 4

Design and analyze a linear time algorithm for computing the intersection of two convex n-gons. Extra credit: Implement it and test it thoroughly.

Problem 5

Let P and Q be two convex polygons with respectively p and q vertices. Find an $O(p + q)$ time algorithm for deciding whether P can be made to fit inside Q, allowing only translations.

Problem 6

Give a simple, yet rigorous proof that if a convex polygon P lies entirely inside another convex polygon Q, the perimeter of P cannot exceed that of Q.

Problem 7

Prove that a disk (i.e. a circle and its interior) cannot be equidecomposed with a square of equal area, using a finite number of pieces. Assume that all cuts are piecewise smooth. You are allowed to make reasonable assumptions to make this problem tractable.

Problem 8

Given $4n$ points in the plane in general position and a line not passing through any of them that has $2n$ points on each side, prove the existence of another line that partitions the plane into 4 regions, each one with exactly n points.