Morphometrics

cs598b, geometric modeling

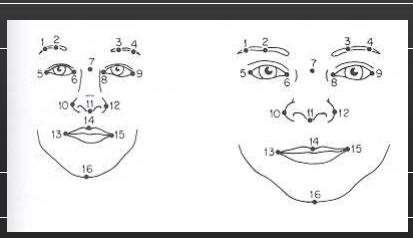
April 18, 2000

Morphometrics

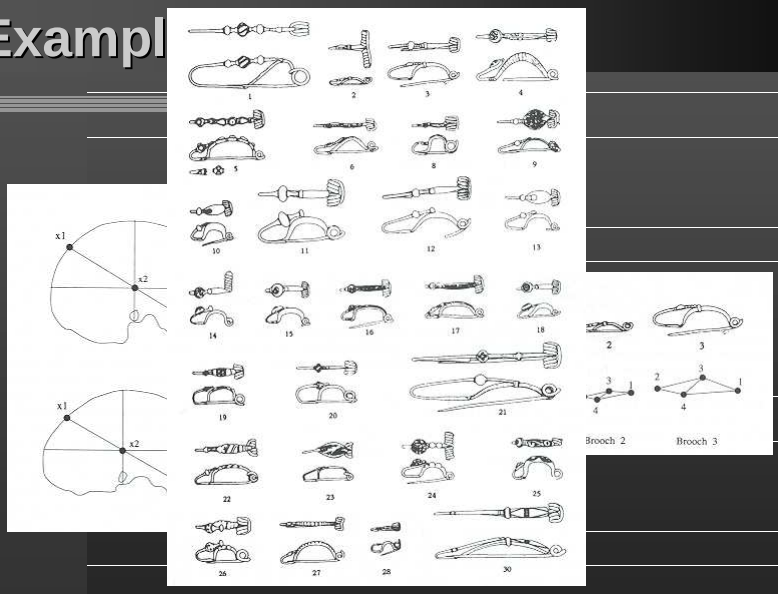
- the study of covariances of biological form
- specialized application of multivariate statistics
- (Fred L. Bookstein, “Morphometric tools for landmark data”)

Landmarks

- feature points
- use to compare changes in biological shape
- carefully chosen

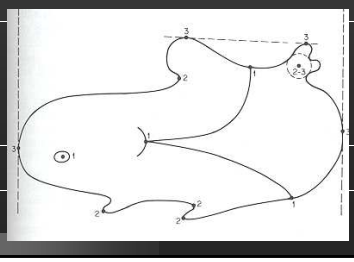


Exampl



Landmarks, II

- example candidates:
  - juxtaposition
  - maximum curvature
  - extremal points (e.g. endpoints of diameters, centroids, ...)

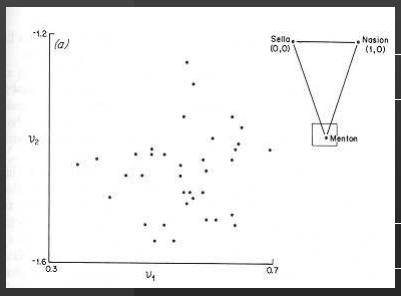


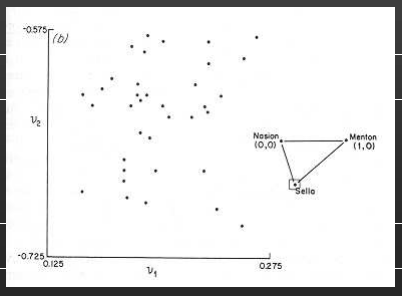
Shape coordinates

- again, we want coordinates independent of translation, scale and rotation
- choose representative set of landmarks as basis
- e.g. in triangle: use one edge as “baseline”

Scatter of shape coordinates

- if we choose a different baseline, the scatter differs mainly by translation, rotation and scale





More shape statistics

- Christopher Small, “The Statistical Theory of Shape”
- shapes as equivalence classes
- shapes are points on a *shape manifold*

Pre-shape

- vector of n normalized landmarks is called *pre-shape*
- lies on (2n - 3) dimensional sphere called *pre-shape space*

Orientation

- there is no continuous orientation function for every shape
- *orbit* is equivalence class of shapes w.r.t. rotation
- want to find distance metric between orbits: minimum angle between all pairs of shapes from two orbits

Shape comparison

- find landmark correspondence which minimizes a distance measure
  - Procrustean distance
  - bending energy of thin-plate spline

Procrustean distance

- view *n* landmarks coordinates as complex numbers:

$$\tau_j = (\tau_{j1}, \tau_{j2}, \ldots, \tau_{jn})$$

- Procrustean distance:

$$d(\sigma_1, \sigma_2) = \cos^{-1}\left(|\sum_{k=1}^{n} \tau_{1k}\tau_{2k}^{*}|\right)$$

- independent of orientation