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★**The discrepancy method. (English summary)**

Randomness and complexity.

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Discrepancy theory originated in the theory of uniform distribution. A typical problem is to quantify the unavoidable limitations on how well a discrete distribution can simulate a continuous one. This monograph on the discrepancy method describes how the tools of discrepancy theory have been successfully used to gain insight into algorithm design and complexity theory. It is an essential reference work, presenting a wealth of applications to a wide variety of topics in computer science, thereby supplementing the superb discrepancy theory monographs of J. Beck and W. W. L. Chen [Irregularities of distribution, Cambridge Univ. Press, Cambridge, 1987; MR 88m:11061], M. Drmota and R. F. Tichy [Sequences, discrepancies and applications, Lecture Notes in Math., 1651, Springer, Berlin, 1997; MR 98j:11057], and J. Matousek [Geometric discrepancy, Springer, Berlin, 1999; MR 2001a:11135].

The first three chapters introduce the fundamentals of discrepancy theory, emphasizing techniques and complete proofs over lists of results. Chapter 1 concerns combinatorial (red-blue) discrepancy, which compares two discrete distributions. Let $\mathcal{S}$ be a family of subsets of a finite set V. A two-coloring of V is a function $\chi\colon X \to \{-1,+1\}$, and the discrepancy $D_\infty(\mathcal{S})$ of $\mathcal{S}$ is the number $\min_\chi \max_{S\in\mathcal{S}} |\chi(S)|$, where $\chi(S) = \sum_{x\in S}\chi(x)$ represents the difference between the numbers of red and blue points in S. Several methods for bounding $D_\infty(\mathcal{S})$ are described, including application of the primal and dual shatter functions to obtain upper bounds. Shatter functions are related to the Vapnik-Chervonenkis dimension. Three algebraic methods for deriving lower bounds are presented, relying on the incidence matrix.

The subject of Chapter 2 is the construction of low-discrepancy sets, including the Halton-Hammersley pointset. There is an extensive discussion of the problem of placing points as uniformly as possible on the unit-radius sphere in three dimensions. To quote the author,"if you ever harbored any doubt about the unity of mathematics, this is required reading. You will witness all branches of mathematics coming together in spectacular fashion." A digression into modular forms follows, touching upon their role in the proof of Fermat's Last Theorem by Wiles. Lower bound techniques form the focus of Chapter 3. These are often approached via the L^2 norm of the discrepancy function. Various analytic techniques are presented, including Haar wavelets, Fourier transforms, and finite differencing.

Geometric sampling is discussed in Chapter 4. The notions of ε-nets (for threshold-based sampling) and ε-approximations (for count-based sampling) are introduced, and the author includes a primer on hyperbolic geometry to explain a construction of a weak ε-net for regular polygons. Chapter 5 treats geometric searching, and Chapter 6, on complexity lower bounds, exploits the correspondence between high discrepancy and high complexity.

Applications to convex hulls and Voronoi diagrams are considered in Chapter 7. This chapter includes the use of sampling techniques from the previous chapters to prove that the convex hull of a set of n points in $\mathbb{R}^d$ can be computed deterministically in $O(n \log n + n^{\lfloor d/2 \rfloor})$ time, for any fixed $d > 1$. This is asymptotically worst-case optimal.

The next three chapters make application of discrepancy theory to linear programming, pseudo-randomness, and communication complexity. Paradoxically, with communication complexity, the theme is the opposite of that in Chapter 6: here, low discrepancy implies high complexity. The final chapter includes a proof that the minimum spanning tree of a connected graph with n vertices and m edges can be computed deterministically in $O(m\alpha(m,n))$ time, where α is the classical inverse of Ackermann's function.

The book contains well over 100 figures and illustrations. Three appendices collect background material from probability theory, harmonic analysis, and convex geometry. The author's elegant writing is personable and engaging, without being chatty, and consistently delivers new insights into the deep underlying principles. Several previously unpublished proofs are presented. There are bibliographic notes at the end of each chapter, with over 300 references.

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Doc. Deliv.