

# A Trustworthy Proof Checker

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**Abstract.** Proof-Carrying Code (PCC) and other applications in computer security require machine-checkable proofs of properties of machine-language programs. The main advantage of the PCC approach is that the amount of code that must be explicitly trusted is very small: it consists of the logic in which predicates and proofs are expressed, the safety predicate, and the proof checker. We have built a minimal proof checker, and we explain its design principles, and the representation issues of the logic, safety predicate, and safety proofs. We show that the trusted computing base (TCB) in such a system can indeed be very small. In our current system the TCB is less than 2,700 lines of code (an order of magnitude smaller even than other PCC systems) which adds to our confidence of its correctness.

## 1. Introduction

Machine-verified proofs have applications in computer security, program verification, and the formalization of mathematics. We are particularly interested in security applications such as proof-carrying code, in which an untrusted program provider must send a proof that the machine-language program is safe to execute; or proof-carrying authorization, in which an untrusted client must send a proof that some desired server operation conforms to an authorization policy. The recipient doesn't trust the proof provider, so the recipient's proof checker is an essential component of the trusted computing base: a bug in the proof checker can be a security hole in the larger system. Therefore, the checker must be trustworthy: it must be small, simple, readable, and based on well-understood engineering and mathematical principles.

In contrast, theorem provers are often large and ugly, as required by the incompleteness results of Gödel and Turing: no prover of bounded size is sufficiently general, but one can always hack more features into the prover until it proves the desired class of theorems. It is difficult to fully trust such software, so some proving systems use technical means to ensure that buggy provers cannot produce invalid proofs: the abstract data type *theorem* of LCF

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(Gordon et al., 1979), or the proof-witness objects of Coq (Barras et al., 1998) or Twelf (Pfenning and Schürmann, 1999). With these means, only a small part of a large system must be examined and trusted.

How large is the proof checker that must be examined and trusted? To answer this question we have tried the experiment of constructing and measuring the *smallest possible* useful proof checker for some real application. Our checker receives, checks the safety of, and executes, proof-carrying code: machine code for the Sparc with an accompanying proof of safety. The proof is in higher-order logic represented in LF notation.

The checker would also be directly useful for proof-carrying authorization (Appel and Felten, 1999; Bauer et al., 2002), that is, checking proofs of authentication and permission according to some distributed policy.

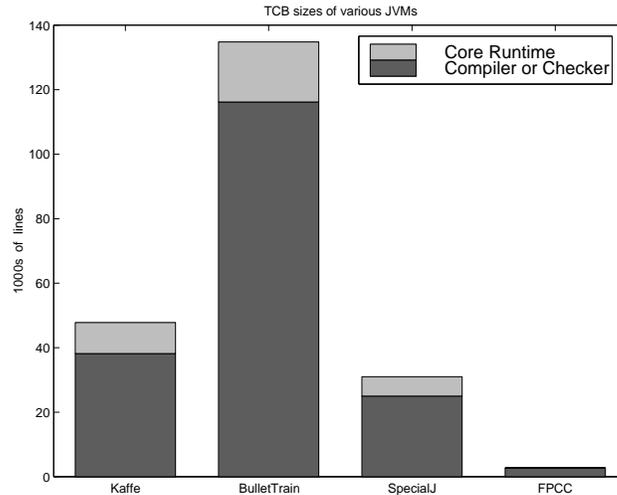
A useful measure of the effort required to examine, understand, and trust a program is its size in (non-blank, non-comment) lines of source code. Although there may be much variation in effort and complexity per line of code, a crude quantitative measure is better than none. We want some measure that corresponds to the difficulty for a human to understand, analyze, and (perhaps) to trust the system. The executable source code is what the human must trust; any comment lines in the code just provide evidence and explanation, not (in principle) additional complexity.

It is also necessary to count, or otherwise account for, any compiler, libraries, or supporting software used to execute the program. We address this issue explicitly by avoiding the use of libraries and by making the checker small enough so that it can be examined in machine language.

The *trusted computing base* (TCB) of a proof-carrying code system consists of all code that must be explicitly trusted as correct by the user of the system. In our case the TCB consists of two pieces: first, the specification of the safety predicate in higher-order logic, and second, the proof checker, a small C program that checks proofs, loads, and executes safe programs.

In their investigation of Java-enabled browsers Dean, et al. (1997), found that the first-generation implementations averaged one security-relevant bug per 3,000 lines of source code (Felten, 2002). These browsers, as mobile-code host platforms that depend on static checking for security, exemplify the kind of application for which proof-carrying code is well suited. Wang and Appel (2002) measured the TCBs of various Java Virtual Machines at between 50,000 and 200,000 lines of code. The SpecialJ JVM (Colby et al., 2000) uses proof-carrying code to reduce the TCB to 36,000 lines.

In this work, we show how to reduce the size of the TCB to under 2,700 lines (see figure 1), and by basing those lines on a well understood logical framework, we have produced a checker which is small enough so that it can be manually verified; and as such it can be relied upon to accept only valid proofs. Since this small checker “knows” only about machine instructions, and nothing about the programming language being compiled and its type



*Figure 1.* The TCB sizes of various JVMs (in thousands of lines of code): Kaffe is an open-source non-optimizing Java JIT, BulletTrain is a highly optimizing Java compiler, SpecialJ is a classical PCC system, and FPCC is our foundational PCC system. Core runtime is the minimal runtime necessary just to run the core of the programming language (without APIs for such things as I/O for example).

system, the semantic techniques for generating the proofs that the TCB will check can be involved and complex (Appel, 2001), but the checker can be simple.

## 2. The LF logical framework

For a proof checker to be simple and correct, it is helpful to use a well designed and well understood representation for logics, theorems, and proofs. We use the LF logical framework.

LF (Harper et al., 1993) provides a means for defining and presenting logics. The framework is general enough to represent a great number of logics of interest in mathematics and computer science (for instance: first-order, higher-order, intuitionistic, classical, modal, temporal, and others). The framework is based on a general treatment of syntax, rules, and proofs by means of a typed first-order  $\lambda$ -calculus with dependent types. The LF type system has three levels of terms: objects, types, and kinds. Types classify objects and kinds classify families of types. The formal notion of definitional equality is taken to be  $\beta\eta$ -conversion.

A logical system is presented by a signature, which assigns types and kinds to a finite set of constants that represent its syntax, its judgments, and its rule schemes. The LF type system ensures that object-logic terms are well formed. At the proof level, the system is based on the *judgments-as-types*

principle: judgments are represented as types, and proofs are represented as terms whose type is the representation of the theorem they prove. Thus, there is a correspondence between type-checked terms and theorems of the object logic. In this way proof checking of the object logic is reduced to type checking of the LF terms.

For developing our proofs, we use Twelf (Pfenning and Schürmann, 1999), an implementation of LF by Frank Pfenning and his students. Twelf is a sophisticated system with many useful features: in addition to an LF type checker, it contains a type-reconstruction algorithm that permits users to omit many explicit parameters, a proof-search algorithm (which is like a higher-order Prolog interpreter), constraint regimes (e.g., linear programming over the exact rational numbers), mode analysis of parameters, a meta-theorem prover, a pretty-printer, a module system, a configuration system, an interactive Emacs mode, and more. We have found many of these features useful in proof development, but Twelf is certainly not a minimal proof checker. However, since Twelf does construct explicit proof objects internally, we can extract these objects to send to our minimal checker.

In LF one declares the operators, axioms, and inference rules of an *object logic* as constructors. For example, we can declare a fragment of first-order logic with the type `form` for formulas and a dependent type constructor `pf` for proofs, so that for any formula `A`, the type `pf (A)` contains values that are proofs of `A`. Then, we can declare an “implies” constructor `imp` (infix, so it appears between its arguments), so that if `A` and `B` are formulas then so is `A imp B`. Finally, we can define introduction and elimination rules for `imp`.<sup>1</sup>

```
form : type.
pf   : form -> type.
imp  : form -> form -> form.
%infix right 10 imp.
imp_i: (pf A -> pf B) -> pf (A imp B).
imp_e: pf (A imp B) -> pf A -> pf B.
```

All the above are defined as constructors. In general, constructors have the form `name :  $\tau$`  and declare that `name` is a value of type  `$\tau$` .

It is easy to declare inconsistent object-logic constructors. For example, `invalid: pf A` is a constructor that acts as a proof of any formula, so using it we could easily prove the false proposition:

```
logic_inconsistent : pf (false) = invalid.
```

So the object logic should be designed carefully and must be trusted.

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<sup>1</sup> Here, for example, the `imp_i` axiom states that if you have an LF function that can transform proofs of `A` to proofs of `B`, then applying the rule produces a proof of `A imp B`. The rule `imp_e` (generally known as *modus ponens*) goes the other way: given a proof of `A imp B` and a proof of `A`, applying the rule produces a proof of `B`.

Once the object logic is defined, theorems can be proved. We can prove for instance that implication is transitive:

```
imp_trans: pf (A imp B) -> pf (B imp C) -> pf (A imp C) =
  [p1: pf (A imp B)][p2: pf (B imp C)]
  imp_i [p3: pf A] imp_e p2 (imp_e p1 p3).
```

In general, definitions (including predicates and theorems) have the form  $name : \tau = exp$ , which means that  $name$  is now to stand for the value  $exp$  whose type is  $\tau$ . In this example, the  $exp$  is a function with formal parameters  $p1$  and  $p2$ , and with body  $imp_i [p3] imp_e p2 (imp_e p1 p3)$ .

Definitions need not be trusted, because the type-checker can verify whether  $exp$  does have type  $\tau$ . In general, if a proof checker is to check the proof  $P$  of theorem  $T$  in a logic  $\mathcal{L}$ , then the constructors (operators and axioms) of  $\mathcal{L}$  must be given to the checker in a trusted way (i.e., the adversary must not be free to install inconsistent axioms). The statement of  $T$  must also be trusted (i.e., the adversary must not be free to substitute an irrelevant or vacuous theorem). The adversary provides only the proof  $P$ , and then the checker does the proof checking (i.e., it type-checks in the LF type system the definition  $t : T = P$ , for some arbitrary name  $t$ ).

### 3. Application: Proof-carrying code

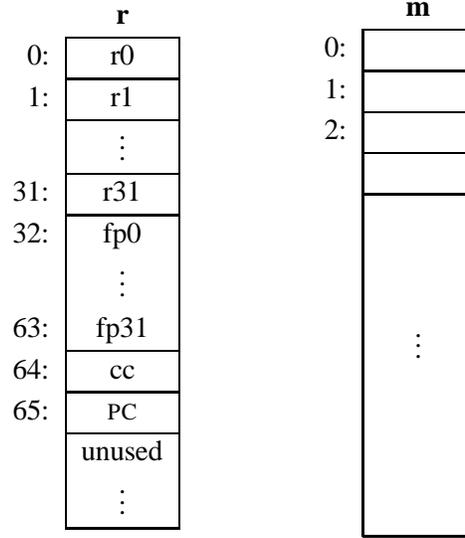
Our checker is intended to serve a purpose: to check safety theorems about machine-language programs. It is important to include application-specific portions of the checker in our measurements to ensure that we have adequately addressed all issues relating to interfacing to the real world.

The most important real-world-interface issue is, “is the proved theorem meaningful?” An accurate checker does no good if it checks the wrong theorem. As we will explain, the specification of the safety theorem is larger than all the other components of our checker combined!

Given a machine-language program  $P$ , that is, a sequence of integers that code for machine instructions (on the Sparc, in our case), the theorem is, “when run on the Sparc,  $P$  never executes an illegal instruction, nor reads or writes from memory outside a given range of addresses.” To formalize this theorem it is necessary to formalize a description of instruction execution on the Sparc processor. We do this in higher-order logic augmented with arithmetic.

In our model (Michael and Appel, 2000), a machine state comprises a *register bank* ( $r$ ), and a *memory* ( $m$ ), each of which is a function from integers (register numbers and addresses) to integers (contents). Every register of the instruction-set architecture (ISA) must be assigned a number in the register bank: the general registers, the floating-point registers, the condition codes,

and the program counter. Where the ISA does not specify a number (such as for the PC) or when the numbers for two registers conflict (such as for the floating point and integer registers) we use an arbitrary unused index:



A single step of the machine is the execution of one instruction. We can specify instruction execution by giving a step relation  $(r, m) \mapsto (r', m')$  that maps the prior state  $(r, m)$  to the next state  $(r', m')$  that holds after the execution of the machine instruction.

For example, to describe the add instruction  $r_1 \leftarrow r_2 + r_3$  we might start by writing,

$$(r, m) \mapsto (r', m') \equiv \\ r'(1) = r(2) + r(3) \wedge (\forall x \neq 1. r'(x) = r(x)) \wedge m' = m$$

In fact, we can parameterize the above on the three registers involved and define  $\text{add}(i, j, k)$  as the following predicate on four arguments  $(r, m, r', m')$ :

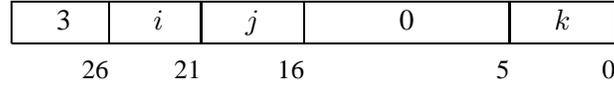
$$\text{add}(i, j, k) \equiv \\ \lambda r, m, r', m'. r'(i) = r(j) + r(k) \wedge (\forall x \neq i. r'(x) = r(x)) \wedge m' = m$$

Similarly, for the load instruction  $r_i \leftarrow m[r_j + c]$  we define its semantics to be the predicate:

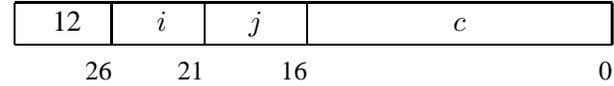
$$\text{load}(i, j, c) \equiv \\ \lambda r, m, r', m'. r'(i) = m(r(j) + c) \\ \wedge (\forall x \neq i. r'(x) = r(x)) \wedge m' = m$$

To enforce memory safety policies, we will modify the definition of  $\text{load}(i, j, c)$  to require that the loaded address is legal (Appel, 2001), but we omit those details here.

But we must also take into account instruction fetch and decode. Suppose, for example, that the add instruction is encoded as a 32-bit word, containing a 6-bit field with opcode 3 denoting *add*, a 5-bit field denoting the destination register *i*, and 5-bit fields denoting the source registers *j*, *k*:



The load instruction might be encoded as:



Then we can say that some number *w* decodes to an instruction *instr* iff,

$$\begin{aligned}
 \text{decode}(w, \text{instr}) \equiv & \\
 & (\exists i, j, k. \\
 & 0 \leq i < 2^5 \wedge 0 \leq j < 2^5 \wedge 0 \leq k < 2^5 \wedge \\
 & w = 3 \cdot 2^{26} + i \cdot 2^{21} + j \cdot 2^{16} + k \cdot 2^0 \wedge \\
 & \text{instr} = \text{add}(i, j, k)) \\
 & \vee (\exists i, j, c. \\
 & 0 \leq i < 2^5 \wedge 0 \leq j < 2^5 \wedge 0 \leq c < 2^{16} \wedge \\
 & w = 12 \cdot 2^{26} + i \cdot 2^{21} + j \cdot 2^{16} + c \cdot 2^0 \wedge \\
 & \text{instr} = \text{load}(i, j, c)) \\
 & \vee \dots
 \end{aligned}$$

with the ellipsis denoting the many other instructions of the machine, which must also be specified in this formula.

We have shown (Michael and Appel, 2000) how to scale this idea up to the instruction set of a real machine. Real machines have large but semi-regular instruction sets; instead of a single global disjunction, the decode relation can be factored into operands, addressing modes, and so on. Real machines don't use integer arithmetic, they use modular arithmetic, which can itself be specified in our higher-order logic. Some real machines have multiple program counters (e.g., Sparc) or variable-length instructions (e.g., Pentium), and these can also be accommodated.

Our description of the decode relation is heavily factored by higher-order predicates (this would not be possible without higher-order logic). We have specified the execution behavior of a large subset of the Sparc architecture, and we have built a prototype proof-generating compiler that targets that subset. For proof-carrying code, it is sufficient to specify a subset of the machine architecture; any unspecified instruction will be treated by the safety policy as illegal. While this may be inconvenient for compilers that want to generate that instruction, it does ensure that safety cannot be compromised.

#### 4. Specifying safety

Our step relation  $(r, m) \mapsto (r', m')$  is deliberately partial; some states have no successor state. In these states the program counter  $r(\text{PC})$  points to an illegal instruction. Using this partial step relation, we can define safety. A safe program is one that will never execute an illegal instruction; that is, a given state is safe if, for any state reachable in the Kleene closure of the step relation, there is a successor state:

$$\text{safe-state}(r, m) \equiv \forall r', m'. (r, m) \mapsto^* (r', m') \Rightarrow \exists r'', m''. (r', m') \mapsto (r'', m'')$$

A program is just a sequence of integers (each representing a machine instruction); we say that a program  $p$  is loaded at a location  $l$  in memory  $m$  if

$$\text{loaded}(p, m, l) \equiv \forall i \in \text{dom}(p). m(i + l) = p(i)$$

Finally (assuming that programs are written in position-independent code), a program is *safe* if, no matter where we load it in memory, we get a safe state:

$$\text{safe}(p) \equiv \forall r, m, \text{start}. \text{loaded}(p, m, \text{start}) \wedge r(\text{PC}) = \text{start} \Rightarrow \text{safe-state}(r, m)$$

Let  $;$  be a “cons” operator for sequences of integers (easily definable in higher-order logic); then for some program

```
8420 ; 2837 ; 2938 ; 2384 ; nil
```

the safety theorem is simply

```
safe ( 8420 ; 2837 ; 2938 ; 2384 ; nil )
```

and, given a proof  $P$ , the LF definition that must be typechecked is

```
t: pf(safe(8420;2837;2938;2384;nil)) = P.
```

Though we wrote in section 2 that definitions need not be trusted because they can be type-checked, this is not strictly true. Any definition that is used (directly or indirectly) in the statement of the theorem must be trusted, because the wrong definition will lead to the proof of the wrong theorem. Thus, all the definitions leading up to the definition of `safe` (including `add`, `load`, `safe-state`, `step`, `loaded`, etc.) must be part of the trusted checker. Since we have approximately 1,600 lines of such definitions, and they are a component of our “minimal” checker, one of the most important issues we faced is the representation of these definitions; we discuss this in section 7.

On the other hand, a large proof will contain hundreds of internal definitions. These are predicates and internal lemmas of the proof (not of the statement of the theorem), and are at the discretion of the proof provider. Since

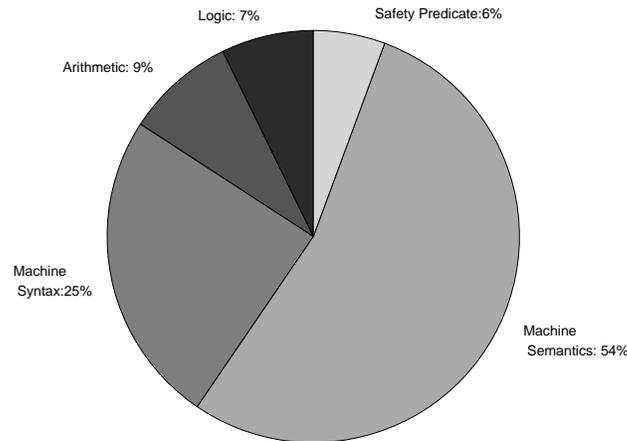


Figure 2. The relative component sizes of the Safety Specification for the Sparc. The Machine Semantics take the lion's share of the Specification.

each is type checked by the checker before it is used in further definitions and proofs, they don't need to be trusted.

In the table below we show the various pieces needed for the specification of the safety theorem in our logic. Every piece in this table is part of the TCB. Column two shows the number of lines of Twelf code needed for the specification and column three the number of definitions in that specification. The first two lines show the size of the logical and arithmetic connectives (in which theorems are specified) as well as the size of the logical and arithmetic axioms (using which theorems are proved). The Sparc specification has two components, a "syntactic" part (the decode relation) and a semantic part (the definitions of *add*, *load*, etc.); these are shown in the next two lines. The size of the safety predicate is shown last. The relative sizes of each component are shown in figure 2.

<i>Safety Specification</i>	<i>Lines</i>	<i>Definitions</i>
Logic	135	61
Arithmetic	160	94
Machine Syntax	460	334
Machine Semantics	1,005	692
Safety Predicate	105	25
Total	1,865	1,206

From this point on we will refer to everything in the table as the *safety specification* or simply the *specification*.

## 5. Eliminating redundancy

Typically an LF signature will contain much redundant information. Consider for example the rules for `imp` presented previously; in fully explicit form, their representation in LF is as follows:

```
imp_i: {A : form}{B : form}
      (pf A -> pf B) -> pf (A imp B).
imp_e: {A : form}{B : form}
      pf (A imp B) -> pf A -> pf B.
```

The fact that both `A` and `B` are formulas can be easily inferred by the fact they are given as arguments to the constructor `imp`, which has been previously declared as an operator over formulas.

On the one hand, eliminating redundancy from the representation of proofs benefits both proof size and type-checking time. On the other hand, it requires performing term reconstruction, and thus it may dramatically increase the complexity of type checking, driving us away from our goal of building a minimal checker.

Twelf deals with redundancy by allowing the user to declare some parameters as *implicit*. More precisely, all variables which are not quantified in the declaration are automatically assumed implicit. Whenever an operator is used, Twelf's term reconstruction will try to determine the correct substitution for all its implicit arguments. For example, in type-checking the lemma

```
imp_refl : pf (A imp A) = imp_i ([p : pf A] p).
```

Twelf will automatically reconstruct the two implicit arguments of `imp_i` to be both equal to `A`.

While Twelf's notion of implicit arguments is effective in eliminating most of the redundancy, type reconstruction adds considerable complexity to the system. Another drawback of Twelf's type reconstruction is its reliance on higher-order unification, which is undecidable. Because of this, type checking of some valid proofs may fail.

Since full type reconstruction is too complex to use in a trusted checker, one might think of sending fully explicit LF proof terms; but a fully explicit proof in Twelf syntax can be exponentially larger than its implicit representation. To avoid these problems, Necula's  $LF_i$  (Necula, 1997) uses partial type reconstruction and a simple algorithm to determine which of the arguments can be made implicit. Implicit arguments are omitted in the representation, and replaced by placeholders. Oracle-based checking (Necula and Rahul, 2001) reduces the proof size even further by allowing the erasure of subterms whose reconstruction is not uniquely determined. Specifically, in cases when the reconstruction of a subterm is not unique, but there is a finite (and usually small) list of candidates, it stores an oracle for the right candidate number instead of storing the entire subterm.

These techniques use clever syntactic representations of proofs that minimize proof size; checking these representations is not as complex as full Twelf-style type reconstruction, but is still more complex than is appropriate for a minimal proof checker. We are willing to tolerate somewhat larger proofs in exchange for a really simple checking algorithm. Instead of using a syntactic representation of proofs, we avoid the need for the checker to parse proofs by using a data-structure representation. However, we still need to avoid exponential blowups, so we reduce redundancy by structure sharing. Therefore, we represent and transmit proofs as LF terms in the form of directed acyclic graphs (DAGs), with structure sharing of common subexpressions to avoid exponential blowup.

A node in the DAG can be one of ten possible types: one for kinds, five for ordinary LF terms, and four for arithmetic expressions. Each node may store up to three integers, `arg1`, `arg2`, and `type`. This last one, if present, will always point to the sub-DAG representing the type of the expression.

	<code>arg1</code>	<code>arg2</code>	<code>type</code>	
<code>n</code>	U	U	U	kind
<code>c</code>	U	U	M	constant
<code>v</code>	M	M	M	variable
<code>a</code>	M	M	O	application
<code>p</code>	M	M	O	product
<code>l</code>	M	M	O	abstraction
<code>#</code>	M	U	O	number
<code>+</code>	M	M	O	addition proof obj
<code>*</code>	M	M	O	mult proof obj
<code>/</code>	M	M	O	div proof obj

M = mandatory

O = optional

U = unused

The content of `arg1` and `arg2` is used in different ways for different node types. For all nodes representing arithmetic expressions (`#`, `+`, `*`, and `/`), they contain integer values; in particular, for the nodes corresponding to meta-level numbers (`#`) only `arg1` is used. For products and abstractions (`p` and `l`), `arg1` points to the bound variable, and `arg2` to the term where the binding takes place. For variable nodes (`v`), `arg1` and `arg2` are used to make sure that the variable always occurs within the scope of a quantifier. For application nodes (`a`), `arg1` and `arg2` point to the function and its argument, respectively. Finally, constant declaration nodes (`c`), and kind declaration nodes (`n`) use neither of the two.

For a concrete example, consider the LF signature:

```
form : type.
pf   : form -> type.
imp  : form -> form -> form.
```

We present below the DAG representation of this signature. We “flattened” the DAG into a numbered list, and, for clarity, we also added a comment on the right showing the corresponding LF term.

```
1| n 0 0 0 ; type Kind
2| c 0 0 1 ; form: type
3| v 0 0 2 ; x: form
4| p 3 1 0 ; {x: form} type
5| c 0 0 4 ; pf: {x: form} type
6| v 0 0 2 ; y: form
7| p 6 2 0 ; {y: form} form
8| v 0 0 2 ; x: form
9| p 8 7 0 ; {x: form}{y: form} form
10| c 0 0 9 ; imp: {x: form}{y: form} form
```

## 6. Dealing with arithmetic

Since our proofs reason about encodings of machine instructions (opcode calculations) and integer values manipulated by programs, the problem of representing arithmetic within our system is a critical one. A purely logical representation based on 0, successor and predecessor is not suitable to us, since it would cause proof size to explode. Even a binary representation (using 0,  $\times 2$ , and  $\times 2 + 1$ ) is not attractive, because in our PCC application there are many 32-bit constants (representations of machine instructions) that would then take 32 operators each.

The latest releases of Twelf offer extensions that deal natively with infinite-precision integers and rationals. While these extensions are very powerful and convenient to use, they offer far more than we need, and because of their generality they have a very complex implementation (the rational extension alone is 1,950 lines of Standard ML). What we would like for our checker is an extension built in the same spirit as those, but much simpler and lighter.

There are essentially two properties that we require from such an extension:

1. LF terms for all the numbers we use; moreover, the size of the LF term for  $n$  should be constant and independent of  $n$ .
2. Proof objects for single-operation arithmetic facts such as “ $10 + 2 = 12$ ”; again, we require that such proof objects have constant size.

Our arithmetic extensions to the checker are the smallest and simplest ones to satisfy (1) and (2) above. We add the `word32` type to the TCB, (representing integers in the range  $[0, 2^{32} - 1]$ ) as well as the following axioms:

```
word32 : type.
+ : word32 -> word32 -> word32 -> type.
* : word32 -> word32 -> word32 -> type.
/ : word32 -> word32 -> word32 -> type.
```

We also modify the checker to accept arithmetic terms such as:

```
567      : word32.
456+25   : + 456 25 481.
32*4     : * 32 4 128.
56/5     : / 56 5 11.
```

This extension does not modify in any way the standard LF type checking: we could have obtained the same result (although much more inefficiently) if we added all these constants to the trusted LF signature by hand. However, granting them special treatment allowed us to save literally millions of lines in the axioms in exchange for an extra 55 lines in the checker.

To embed and use these new constants in our object logic, we also declare:

```
c: word32 -> tm num.
eval_plus: + A B C -> pf (eq (plus (c A) (c B)) (c C)).
eval_times: * A B C -> pf (eq (times (c A) (c B)) (c C)).
eval_div: / M N Q ->
  pf ((geq (c M) (times (c N) (c Q))) and
    (not (geq (c M) (times (c N) (plus one (c Q)))))).
```

This embedding from `word32` to numbers in our object logic is not surjective. Numbers in our object logic are still unbounded; `word32` merely provides us with handy names for the ones used most often.

With this “glue” to connect object logic to meta logic, numbers and proofs of elementary arithmetic properties, are just terms of size two. For example, the representation of 5634 in the object logic, is `(c 5634)`, which, by virtue of these additional axioms, can be easily verified to be a valid term of type `num`.

## 7. Representing axioms and trusted definitions

Since we can represent axioms, theorems, and proofs as DAGs, it might seem that we need neither a parser nor a pretty-printer in our minimal checker. In principle, we could provide our checker with an initial trusted DAG representing the axioms and the theorem to be proved, and then it could receive

and check an untrusted DAG representing the proof. The trusted DAG could be represented in the C language as an initialized array of graph nodes.

This might work if we had a very small number of axioms and trusted definitions, and if the statement of the theorem to be proved were very small. We would have to read and trust the initialized-array statements in C, and understand their correspondence to the axioms (etc.) as we would write them in LF notation. For a sufficiently small DAG, this might be simpler than reading and trusting a parser for LF notation.

However, even a small set of operators and axioms (especially once the axioms of arithmetic are included) requires hundreds of graph nodes. In addition, as explained in section 4, our trusted definitions include the machine-instruction step relation of the Sparc processor. These 1,865 lines of Twelf expand to 22,270 DAG nodes. Clearly it is impossible for a human to directly read and trust a graph that large.

Therefore, we require a parser or pretty-printer in the minimal checker; we choose to use a parser. Our C program will parse the 1,865 lines of axioms and trusted definitions, translating the LF notation into DAG nodes. The axioms and definitions are also part of the C program: they are a constant string to which the parser is applied on startup.

This parser is 428 lines of C code; adding these lines to the minimal checker means our minimal checker can use 1,865 lines of LF instead of 22,270 lines of graph-node initializers, clearly a good tradeoff.

Our parser accepts valid LF expressions, written in the same syntax used by Twelf. In addition to those, it will also parse nonnegative numbers below  $2^{32}$  and type them as `word32` objects.

Two extra features we added to our parser are infix operators (the fixity information is passed to the parser using the same `%infix` directive used by Twelf), and single-line comments. While these are not strictly necessary, implementing them did not significantly affect the complexity of the parser, and their use greatly improved the readability of the specification, and therefore its trustworthiness.

## 7.1. ENCODING HIGHER-ORDER LOGIC IN LF

Our encoding of higher-order logic in LF follows that of Harper et al. (1993) and is shown in figure 7. The *constructors* generate the syntax of the object logic and the *axioms* generate its proofs. A meta-logical type is `type` and an object-logic type is `tp`. Object-logic types are constructed from `form` (the type of formulas), `num` (the type of integers), and the `arrow` constructor. So the object-logic predicate `even?`, for instance, would have object type `(num arrow form)`. The LF term `tm` maps an object type to a meta type, so an object-level term of type `T` has type `(tm T)` in the meta logic.

<u>Logic Constructors</u>	
tp	: type.
tm	: tp -> type.
form	: tp.
num	: tp.
arrow	: tp -> tp -> tp.
pf	: tm form -> type.
lam	: (tm T1 -> tm T2) -> tm (T1 arrow T2).
@	: tm (T1 arrow T2) -> tm T1 -> tm T2.
forall	: (tm T -> tm form) -> tm form.
imp	: tm form -> tm form -> tm form.
<u>Logic Axioms</u>	
beta_e	: pf (P ((lam F) @ X)) -> pf (P (F X)).
beta_i	: pf (P (F X)) -> pf (P (lam F) @ X).
imp_i	: (pf A -> pf B) -> pf (A imp B).
imp_e	: pf (A imp B) -> pf A -> pf B.
forall_i	: ({X : tm T} pf (A X)) -> pf (forall A).
forall_e	: pf (forall A) -> {X : tm T} pf (A X).

Figure 3. Higher-Order Logic in Twelf

Abstraction in the object logic is expressed by the `lam` term. The term `(lam [x] (F x))` is the object-logic function that maps `x` to `(F x)`. Application for such lambda terms is expressed via the `@` operator. The quantifier `forall` is defined to take as input a meta-level (LF) function of type `(tm T -> tm form)` and produce a `tm form`. The use of the LF functions here makes it easy to perform substitution when a proof of `forall` needs to be discharged, since equality in LF is just  $\beta\eta$ -conversion.

Notice that most of the standard logical connectives are absent from figure 7. This is because we can produce them as definitions from the constructors we already have. For instance, the `and` and `or` connectives can be defined as follows:

```
and = [A][B] forall [C] (A imp B imp C) imp C.
or  = [A][B] forall [C] (A imp C) imp (B imp C) imp C.
```

It is easy to see that the above formulae are equivalent to the standard definitions of `and` and `or`. What is in fact required in order to assert this, is to

derive the standard introduction and elimination rules for each of them. These rules are proven as lemmas (from the definitions of the connectives and the existing axioms), so they need not be part of the trusted computing base. On the other hand, since the constructors are used in the definition of the safety predicate, they must be part of the TCB.

Object-level equality<sup>2</sup> is also easy to define:

```
eq : tm T -> tm T -> tm form =
  [A : tm T][B : tm T] forall [P] P @ B imp P @ A.
```

This states that objects  $A$  and  $B$  are considered equal iff any predicate  $P$  that holds on  $B$  also holds on  $A$ .

Terms of type  $(\text{pf } A)$  are terms representing proofs of object formula  $A$ . Such terms are constructed using the axioms of figure 7. Axioms `beta_e` and `beta_i` are used to prove  $\beta$ -equivalence in the object logic. The first states that for any meta-level predicate  $P$  (of type  $\text{tm } T \rightarrow \text{tm form}$ ), if  $P$  holds on the term  $((\text{lam } F) @ X)$  then it also holds on  $(F X)$ . Axiom `beta_i` takes one in the other direction. Axioms `imp_i` and `imp_e` transform a meta-level proof function to the object level and vice-versa. Finally, `forall_i` introduces the `forall` statement if it is presented with a meta-level function from  $X$  to  $\text{pf } (A X)$ , and `forall_e` discharges a `forall` by applying the meta-level function  $A$  to an instance  $X$  of its domain to produce  $\text{pf } (A X)$ . Notice how the higher-order abstract syntax of LF makes this discharging (and the term substitution of  $X$  in the body of  $A$ ) completely painless for the designer of the logic.

As an example of a simple proof we show how to prove the reflexivity lemma `refl`, which states that all objects equal themselves. By the definition of `eq`, if  $X$  is to equal itself, we must to show that for any predicate  $P$  if  $(P @ X)$  then  $(P @ X)$  – which is obvious. Here is the proof:

```
refl : pf (eq X X) =
  forall_i [P] imp_i [q : pf (P @ X)] q.
```

## 7.2. “POLYMORPHIC” PROGRAMMING IN TWELF

ML-style implicit polymorphism allows one to write a function usable at many different argument types, and ML-style type inference does this with

---

<sup>2</sup> The equality predicate is polymorphic in  $T$ . The objects  $A$  and  $B$  have object type  $T$  and so they could be `nums`, `forms` or even object level functions (`arrow` types). The object type  $T$  is implicit in the sense that when we use the `eq` predicate we do not have to specify it; Twelf can automatically infer it. So internally, the meta-level type of `eq` is not what we have specified above but the following:

```
{T : tp} tm T -> tm T -> tm form.
```

We will have more to say about this in section 7.2.

a low syntactic overhead. We are writing proofs, not programs, but we would still like to have polymorphic predicates and polymorphic lemmas. LF is not polymorphic in the ML sense, but Harper et al. (1993) show how to use LF's dependent type system to get the effect and (most of) the convenience of implicit parametric polymorphism with an encoding trick, which we will illustrate with an example.

Suppose we wish to write the lemma `congr` that would allow us to substitute equals for equals:

```
congr : {H : type -> tm form}
       pf (eq X Z) -> pf (H Z) -> pf (H X) = ...
```

The lemma states that for any predicate `H`, if `H` holds on `Z` and `Z = X` then `H` also holds on `X`. Unfortunately this is ill-typed in LF since LF does not allow polymorphism. Fortunately though, there is way to get polymorphism at the object level. We rewrite `congr` as:

```
congr : {H : tm T -> tm form}
       pf (eq X Z) -> pf (H Z) -> pf (H X) = ...
```

and this is now acceptable to Twelf. Function `H` now judges objects of meta-type `(tm T)` for any object-level type `T`, and so `congr` is now “polymorphic” in `T`. We can apply it on any object-level type, such as `num`, `form`, `num arrow num`, `num arrow form`, etc. This solution is general enough to allow us to express any polymorphic term or lemma with ease. Axioms `forall_i` and `forall_e` in figure 7 are likewise polymorphic in `T`.

### 7.3. HOW TO WRITE EXPLICIT TWELF

In the definition of lemma `congr` above, we have left out many explicit parameters since Twelf's type-reconstruction algorithm can infer them. The actual LF type of the term `congr` is:

```
congr : {T: tp}{X: tm T}{Z: tm T}{H: tm T -> tm form}
       pf (_eq T X Z) -> pf (H Z) -> pf (H X) = ...
```

(here we also make use of the explicit version of the equality predicate `_eq`). Type reconstruction in Twelf is extremely useful, especially in a large system like ours, where literally hundreds of definitions and lemmas have to be stated and proved.

Our safety specification was originally written to take advantage of Twelf's ability to infer missing arguments. Before proof checking can begin, this specification needs to be fed to our proof checker. In choosing then what would be in our TCB we had to decide between the following alternatives:

1. Keep the implicitly-typed specification in the TCB and run it through Twelf to produce an explicit version (with no missing arguments or types).

Object Logic Abstraction/Application

```
fld2 = [T1:tp][T2:tp][T3:tp][T4:tp]
lam6 (arrow T1 (arrow T2 form))
      (arrow T3 (arrow T2 form))
      (arrow T2 form)
      (arrow T1 (arrow T3 T4))
      T4 T2 form
[f0][f1][p_pi][icons][ins][w]
(@ T2 form p_pi w) and
(exists2 T1 T3 [g0:tm T1] [g1:tm T3]
 (@ T2 form
  (&& T2 (@ T1 (arrow T2 form) f0 g0)
   (@ T3 (arrow T2 form) f1 g1)) w) and
 (eq T4 ins (@ T3 T4 (@ T1 (arrow T3 T4) icons g0) g1))).
```

Meta Logic Abstraction/Application

```
fld2 = [T1:tp][T2:tp][T3:tp][T4:tp]
[f0][f1][p_pi][icons][ins][w]
(p_pi w) and
(exists2 [g0:tm T1][g1:tm T3]
 (f0 g0 && f1 g1) w) and (eq ins (icons g0 g1)).
```

Figure 4. Abstraction & Application in the Object versus Meta Logic.

This explicit version would be fed to our proof checker. This approach allows the specification to remain in the implicit style. Also our proof checker would remain simple (with no type reconstruction/inference capabilities) but unfortunately we now have to add to the TCB Twelf's type-reconstruction and unification algorithms, which are about 5,000 lines of ML code.

2. Run the implicitly typed specification through Twelf to get an explicit version. Now instead of trusting the implicit specification and Twelf's type-reconstruction algorithms, we keep them out of the TCB and proceed to manually verify the explicit version. This approach also keeps the checker simple (without type-reconstruction capabilities). Unfortunately the explicit specification produced by Twelf explodes in size from 1,700 to 11,000 lines, and thus the code that needs to be verified correct is huge. The TCB would grow by a lot.
3. Rewrite the trusted definitions in an explicit style. Now we do not need type reconstruction in the TCB (the problem of choice 1), and if the rewrite from the implicit to the explicit style can avoid the size explosion (the problem of choice 2), then we have achieved the best of both worlds.

Since neither of choices 1 and 2 above were consistent with our goal of a small trusted computing base, we followed choice 3 and rewrote the trusted definitions in an explicit style while managing to avoid the size explosion. The new safety specification is only 1,865 lines of explicitly-typed Twelf. It contains no terms with implicit arguments – everything is explicit and every quantified variable is explicitly typed. Thus, we do not need a type-reconstruction/type-inference algorithm in the proof checker. The rewrite solves the problem while maintaining the succinctness and brevity of the original TCB, the penalty of the explicit style being an increase in size of 124 lines. The remainder of this section explains the problem in detail and the method we used to bypass it.

To see why there is such an enormous difference in size (1,700 lines vs 11,000) between the implicit specification and its explicit representation generated by Twelf’s type-reconstruction algorithm, consider the following example. Let  $F : \text{tm} \ (\text{num} \ \text{arrow} \ \text{num} \ \text{arrow} \ \text{form})$  be a two-argument object-level predicate (a typical case when describing unary operators in an instruction set). When such a predicate is applied, as in  $(F \ @ \ X \ @ \ Y)$ , Twelf has to infer the implicit arguments to the two instances of operator  $@$ . The explicit representation of the application then becomes:

```
@ num form (@ num (num arrow form) F X) Y
```

It is easy to see how the explicit representation explodes in size for terms of higher order. Since the use of higher-order terms was essential in achieving maximal factoring in the machine descriptions (Michael and Appel, 2000), the size of the explicit representation quickly becomes unmanageable.

Here is another more concrete example from the `decode` relation of section 3. This one shows how the abstraction operator `lam` suffers from the same problem. The predicate below (given in implicit form) is used in specifying the syntax of all Sparc instructions of two arguments.

```
fld2 = lam6 [f0][f1][p_pi][icons][ins][w]
  p_pi @ w and
  exists2 [g0][g1] (f0 @ g0 && f1 @ g1) @ w and
  eq ins (icons @ g0 @ g1).
```

Predicates `f0` and `f1` specify the input and the output registers, `p_pi` decides the instruction opcode, `icons` is the instruction constructor, `ins` is the instruction we are decoding, and `w` is the machine-code word.<sup>3</sup> In explicit form

<sup>3</sup> As we mentioned before, our specifications are highly factored and this is an example of such factoring – any instruction of two arguments can be specified using this predicate. To define, for instance, the predicate that decides the FMOVs (floating-point move) instruction on the Sparc, we would say:

```
ins_MOVs = fld2 @ f_fs2 @ f_fd @ p_FMOVs @ i_FMOVs.
```

where the arguments are as described above.

this turns into what we see on the left-hand side of figure 7.3 – an explicitly typed definition 16 lines long.

The way around this problem is the following: We avoid using object-logic predicates whenever possible. This way we need not specify the types on which object-logic application and abstraction are used. For example, the `fld2` predicate above now becomes what we see on the right-hand side of figure 7.3. This new predicate has shrunk in size by more than half.

Sometimes moving predicates to the meta-logic is not possible. For instance, we represent *instructions* as predicates from machine states to machine states (see section 3). Such predicates must be in the object logic since we need to be able to use them in quantifiers (`exists [ins : tm instr] ...`). Thus, we face the problem of having to supply all the implicit types when defining such predicates and when applying them. But since these types are always fixed we can factor the partial applications and avoid the repetition. So, for example, when defining some Sparc machine instruction as in:

```
i_anyInstr = lam2 [rs : tnum][rd : tnum]
  lam4 registers memory registers memory form
    [r : tregs][m : tmem] [r' : tregs][m' : tmem]
    ...
```

we define the predicate `instr_lam` as:

```
instr_lam = lam4 registers memory registers memory form.
```

and then use it in defining each of the 250 or so Sparc instructions as below:

```
i_anyInstr = [rs : tnum][rd : tnum]
  instr_lam [r : tregs][m : tmem]
    [r' : tregs][m' : tmem]
  ...
```

This technique turns out to be very effective because our machine syntax and semantics specifications were highly factored to begin with (Michael and Appel, 2000).

So by moving to the meta logic and by clever factoring we have moved the TCB from implicit to explicit style with only a minor increase in size. Now we don't have to trust a complicated type-reconstruction/type-inference algorithm. What we feed to our proof checker is precisely the set of axioms we explicitly trust.

#### 7.4. THE IMPLICIT LAYER

When we are building proofs, we still wish to use the implicit style because of its brevity and convenience. For this reason we have built an implicit layer on top of our explicit TCB. This allows us to write proofs and definitions in the

implicit style and LF's  $\beta\eta$ -conversion takes care of establishing meta-level term equality. For instance, consider the object-logic application operator  $\_@$  given below:

$$\_@ : \{T1: tp\}\{T2: tp\} \\ tm (T1 \text{ arrow } T2) \rightarrow tm T1 \rightarrow tm T2.$$

In the implicit layer we now define a corresponding application operator  $@$  in terms of  $\_@$  as follows:

$$@ : tm (T1 \text{ arrow } T2) \rightarrow tm T1 \rightarrow tm T2 = \_@ T1 T2.$$

In this term the type variables  $T1$  and  $T2$  are implicit and need not be specified when  $@$  is used. Because  $@$  is a definition used only in proofs (not in the statement of the safety predicate), it does not have to be trusted.

## 8. The proof checker

The total number of lines of code that form our checker is 2,668. Of these, 1,865 are used to represent the LF signature containing the core axioms and definition, which is stored as a static constant string.

The remaining 803 lines of C source code, can be broken down as follows:

<i>Component</i>	<i>Lines</i>
Error messaging	14
Input/Output	29
Parser	428
DAG creation and manipulation	111
Type checking and term equality	167
Main program	54
Total	803

In designing the components listed above, we make sure not to use any library functions. Libraries often have bugs, and by avoiding their use we eliminate the possibility that some adversary may exploit one of these bugs to disable or subvert proof checking. However, we do make use of two POSIX calls: `read`, to read the program and proof, and `_exit`, to quit if the proof is invalid. This seems to be the minimum possible use of external libraries.

### 8.1. TYPE CHECKING AND TERM EQUALITY

The checker implements the syntax-directed algorithm given by the inference rules of Figure 8.1, which is essentially the same as a standard algorithm

(Harper and Pfenning, 2000, Section 6). The rules derive sequents of the form  $\Sigma, \Gamma \vdash X : A$ , where  $\Sigma$  is the trusted signature;  $\Gamma$  is a typing context holding variables introduced by  $\lambda$ - or  $\Pi$ -abstractions when computing the types of subexpressions of such abstractions;  $X$  is a term whose classifier we must compute; and  $A$  is the classifier computed for  $X$ . Given  $X$ , the rules are applied bottom-up to compute  $A$ , where  $\Gamma$  is initially empty. Note that the signature  $\Sigma$  is not type checked, which is reasonable since it is trusted. Symbols may be bound more than once by  $\Gamma$  in sequents; the *lookup* function is used to find the most recently added binding in  $\Gamma$  for a symbol. Thus,  $\Gamma$  holds a stack of bindings. This stack is implemented by the call stack created during recursive calls in the checker. In several rules,  $\alpha$  is a meta-variable ranging over  $\{type, kind\}$ . In (app),  $[N/x]B$  is written to denote the result of safely substituting  $N$  for  $x$  in  $B$ .

While the algorithm of Figure 8.1 is essentially the same as Harper and Pfenning’s, there are a few differences that should be noted. First, the algorithm is presented slightly more concisely here, by combining several rules from Harper and Pfenning’s presentation of their algorithm. Second, Harper and Pfenning’s algorithm compares expressions for equivalence uses type information, where here we use Coquand’s algorithm, which does not use any type information, for our  $\simeq$  relation (Coquand, 1991). The one significant difference is that Harper and Pfenning’s algorithm is for LF without type-level  $\lambda$ -abstractions. The PCC application makes use of type-level  $\lambda$ -abstractions, so an extension to the algorithm is needed. In (app), we need to reduce the type of  $M$  to weak head normal form. This is indicated by  $\xrightarrow{whr}$ . The weak head normal form of  $X$  is  $X$  if  $X$  is not of the form  $(\lambda x : E. F) G$ , and otherwise, it is the weak head normal form of  $[G/x]F$ . This is similar to what is done in Huet’s Constructive Engine for the Calculus of Constructions (Huet, 1989). Note that the resulting algorithm is complete for LF, since this change to the (app) rule merely allows more classifications to be derived than in LF.

Given a proof term  $X$  and a theorem  $A$ , the checker first computes the classifier of  $X$  with respect to the trusted signature  $\Sigma$  in the empty context. If  $X$  indeed has a classifier,  $A'$ , then the checker checks  $A' \simeq A$ . If  $X$  is not classifiable or its classifier is not equivalent to  $A$ , the checker reports that the proof is invalid and exits. Otherwise, it returns 0, indicating that  $X$  is indeed a valid proof of  $A$ .

## 8.2. TRUSTING THE C COMPILER

These 803 lines of C need to be compiled by a C compiler, and so it would appear that this compiler would need to be included in our TCB. The C compiler could have bugs that may potentially be exploited by an adversary to circumvent proof checking. More dangerously perhaps, the C compiler could have been written by the adversary so while compiling our checker, it could

$$\begin{array}{l}
\text{(type)} \quad \overline{\Sigma, \Gamma \vdash \text{type} : \text{kind}} \\
\\
\text{(ax)} \quad \overline{\Sigma, \Gamma \vdash v : A} \quad A = \text{lookup}(v, \Sigma, \Gamma) \\
\\
\text{(lam)} \quad \frac{\Sigma, \Gamma, x : A \vdash M : B \quad \Sigma, \Gamma \vdash A : \text{type} \quad \Sigma, \Gamma, x : A \vdash B : \alpha}{\Sigma, \Gamma \vdash \lambda x : A. M : \Pi x : A. B} \\
\\
\text{(app)} \quad \frac{\Sigma, \Gamma \vdash M : X \quad X \xrightarrow{\text{whr}} \Pi x : A. B \quad \Sigma, \Gamma \vdash N : A' \quad A \simeq A'}{\Sigma, \Gamma \vdash (M N) : [N/x]B} \\
\\
\text{(pi)} \quad \frac{\Sigma, \Gamma \vdash A : \text{type} \quad \Sigma, \Gamma, x : A \vdash B : \alpha}{\Sigma, \Gamma \vdash \Pi x : A. B : \alpha}
\end{array}$$

Figure 5. The LF type-checking algorithm

insert a Thompson-style (Thompson, 1984) Trojan horse into the executable of the checker.

All proof verification systems suffer from this problem. One solution (as suggested by Pollack (1996)) is that of independent checking: write the proof checker in a widely used programming language and then use different compilers of that language to compile the checker. Then, run all your checkers on the proof in question. This is similar to the way mathematical proofs are “verified” as such by mathematicians today.

The small size of our checker suggests another solution. Given enough time one may read the output of the C compiler (assembly language or machine code) and verify that this output faithfully implements the C program given to the compiler. Such an examination would be tedious but it is not out of the question for a C program the size of our checker (3900 Sparc instructions, as compiled), and it could be carried out if such a high level of assurance was necessary. Such an investigation would certainly uncover Thompson-style Trojan horses inserted by a malicious compiler. This approach would not be feasible for the JVMs mentioned in the introduction; they are simply too big.

One might consider writing the checker in a more sophisticated programming language. For example, it might be easier to reason about the correctness of ML programs than C programs. However, there would still be the need to trust an ML compiler and its associated runtime system, including garbage collector. We could take the same approach with ML as with C – don’t trust the compiler but examine the compiled code to manually verify the faithfulness of the translation – but this might be more difficult with ML

than with C. In this context, the advantage of C is its low level of abstraction – the machine language corresponds closely to the source code.

Pollack (1996) writes, “it is unlikely that a bug in the computing platform [e.g., the compiler] causes a proof checker to erroneously accept a proof.” However, the application of machine-checked proof in computer security is an adversarial situation. For example, Proof-Carrying Code is designed so that an adversary can send a program to execute, with a proof that it respects its interfaces. The adversary will have had the opportunity to study a copy of the proof checker, and the ML or C compiler that compiles the proof checker. If the adversary finds a bug, he can try to design a proof that exercises this bug, in such a way that a proof of a false safety predicate is accepted. Though this scenario may seem far-fetched, it is similar to some of the attacks on Java Virtual Machines reported by Dean et al. (1997).

*Independent implementations.* Another way that we can gain confidence in a proof is to check it in independently implemented proof checkers. For several years we have used Twelf to check our proofs, and while we have found bugs in Twelf from time to time, these have always been incompleteness bugs rather than soundness bugs: we have never observed Twelf to accept an invalid proof. However, we have not tested Twelf in the adversarial way described in the previous paragraph (such adversarial attacks are not easy!). By using our own independent checker (perhaps in addition to Twelf) we could gain confidence in adversarially supplied proofs.

It’s not difficult to send the same proof through both Twelf and through our checker. But the LF proof-checker is only 1/3 of our trusted base; the other 2/3 is the specification of the logical inference rules and of the theorem to be proved, and we have no independent implementation of those. Furthermore, proofs with respect to the original specification will not be valid for the new logic and theorem, unless adequacy results are proved and checked.

### 8.3. PROOF-CHECKING MEASUREMENTS

In order to test the proof checker, and measure its performance, we wrote a small Standard ML program that converts Twelf internal data structures into DAG format, and dumps the output of this conversion to a file, ready for consumption by the checker.

We performed our measurements on a sample proof of nontrivial size, that proves a substantial set of lemmas that will be used in proofs of real Sparc programs. The proof in its fully explicit form is 287,285 lines of Twelf. Its DAG representation consists of 4,212,104 nodes. The Twelf system can check this proof in 43.8 seconds.

Checking the sample proof consists of the four steps: parsing and type-checking the TCB (0.2 sec), loading the proof from disk (0.2 sec), checking

the DAG for well-formedness, and type-checking the proof itself (9.4 sec). Flit is about four times faster than Twelf—although it relies on Twelf to digest the proof and construct the DAG. During type checking of this proof the number of temporary nodes generated is 6,744,899. The measurements above were made on a 1 GHz Pentium III PC with 256MB of memory.

In our original implementation of the proof-checker we found that most of proof-checking time was spent performing substitutions. All the lemmas and definitions we use in our proofs are closed expressions, and therefore they do not need to be traversed when substituting for a variable. We modified our checker to keep track of such closed terms and to avoid their traversal during substitutions. This simple optimization drastically reduced type-checking time by more than an order of magnitude.

## 9. Related Work

There has been a large amount of work in the area of proofs of machine language programs using both first order (Boyer and Yu, 1992) and higher order logics (Wahab, 1998; Gordon, 1994). Some of this work was focused on proving the correctness of the compiler or the code generator (see for instance Milner and Weyhrauch, 1972). For a historical survey see Stringer-Calvert (1998).

*Specifying machine architectures.* Two pieces of work are most related to our work as far as the safety specification is concerned. Wahab (1998) is concerned with correctness (not just safety) of assembly language programs. He defines a flow-graph language expressive enough to describe sequential machine code programs (he deals with the Alpha AXP processor). His logic is non-standard in that substitution is taken as a primitive operator and the logic contains rules detailing term equality under it. He proves the Hoare-logic rules as theorems and uses abstraction in order to massage the code stream and get shorter correctness proofs. The translation from machine code to the flow-graph language does not go through a “decode” relation, i.e., he proves the safety of assembly language programs not machine code programs. Thus he assumes that the assembler (translating assembly language to machine code) is correct (it is part of the trusted base) which grows his TCB substantially.

Boyer and Yu (1992) formally specify a subset of the MC68020 micro-processor within the logic of the Boyer-Moore Theorem Prover (Boyer and Moore, 1988), a quantifier-free first order logic with equality. Their specification of the step relation is similar to ours (they also include a decode relation) but in their approach these relations are functions. The theorem prover they use allows them to “run” the step function on concrete data (i.e. once the step

function is specified they automatically have a simulator for the CPU). Their logic, albeit first-order, is much larger than ours mainly because of its wealth of arithmetic operators (decoding can be done directly from the specification). Also their machine descriptions are larger than ours; the subset of the 68020 machine description is about 128K bytes while our description of the Sparc is less than half that size. Admittedly, the Motorola chip is much more complex than the Sparc, but we suspect that most of the size difference is attributed to our extensive use of factoring facilitated by higher order logic.

*Small, trustworthy proof checkers.* Watson (2001) surveys proof representations in 16 different theorem-proving systems. Relatively few of these systems support the export of proofs for independent checking. “This feature has generally been added to the prover when mature.” One example of a prover that supports proof export is HOL.

The HOL system is an theorem prover for higher-order logic (Gordon, 2000). Historically, HOL did not have a facility to produce proof objects that could be independently verified; it relies on abstract data types to prevent confusing formulas with theorems.

Harrison and Slind (1994) implemented a reference version of HOL, called `gtt`. The `gtt` kernel, plus a parser and other code to handle I/O issues, could in principle be used to check HOL proofs, though HOL at that time did not have an external proof representation. The kernel is about 750 lines of Standard ML (compared to our 278 lines of C for DAG manipulation, type checking, term equality). The size of the parser is not given.

Wright (1994) implemented and formally verified a proof checker for a higher-order logic (it was “intended to model proofs that could be output by the HOL system”). The program was written in ML; the components that would be relevant to the independent checking of proofs are the parser (for parsing axioms), i/o interface, and the “core.” The core is approximately 530 lines of ML; the size of the other components are not given.

Wong (1995) designed a representation language for HOL proofs, and built an independent checker for HOL, larger than Wright’s but more efficient. His parser is 2297 lines of ML, his core checker is 5715 lines, and there are 2905 lines of auxiliary modules, a total of 10,917 lines of code (Wong, 2003).

It is not clear why these checkers for HOL are larger than our checker for LF (which, in principle, is more general than HOL). It is certainly not the conciseness and expressiveness of C (which we used) versus ML (which they used); a general rule of thumb is that for symbolic applications such as this, ML is about twice as concise as C. HOL just has more operators and concepts than LF, and it appears to be more efficient to represent HOL in LF and then check the LF. Or it may be that we pursued the (perhaps artificial) design goal of minimizing the size of our checker, and others had different primary design goals.

## 10. Future work

The DAG representation of proofs is quite large, and we would like to do better. One approach would be to compress the DAGs in some way; another approach is to use a compressed form of the LF syntactic notation. However, we believe that the most promising approach is neither of these.

Our proofs of program safety are structured as follows: first we prove (by hand, and check by machine) many structural lemmas about machine instructions and semantics of types (Appel and Felty, 2000; Appel and McAllester, 2001; Ahmed et al., 2002). Then we use these lemmas to prove (by hand, as derived lemmas) the rules of a low-level typed assembly language (TAL). Our TAL has several important properties:

1. Each TAL operator corresponds to exactly 0 or 1 machine instructions (0-instruction operators are coercions that serve as hints to the TAL type checker and do nothing at runtime).
2. The TAL rules prescribe Sparc instruction encodings as well as the more conventional (Morrisett et al., 1998) register names, types, and so on.
3. The TAL typing rules are syntax-directed, so type-checking a TAL expression is decidable by a simple tree-walk without backtracking.
4. The TAL rules can be expressed as a set of Horn clauses.

Although we use higher-order logic to state and prove lemmas leading up to the proofs of the TAL typing rules, and we use higher-order logic in the proofs of the TAL rules, we take care to make all the statements of the TAL rules first-order Horn clauses. Consider such a clause,

```
head :- goal1 , goal2 , goal3.
```

In LF (using our object logic) we could express this as a lemma:

```
n: pf(goal3) -> pf(goal2) -> pf(goal1) -> pf(head) =
    proof.
```

Inside *proof* there may be higher-order abstract syntax, quantification, and so on, but the *goals* are all Prolog-style. The name *n* identifies the clause.

Our compiler produces Sparc machine code using a series of typed intermediate languages, the last of which is our TAL. Our prover constructs the safety proof for the Sparc program by “executing” the TAL Horn clauses as a logic program, using the TAL expression as input data. The proof is then a tree of clause names, corresponding to the TAL typing derivation.

We can make a checker that takes smaller proofs by just implementing a simple Prolog interpreter (without backtracking, since TAL is syntax-directed).

But then we would need to trust the Prolog interpreter and the Prolog program itself (all the TAL rules). This is similar to what Necula (1997) and Morrisett et al. (1998) do. The problem is that in a full-scale system, the TAL comprises about a thousand fairly complex rules. Necula and Morrisett have given informal (i.e., mathematical) proofs of the soundness of their type systems for prototype languages, but no machine-checked proof, and no proof of a full-scale system.

The solution is to use the technology we have described in the previous sections of this paper to check the derivations of the TAL rules from the axioms of logic (and the specification of the Sparc). Then can add a simple (non-backtracking) Prolog interpreter to our minimal checker, which will no longer be minimal. In their preliminary report on this extension, Wu, Appel, and Stump (2003) report that an additional 366 lines of C code are required.

The proof producer (adversary) will first send to our checker, as a DAG, the definitions of Horn clauses for the TAL rules, which will be LF-typechecked. Then, the “proofs” sent for machine-language programs will be in the form of TAL expressions, which are much smaller than the proof DAGs we measured in section 8.

A further useful extension would be to implement oracle-based checking (Necula and Rahul, 2001). In this scheme, a stream of “oracle bits” guides the application of a set of Horn clauses, so that it would not be necessary to send the TAL expression – it would be re-derived by consulting the oracle. This would probably give the most concise safety proofs for machine-language programs, and the implementation of the oracle-stream decoder would not be too large. Again, in this solution the Horn clauses are first checked (using a proof DAG), and then they can be used for checking many successive TAL programs.

Although this approach seems very specific to our application in proof-carrying code, it probably applies in other domains as well. Our semantic approach to distributed authentication frameworks (Appel and Felten, 1999) takes the form of axioms in higher-order logic, which are then used to prove (as derived lemmas) first-order protocol-specific rules. While in that work we did not structure those rules as Horn clauses, more recent work in distributed authentication (DeTreville, 2002) does express security policies as sets of Horn clauses. By combining the approaches, we could have our checker first verify the soundness of a set of rules (using a DAG of higher-order logic) and then interpret these rules as a Prolog program.

### 10.1. CHECKING THE CHECKER

Can we mechanically prove that our checker is sound? Any such proof would have to be checked by some mechanical checker, with respect to some set of axioms and specifications. But we believe that we have reduced the size of the

trusted base (specifications and proof checking algorithm) to approximately the smallest possible size, so any mechanical soundness proof would have to be with respect to a larger trusted base.

However, mechanical validation of our trusted base could still be useful. For example, one could derive a simulator from the specification of machine instructions, and compare the execution of programs in the simulator to programs running on the real machine. One could compare the output of a Sparc disassembler to the evaluation of our instruction `decode` relation. And, indeed, one could construct a program correctness proof and check it in a standard system such as Coq or HOL. These activities would not increase the size of the trusted base; instead, they would give more reasons to trust the trusted base.

What we have done is not to mechanically prove the trustworthiness of our trusted base; instead, we have, by construction, measured an (approximate) lower bound for the size of what needs to be validated.

## 11. Conclusion

Proof-carrying code has a number of technical advantages over other approaches to the security problem of mobile code. One of the most important of these is that the trusted code of such a system can be made small. We have quantified this and have shown that the trusted code can be orders of magnitude smaller than in competing systems (e.g., Java Virtual Machines).

We have also analyzed the representation issues of the logical specification and shown how they relate to the size of the safety predicate and the proof checker. In our system the trusted code itself is based on a well understood and analyzed logical framework, which adds to our confidence of its correctness.

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