Effective audit policy for voter-verified paper ballots

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Abstract

Scientists and politicians are reaching consensus that elections must be countable independently of the need to trust the computer software in a DRE voting machine or in an optical scanner. Public trust in elections requires a Voter-Verified Paper Ballot (VVPB), printed by a DRE or scanned by a scanner. But producing the VVPBs is not enough; to do any good they must also be audited. A recount of 1% of randomly selected precincts (as performed in some states) is *not* sufficient to detect fraud with high confidence, except in statewide races. A recount of 5% would be barely adequate, but would be quite expensive. I propose a new statutory framework that will be as affordable as a 1% recount but more effective than a 5% recount. It requires a mandatory audit of 1% of precincts, and permits any candidate to demand (and pay actual costs for) an audit of up to 7 precincts. The rule of 1%+7 will give high confidence for both large and small elections.

1   Introduction

Direct-Recording Electronic (DRE) voting machines contain general-purpose computer chips that are programmed to count votes. Studies have shown [16, 1, 8, 9, 4, 11, 3, 2] that it is not difficult to replace the vote-counting software in these machines with fraudulent software that silently shifts

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votes from one candidate to another without any outward sign to the voter that it is doing so. Malfunction of the computer program, or fraudulent replacement or manipulation of the computer program, can be undetectable in pre-election "logic-and-accuracy" testing.

Scientific and political consensus has been converging on the need to be able to trust the results of elections independently of the need to trust any particular computer software. Therefore, in 2004-2007 many states are either enacting laws requiring that all voting equipment shall produce voter-verified paper ballots (VVPB), or abandoning DREs and moving to optical-scan voting.

Voter-verified paper ballots can be used with, or occur naturally in, several technologies:

- DRE machines equipped with a printer to print a voter-verified paper ballot. After the voter inspects the printout to make sure that it agrees with the selections that she made electronically, she presses a button and the paper ballot is dropped automatically into a ballot box. She does not take the ballot with her—it is not a "receipt"—because the voter must not be able to prove to someone else how she voted, since that would enable vote-selling. Also, the purpose of the VVPB is to be recounted later (if necessary), which would not be possible if the voter takes it away.

- Optical-scan machines, with op-scan forms filled out by hand and counted by machine. Preferably this machine should be right at the polling place, and the voter should feed her ballot directly to the machine. This is called "precinct-count optical scan"; it is preferable to off-site counting because a precinct-count machine can tell the voter right away if she has made a mistake (such as an overvote) that voids the ballot, so she can have another chance to prepare an accurate ballot. The optical-scan form filled out by the voter is *itself* the voter-verified paper ballot.

- Hand-counted paper ballots.

With either printer-equipped DRE or optical-scan machines, it is possible that software malfunction or fraud can make the reported totals disagree with the paper ballots. Because the voter has seen the paper ballots directly, without any computer interpreting their contents to her, we presume (as do many state statutes) that the paper ballot is the more accurate record of the vote.

Requiring the VVPB is a necessary first step; but adopting new VVPB-capable equipment is not enough, if the paper ballots are never examined. In many states there are no laws requiring an audit of the paper ballots, and those states' recount statutes are ill-adapted to audits.

In this paper I will take the example of New Jersey, but my conclusions apply broadly to elections as conducted by most states. In 2005 the New Jersey legislature enacted a law requiring that every voting machine be equipped to produce a voter-verified paper record of each vote cast, to be considered the ballot of record in case of a disagreement with the machine count. However, the legislature did not require any system of audit of the VVPBs.

In New Jersey we cannot achieve an audit through the existing recount procedures, because under existing procedures, recounts tend to be performed only if there is a very close election or if a judge is convinced there is "reason to believe" that the results are inaccurate. Software-based fraud or malfunction does not necessarily result in a close election *or* in evidence that gives specific reason to believe that something has gone wrong.

Therefore, several states that have recently moved to voter-verified paper ballots have in addition specified a system of routinely auditing (hand-recounting) a small percentage of randomly selected precincts. If machine malfunction or software-based vote fraud in an election is sufficiently widespread to change the result by several percent, then (if there is a great number of precincts in the election) in many of the precincts the paper ballots won't match the electronic totals. If there is a mismatch in many of the precincts, then an hand recount of just a few precincts will detect the existence of the fraud or malfunction. That is, there is a pretty good chance that the precincts selected for audit will overlap with the precincts in which fraud or malfunction occurred.

In addition to serving the purpose of detecting fraud or malfunction, a mandatory random audit can identify whether election and post-election procedures are functioning well enough to guarantee that, in the event of a full-scale contested recount, the voter-verified paper trail would be usable, accountable, and reliable. I will discuss this later, in Section 5.

States should adopt a system of audit that addresses three principal goals:

1. The audit system should *deter fraud* by providing a significant chance that fraud will be detected.

2. The audit system should provide a way that a losing candidate can gain *confidence* (in the statistical sense) that his own election result was accurate.

3. The system should guarantee that, in the event that a court orders a full hand recount of an election, the voter-verified paper ballots will function well enough to make such a recount possible.

The system should be implementable without excessive or unnecessary cost and effort.

Proposals have been floated for an audit of 1% of precincts after each election. As I will show, an audit of 1% or 2% of randomly selected precincts is not enough to achieve all of the goals I list above.

The solution. I propose a two-part solution that will be effective (and cost-effective). Neither part by itself will be effective enough; both are needed.

- A mandatory audit (hand recount) of the voter-verified paper ballots from a random 1% of the precincts after each election.

- A (new) statutory provision that allows candidates to request (and pay for) *targeted* audits of up to 7 selected precincts, *without* going before a judge and *without* demonstrating "reason to believe" that the election was inaccurate. I will demonstrate mathematically where the "magic number" 7 comes from.

Summary of conclusions. Taken together, these two prongs will be much more effective and cheaper than measures adopted by some states that require 5% of precincts to be recounted by hand. In particular, for statewide elections (for Governor or U.S. Senator) the 1% audits are generally sufficient. For smaller elections (mayor, U.S. Representative, state legislator), the 1% audits will help to ensure that no machine error or software-based fraud is occurring repeatedly and on a wide scale, but they cannot effectively protect against individual problems at the local level. This because there simply aren't enough precincts in a local (legislative district or smaller) election for a 1% sample to be meaningful. An on-demand audit of 7 precincts is enough to provide

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assurance for local elections. The right to an on-demand targeted audit (even though it is rarely exercised) in combination with the 1% mandatory audit, will deter fraud.

These conclusions apply to states, like New Jersey, in which precincts all have approximately the same number of voters. The mathematics (and therefore, the necessary statutory language) is a bit more complicated for nonuniform precincts.

Related work. A recent comprehensive survey by Norden *et al.* [17] discusses strengths and weaknesses of a wide variety of post-election audit procedures and provides a good roadmap for election policy decisions.

2 What can random audits achieve?

If New Jersey is to audit its elections to detect and deter software malfunction or software-based vote fraud in its DRE or optical-scan machines, how should the system of auditing be designed? Is it sufficient to recount 1% of the precincts after each election (as Colorado and California do), or 2% (as do Pennsylvania and Arizona), or 3–5% (Kentucky, Illinois, New York, West Virginia, Alaska), or to specify the number of machines per county or district (Connecticut, Minnesota)? The Voter Confidence and Increased Accessibility Act of 2007 (H.R. 811, introduced by Rush Holt) calls for up to 10% of precincts to be audited, depending on the margin of victory.

The number of precincts one must recount depends on many factors: how many precincts there are in the election, how small a level of malfunction or fraud we insist on detecting, and so on. Suppose we want 95% confidence that, if fraud or error of 2% has occurred, we will detect it. (The Appendix describes methods of calculation.) In an election for Governor of New Jersey (with 6320 precincts) it suffices to recount 1% of the precincts, randomly selected. In one Congressional race (with a Congressional district, in NJ, of 486 precincts), it would be necessary to audit 12% of the precincts. In an election for mayor of a small borough (with 10 precincts) it would be necessary to recount 100% of the precincts!

To remain confident of the integrity of elections conducted by DRE (or op-scan) machines, we need manual audits (hand recounts). But we must recognize that hand recounts are expensive,

Table 1. Percentage of precincts to audit to detect vote fraud of 1%
(Precinct counts and city sizes representing New Jersey)

Race	Precincts	95% detection	50% detection
Governor, U.S. Senator	6320	0.7%	0.2%
U.S. Representative	486	8%	3%
State Legislator	158	24%	9%
Large city mayor	170	12%	8%
Small town mayor	10	100%	80%

Table 2. Percentage of precincts to audit to detect vote fraud of 5%

Race	Precincts	95% detection	50% detection
Governor, U.S. Senator	6320	0.1%	0.05%
U.S. Representative	486	1.7%	0.6%
State Legislator	158	5%	1.7%
Large city mayor	170	5%	1.7%
Small town mayor	10	60%	30%

Tables 1 & 2. What can be achieved by recounting a fixed percentage of the precincts. The fact that 12%, 24% or even 60% of precincts need to be recounted (in some cases) for 95% confidence is a problem with the fixed-percentage method. See the Appendix for methods of calculating these tables.

labor-intensive, and time consuming. Furthermore, these recounts will best serve their purpose (convincing even the losers of elections that the elections are legitimate) if they are done in the presence of witnesses representing the candidates and parties. Even if the State could find the resources to hand-count a large percentage of the precincts, it can be difficult for the candidates and parties to find enough volunteers to witness them. For these reasons, it is worth considering methods of assurance that are less arduous than a mandatory audit of 12% of all the precincts in the state after every Congressional election.

The solution to this problem is to perform a mandatory audit of a small percentage (1%) of the precincts, but to provide new statutory safeguards so that a greater percentage of precincts can be recounted *as necessary in specific elections.*

A scenario. Consider an election for mayor of a city about the size of Newark with approximately 170 election districts (precincts). Each precinct has about 700 registered voters, but the expected turnout is about 350 voters per precinct. To win the election one needs about 30,000 votes (this is 50.4% of the votes). Let us suppose a cheater wants to steal the election, and he calculates

6300 beads representing the precincts in a New Jersey Governor election; 10% of the beads are blue, representing fraudulent voting machines. A 1% sample (63 beads) is shown; it is extremely likely to include at least one blue bead (in this case the sample has 7 blue beads), and thus the audit will catch some of the fraudulent machines (triggering, in principle, a wider recount and a forensic investigation).

100 marbles representing the precincts of a city mayoral election; 10% of the marbles are blue representing fraudulent voting machines. A 1% sample is shown (one marble); it's unlikely that a 1% sample will include any blue marbles. While a 1% audit works well for statewide races, it does not suffice for local or legislative-district elections.

that he needs to steal 3.3% of the votes (that is, flip 2000 votes from one candidate to another). By fraudulently reprogramming DRE or op-scan machines, he can steal 200 votes from each of 10 precincts, or 20 votes from each of 100 precincts. Of course, if there is a VVPB, then the machines will report fraudulent results and a hand recount would detect the fraud. But is it possible that none of the hand recounts will be done in precincts where the machines are cheating?

Stealing many votes in a few precincts. If the cheater steals 200 votes from a precinct with only 350 voters, then it will be "obvious" that something is wrong. "Everyone knows" that this precinct's voters lean heavily toward one party, but the vote is lopsided the other way. In such a case, some authors [18][19, p. 15] assume that a criminal candidate would not dare perpetrate a fraud of more than 20% in any single precinct because this would be noticeably different from what is expected in the precinct-by-precinct results.

Is it true that no one would dare steal more than 20% of the votes from a single precinct? Election history in the United States shows many cases where people have dared. [5] But still, it might seem that the NJ Statutes provide a remedy:

NJ 19:28-1. When any candidate at any election shall have reason to believe that an error has been made in counting the votes of that election, the candidate may, within a period of 15 days following such election, apply to a judge of the Superior Court assigned to the county wherein such district[1] or districts are located, for a recount of the votes cast at the election in any district or districts. ...

Although Statute 19:28 *seems* to provide a remedy, in fact judges in New Jersey have historically been very reluctant to order recounts unless the plaintiffs can demonstrate a very strong reason to believe that the vote count is in error. The fact that "everyone knows" something does not mean it's easy to prove in court. In fact, "everyone knows" that Frank Hague's political machine manipulated elections in Jersey City from 1917 to 1947 [15]. Newark and other New Jersey cities had political machines run by corrupt mayors [7] (with voting-place irregularities [13]) during the 1960s and perhaps at other times before and since; and so on. But that does not necessarily give a legal "reason to believe" that an election in Newark in recent times should be examined more closely.

Therefore, although Rivest [18] assumes that one "wouldn't dare to change more than 20% of the votes in one precinct" using software-based voting-machine fraud, and so does Stanislevic [19, p. 15], I argue that the New Jersey recount statute does not support this assumption, at least the way it is interpreted by judges. Later in this paper I will propose that *targeted recounts* provide a solution.

Stealing a few votes in many precincts. Suppose on the other hand the cheater rigs the software of the voting machines to steal 20 votes (5% or 6% of 350) from each of 100 precincts (out of the 170 precincts in the election overall). Then it will not be obvious that the election was stolen, and the losing candidate will have not have any reason to believe that he should go to court and ask for a massive recount. However, since so many different voting machines have been rigged, a recount of just a few precincts would be likely to find one machine whose electronic total doesn't match the hand recount. In this case, a random audit of 5 precincts (3% of the 170 precincts) will

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[1]NJ Statutes use the term "election district" for what I am informally calling "precinct."

be 95% likely to detect the fraud. A random audit of 2 precincts (just over 1% of the precincts) will be 50% likely to detect the fraud.

This is the main purpose of a mandatory random hand-recount of a fixed percentage of the precincts—if there's error or fraud in many voting machines in many precincts, a small random sample will find it.

Tables 1 and 2 provide several more examples. In each case I assume that the cheater does not dare to steal more than $\frac{1}{7}$ of the votes in any one precinct (and I continue to defer the answer to the question, "*Why* would he not dare?"). There is nothing magic about the fraction $\frac{1}{7}$, or 14%; we simply need to assume some percentage of per-precinct vote theft beyond which it will be *noticeable* or *suspicious* that the results are unexpected. If we were to assume a slightly bigger or smaller percentage, then we could calculate a slightly smaller or bigger confidence in detecting fraud, but the pattern of results would be about the same.

The last line of Table 1 shows something surprising. If we must design a random recount procedure to detect even the smallest amounts of vote-stealing (1%) even in the smallest elections (small-town mayor, school board), then in fact we must always manually recount *all* precincts.

Clearly it is unrealistic to manually recount every precinct. Large-scale hand-counting of paper ballots requires time and therefore money. Hand-counting works well only in the presence of witnesses representing the parties or candidates, and it is hard to find enough witnesses to watch hand counts of all the precincts in every election.

As Tables 1 and 2 show, recounting of just 1% of precincts gives very high confidence in statewide elections, but does not (in the absence of other measures) help much in very small elections. For medium-size elections (legislative district, large city mayor, U.S. Congressional district), a 1.7% rate of random audit is enough to have a 50% detection rate for massive vote fraud. This will certainly help to *deter* fraud; but a 50% chance of detecting fraud is not very reassuring to a losing candidate! We should be able to guarantee to this candidate the legitimacy of the election with a higher degree of confidence.

3   Candidate-paid targeted recounts

Suppose the State implements automatic audits of 1% of all precincts. In a less-than-statewide election (legislature, mayor, or congressman); the 1% recount means that if fraud was perpetrated, there's a 50% chance (or less) that it will be detected by the audit.

In most cases, candidates and parties have respect for each others' integrity, and for the integrity of the election officials. They—very reasonably—believe that there's no fraud in their district. Therefore, that the 1% audit provides only 50% chance of finding fraud (if it was committed in this district in this election) is not bothersome—the audit is simply a welcome layer of extra assurance.

In contrast, consider a locale where candidates are suspicious of the integrity of election procedures. A robust and successful system of elections provides legitimacy to election results even in those difficult circumstances. An audit that provides less than a 50% chance of detecting fraud—if fraud was committed—will not reassure the losing candidate enough to provide the legitimacy we want for all elections.

For example, suppose a candidate for the State Legislature who expected to win by 53–47%, is surprised by a reported result that he lost by 48–52%. Should the candidate demand a full recount of all 158 precincts? Fortunately, there is a more efficient solution. Suppose the candidate could simply request (and pay for) a hand audit of 7 randomly selected precincts. If any fraud (or malfunction) has actually occurred, this will give a 95% chance of detecting it.

Although New Jersey Statute 19:28 (excerpted above) allows candidates to ask for recounts, the existing statute does not really facilitate this kind of recount; there is no *specific* "reason to believe that an error has been made in counting the votes," and a judge would very likely deny the request for a recount in this circumstance.

If the State adopts only a 1% mandatory random-audit rate, then without other provisions it will be *impossible* to achieve better than a 50% chance of detecting fraud in individual legislative races, or to achieve better than a 10% chance of detecting fraud in individual small-town races.

Because a mandatory audit of a small percentage of (randomly selected) precincts is inadequate to give high confidence in local and legislative elections, I propose *targeted audits of precincts chosen*

by the candidate or party.

The idea of targeted recounts is not entirely new. In the context of mandatory audits of VVPBs, Hyde [10] suggested that instead selecting a random 1% or 2% of precincts, each of the stakeholders in an election should be allowed to select some of the precincts, for a total of about 5 precincts per county. I suggest nonmandatory targeted audits because I believe that the random 1% audits will be simpler to administer, and that additional candidate-paid targeted audits will be rare and limited to cases where the candidate has grounds for unease.

The statuory language would read as follows.

1. In any election conducted by the State in which 50 or more election districts participate, immediately after the election the County Clerk[2] shall conduct a public random drawing to select 1% of the participating election districts (rounded up to the nearest whole number). If the election comprises several legislative or congressional districts, this selection will be 1% of the entirety of the legislative or congressional districts in the election overall, not 1% of the election districts in each legislative or congressional district.[3] The voter-verified paper records from these election districts will be recounted by hand in the presence of representatives of the candidates and parties.

2. The County Clerk shall make public the number of votes cast for each candidate in each individual election district, as promptly as feasible after each election. Up to 7 days after these results are made public, any candidate in an election, or that candidate's party, may apply to the County Clerk for an audit of up to 7 election districts from that election, to be chosen by the candidate or party making the application. The candidate

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[2]I have suggested here the County Clerk because that is the official charged in existing Statutes with supervising recounts. It is not clear that this is the best choice. County clerks already conduct random drawings to determine which party will be listed first on the ballot at each election, and there have been rumors that this drawing has been routinely manipulated. Even if these rumors are unfounded, their existence highlights the importance of rulemaking to specify a protocol for the random selection of 1% of the precincts in such a way that the results can be considered legitimate by all. This need is also highlighted by the recent felony convictions of election workers in Ohio who manipulated the drawing of precincts to audit in Ohio's mandatory recount of 3% of the precincts in the 2004 election. [14] Norden *et al.* [17] present cogent arguments for making the random selection at the statewide level, because it is easier to gather witnesses to monitor the proceedings at a single location than scattered throughout many counties.

[3]This is mostly to avoid rounding up to the nearest whole number in *each* legislative district.

or party shall pay a reasonable fee to cover the cost of this audit, based on the number of election districts counted and according to a fee schedule to be determined and published from time to time by the Attorney General. The County Clerk must promptly conduct an audit of these election districts, with the voter-verified paper records from these election districts to be recounted by hand in the presence of representatives of the candidates and parties. If there is a significant discrepancy between the originally announced totals and the hand recount, then the fee paid by the candidate shall be refunded.

3. The absentee and provisional ballots cast in any county, if they are not more numerous than the maximum number of registered voters in any election district, shall constititute one precinct for the purposes of selecting election districts under paragraphs (1) and (2). If the absentee and provisional ballots are more numerous than the maximum number of registered voters permitted in one election district under 19:4-13, then the absentee ballots in each county shall be divided physically into batches no larger than the maximum number of voters permitted in one election district, and if possible no smaller than the minimumm number permitted under 19:4-13. This division may be made by separating the absentee and provisional ballots into appropriate political subdivisions such as boroughs and townships.[4]

4. If there is a discrepancy between the totals originally made public by the County Clerk and the results of any audit under paragraph (1) or (2) above, then for purposes of adjudicating requests for wider recounts under 19:28-1, this constitutes reason to believe that, in election districts other than those audited, errors have been made in the counting of votes of that election.

5. Statute 19:28-1 to be amended: after "within a period of 15 days following

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[4]It appears to be the practice in at least one New Jersey county [6] (perhaps in all) to list the absentee ballots for each town and the provisional ballots for each town as separate entries as if the absentee ballots were one precinct of the town and the provisionals were one precinct of the town. This paragraph of the proposed statute should be adjusted as necessary to match the chain-of-custody and physical separation of absentee ballots used in actual practice by county election officials. Absentee ballots are little-used in New Jersey; the "virtual precinct" of absentee ballots typically has many fewer votes than an ordinary precinct. Only in 2005 did it become legal in New Jersey to vote absentee without specifying a reason (such as being absent on election day).

such election," add "or within 7 days after the results are announced of audits under [paragraphs (1) and (2) above], whichever is later,"....

Rationale. This proposed statute achieves several goals at a reasonable cost.

- It gives a high confidence in the accuracy of voting machines in statewide elections, without even the need to invoke provision 2.

- It deters fraud by giving high confidence that, if there is systematic voting-machine fraud or malfunction in one city or another, it will eventually be detected.

- It gives a way to justify the assumption that "no one would dare to steal more than 20% of the votes in any one precinct." If such a theft was attempted, the losing candidate would have a strong enough suspicion to justify recounting 7 precincts, *and would know which precincts to specify in his application.* I have seen no other proposal that justifies this assumption in a way actually related to recount laws in any particular State.

- It gives each candidate a way to achieve high confidence that there is not substantial systematic voting-machine fraud. Suppose a candidate in a legislative race (158 precincts) would like to increase his confidence in the integrity of the voting machines, but does not suspect any particular precincts. He can then make a random selection of 7 precincts, and ask for these to be recounted in addition to the (approximately) 2 precincts that comprise the State's automatic 1% recount. The resulting 9 precincts is enough to guarantee a 95% confidence that, if there was substantial software-based fraud or malfunction, it will be detected.

Table 3 shows that the rule of 1%+7 precincts is sufficient to guarantee confidence in a wide variety of cases. That is, the column (D) is at least as large as column (A) or (B). The rule of 1% alone (column C) is insufficient.[5] The "magic number" 7 is not a universal law of nature— it's simply just enough to give 95% confidence of detecting a 5% vote theft, or 50% confidence

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[5]For the statewide races, it appears that 1% (that is, 64 precincts in column (C)) is more than the minimum neede (that is, 8.6 precincts in column A). But every New Jersey gubernatorial election of 6320 precincts is also 40 legislative elections of 158 precincts, and every senatorial election is also 14 U.S.-representative elections of 486 precincts. Therefore 1% is really a minimum for useful audits of these elections.

		(A) How many precincts to recount in so that, if 5% of votes were stolen (no more than 1/7 of votes in any one precinct), there's a 95% chance of detecting	(B) How many precincts to recount in so that, if 2% of votes were stolen (no more than 1/7 of votes in any one precinct), there's a 50% chance of detecting	(C) How many precincts would be recounted if just 1% of precincts are selected for recount	(D) How many precincts would be recounted if just 1% plus 7 additional are selected for recount
Race	Precincts				
Governor, U.S. Senator	6320	8.6 precincts	7.1	64	71
U.S. Representative	486	8.5	7.1	5	12
Large city mayor	170	8.4	7.0	2	9
State Legislator	158	8.3	7.0	2	9
Small town mayor	10	5.6	6.7	0	7

Table 3.

of detecting a 2% vote theft. Lower numbers (such as 5 or 6 precincts) would give slightly lower confidence (but would still be far better than a simple recount of 1% of precincts); higher numbers (9 or 10 precincts) would give even more confidence.

The effort of recounting paper ballots in 7 precincts will likely be manageable for whatever authority (perhaps the Superintendent of Elections) must conduct it; the cost of recounting 7 precincts will likely be affordable to the candidate. Furthermore, in the vast majority of districts where there is a reputation for clean elections and where the machinery is understood to work reliably, there will be few demands to recount an additional 7 precincts.

Experience in New Hampshire with recounts on demand. One might think that it would be burdensome upon the State and its election workers to be subject to the whim of candidates in their demands for extra recounts in every election. Can on-demand recounts be manipulated to produce chaos? But we can look to other States for evidence that, even in those States where laws permit *unlimited* on-demand recounts, it is rare to have frivolous demands for recounts.

The statutes of the State of New Hampshire permit candidates to demand recounts after any State general election. For close elections there is a set fee schedule (from $40 to $2,000) that depends on the closeness of the election and whether it is a statewide or legislative-district election. When the election is not at all close, the candidate may still demand a recount, and the Secretary of State charges a fee to cover the costs. If the recount changes the result of the election, the fee is

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refunded or waived.[6]

Candidates apply to the Secretary of State, who must perform the recount and who has no discretion to deny it.[7] In a legislative election it is typical that there will be about one legislative district in which a candidate demands a recount, for which a typical fee is $1000. In 2004 the Green Party (whose candidate had received a tiny percentage of the Presidential vote) requested that approximately 17 towns be recounted, as a way of monitoring the accuracy of optical-scan machines; the Secretary of State charged a fee (based on actual cost) of approximately $10,000. The Secretary of State is satisfied that the fees are adequate to cover the actual costs incurred. According to the Secretary of State, candidates do not complain that the fees are unreasonably high. The Secretary of State is satisfied that there are not substantial numbers of frivolous recount requests, and that the system of recount-on-demand works well in New Hampshire. In addition to the recounts on demand, the State of New Hampshire is moving to institute mandatory recounts of randomly selected legislative districts.

Third party candidates and referenda. This proposed statute gives third-party or independent candidates the same rights as major-party candidates: 7 precincts. This will protect them against fraud in the same way that it protects the major-party candidates. On the other hand, what is missing from this proposal is a mechanism for on-demand audits of ballot questions, since there is no *candidate* designated to demand an audit. Perhaps any group of citizens (willing to pay) should per permitted to demand an audit of 7 precincts, on ballot questions.

4 What is a "recount"?

New Jersey Statute 19:28 is unclear about what a "recount" actually means: is it a hand count of individual voter-verified paper ballots, or just a re-examination of the totals printed by each machine? The latter is obviously inadequate for the purpose of detecting internal malfunction in the voting machine (or op-scan counter) itself.

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[6]New Hampshire Revised Statutes, section 660.

[7]David M. Scanlan, Deputy Secretary of State of New Hampshire, telephone interview, February 5, 2007; source for all facts in this paragraph.

During the last 70 or more years in New Jersey, when the State has primarily used lever-action and DRE voting machines with no voter-verified paper ballots, it has not been possible to actually recount the individual votes. Malfunctions in lever-action or DRE voting machines have caused votes to be irretrievably lost or misrecorded. Therefore, when "recounts" are performed, what really happens is that the vote totals from individual machines are reinspected, the absentee ballots are hand-counted, and the numbers from all the precincts are carefully re-added.

In fact it is often the case that errors in addition are found and corrected during this process. This is so even though the addition of precinct numbers is performed by computer programs (election tabulation software) that are certified as reliable; often the problem is that the wrong data is fed into these programs. Clearly this kind of "recount"—adding up the numbers again—has significant value. But it will not detect problems inside the voting machines themselves.

With *individual* voter-verified paper ballots, a different kind of recount is possible: the examination of each individual paper ballot in one or more precincts. This kind of recount is more time-consuming and expensive than a simple re-addition of precincts.

On what basis should a candidate, or a judge, decide which kind of recount to request or to order? The answer is simple. If a candidate, or judge, wishes to detect whether one or more *voting machines* may be malfunctioning or fraudulent, then hand recounts are useful. On the other hand, if one wants to detect a mistake in the addition of the precinct numbers, then a simpler (and cheaper) audit of the addition will be useful.

New Jersey Statute 19:28 sets a fee of $25 per precinct for a recount. This dollar amount, like many to be found in the Statutes, was set many decades ago and is far less than the actual expense that would be incurred by the State or by a County in hand-recounting a precinct. In order to discourage frivolous requests for hand recounts, it would be reasonable to distinguish between three kinds of audits:

- **Audit without hand recount,** which follows the paper trail starting from the vote total printed by each voting machine at the close of the polls. The physical printout from each machine (which is not a VVPB, since it is printed just after the last voter has departed) should be compared against removable memory ("results cartridges") and against the numbers

recorded in the central tabulation system. This audit will catch errors of addition and errors of including the wrong machines in the overall total.

- **Hand recount of one race** from the voter-verified paper ballots, in a specified set of precincts.

- **Hand recount of all races** from the voter-verified paper ballots, in a specified set of precincts.

The mandatory recount of 1% of all precincts should be a *hand recount of all races.* A candidate or party exercising the right to pay for a recount of selected precincts should be permitted any of these three kinds of recounts. It would be reasonable to charge a different fee for the different kinds of recounts, and the chief election official of the state[8] should make this clear in the fee schedule.

Audits or recounts? Mandatory audits of a percentage of precincts, or on-demand audits of selected precincts, are not recounts that directly change official election results. They are evidence to be taken to a court who may order a wider recount. That court, in deciding whether to recount all precincts or just some precincts (for example, those that use a certain technology), can be guided by evidence, statute, and precedent.

5   Auditing the audit process

It is often overlooked that an important purpose of a mandatory random audit is to continually monitor the effectiveness of the paper-trail process. A wide variety of errors and mistakes can occur, any of which would compromise the ability to count elections accurately.

Not only can audits detect fraud, but they can also identify where post-election procedures are in need of improvement. The first formal study of a county that has put VVPB equipment into use was in Cuyahoga County, Ohio (Cleveland). [12] There it was found that, through a combination of unreliable equipment, weak polling-place procedures, and weak post-election chain-

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[8]In most states, the Secretary of State; in New Jersey, the Attorney General.

of-custody procedures, "The machines' four sources of vote totals—VVPAT[9] individual ballots, VVPAT summary, election archive, and memory cards—did not agree with one another."

The VVPAT was not the only source of problems: there were problems in chain-of-custody, management, and aggregation of individual precinct totals. Even if we did not have a voter-verified paper ballot, the integrity of elections relies on the trustworthy aggregation of precinct totals.

In New Jersey, as in Ohio, the totals from each machine are transmitted in at least two different (redundant) ways to the central tabulation facility: on electronic cartridges removed from the voting machines at the end of the day, and on paper summary printouts (not the same as VVPBs) removed from the voting machines at the end of the day. These two numbers should agree; in Cuyahoga county, they did not. The study mentioned above allowed Cuyahoga County to recognize weaknesses in its equipment and procedures, and begin addressing them. The study illustrates another benefit of random audits—not just the deterrence of fraud, but the diagnosis and improvement of election procedures.

If the State has a mandatory random recount procedure, then such a procedure will uncover these problems (if they exist) on a routine basis. Then the equipment and procedures can be tuned and improved, *before* there is a bitterly contested fight with allegations of fraud in a close election. A 1% rate of random audit is entirely sufficient for the purpose of monitering the effectiveness of the paper-trail process.

6   Conclusion

In the auditing of voter-verified paper ballots, states should rely on a two-pronged approach: mandatory hand recounts of a small percentage of precincts, and candidate-paid recounts on demand of up to 7 precincts to gain confidence in specific circumstances. Both parts are necessary, as a mandatory 1% or 2% audit alone is not sufficient.

Most existing statutes (such as New Jersey 19:28) do not adequately support recounts for the

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[9]VVPAT (Voter-verified paper audit trail) is similar in meaning to VVPB (voter-verified paper ballot), but does not carry the connotation that the VVPAT is necessarily the ballot of record. The 2005 New Jersey statute uses the term "individual voter-verified paper record" and makes it clear that it is the ballot of record in case of disagreement with the machine total; thus it is effectively a VVPB.

purposes of sampling for error in smaller elections, so new legislation is needed.

Appendix: Methods of calculation

Suppose an election has voters divided into n precincts in each of which there are p active voters. Let us suppose that a cheater changes no more than $g = \frac{1}{7}$ of the votes in any single precinct, because if he changes more than that, the losing candidate will notice an anomalous precinct and demand a recount of that precinct. Let us suppose the cheater changes an overall proportion c of the votes cast; in Table 1 I used $c = 0.01$ and in Table 2 I used $c = 0.05$. Then the number b of fraudulent precincts is $b = c/g$.

Let u_{95} be the number of precincts recounted. To achieve a 95% confidence of detecting a fraudulent precinct (if cheating has occurred at this level), Rivest [18] gives the "Improved Rule of Three,"

$$u_{95} \geq n(1 - \exp(-3/b))$$

To achieve a 50% confidence level, the formula is

$$u_{50} \geq n(1 - \exp(-1/b))$$

The fraction of precincts to recount is simply

$$r_{95} = u_{95}/n \qquad r_{50} = u_{50}/n$$

We can relabel Table 1 as follows:

Pct. precincts to audit to detect fraud of $c = 1\%$

Race	Precincts	95% detection	50% detection
Gov.	$n = 6320$	$r_{95} = 0.007$	$r_{50} = 0.002$

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