PRINCETON UNIVERSITY COS 522: COMPUTATIONAL COMPLEXITY

Homework 1

Out: *February 14* Due: *February 26*

You can collaborate with your classmates, but be sure to list your collaborators with your answer. Also, limit your answers to one page or less —you just need to give enough detail to convince me. If you suspect a problem is open, just say so and give reasons for your suspicion.

§1 Show that $\Sigma_2^p = \mathbf{NP}^{\mathrm{SAT}}$.

§2 Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$. (Hint: Use padding, mentioned in the notes for Lecture 1.)

§3 Can you give a definition of **NEXPTIME** in terms of certificates as we did for **NP**? If not, report your best attempt.

§4 Say that a class C_1 is *superior to* a class C_2 if there is a machine M_1 in class C_1 such that for every machine M_2 in class C_2 and every large enough n, there is an input of size between n and n^2 on which M_1 and M_2 answer differently.

(a) Is $\mathbf{DTIME}(n^{1.1})$ superior to $\mathbf{DTIME}(n)$?

(b) Is $\mathbf{NTIME}(n^{1.1})$ superior to $\mathbf{NTIME}(n)$?

§5 Suppose we define the *logspace hierarchy* in analogy with the polynomial hierarchy using logspace machines that can use alternation. Does this hierarchy collapse by Immerman's theorem ($\mathbf{NL} = \mathbf{coNL}$)?