MIMO III: Channel Capacity, Interference Alignment

COS 463: *Wireless Networks*
Lecture 18
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[Parts adapted from D. Tse]

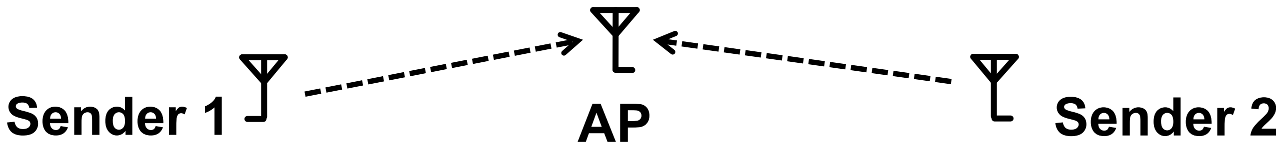
Today

1. **Interference Channel Capacity**
   - **Single-Antenna Context**

2. MIMO Channel Capacity

3. Interference Alignment

Two-User Interference Channel



- On the **same frequency channel** at the **same time:**
  - **Sender 1** sends signal x_1 with power P_1
  - **Sender 2** sends signal x_2 with power P_2

- **AP receives:** $y[m] = x_1[m] + x_2[m] + w[m]$
  - w[m] is background Gaussian Noise with variance σ^2

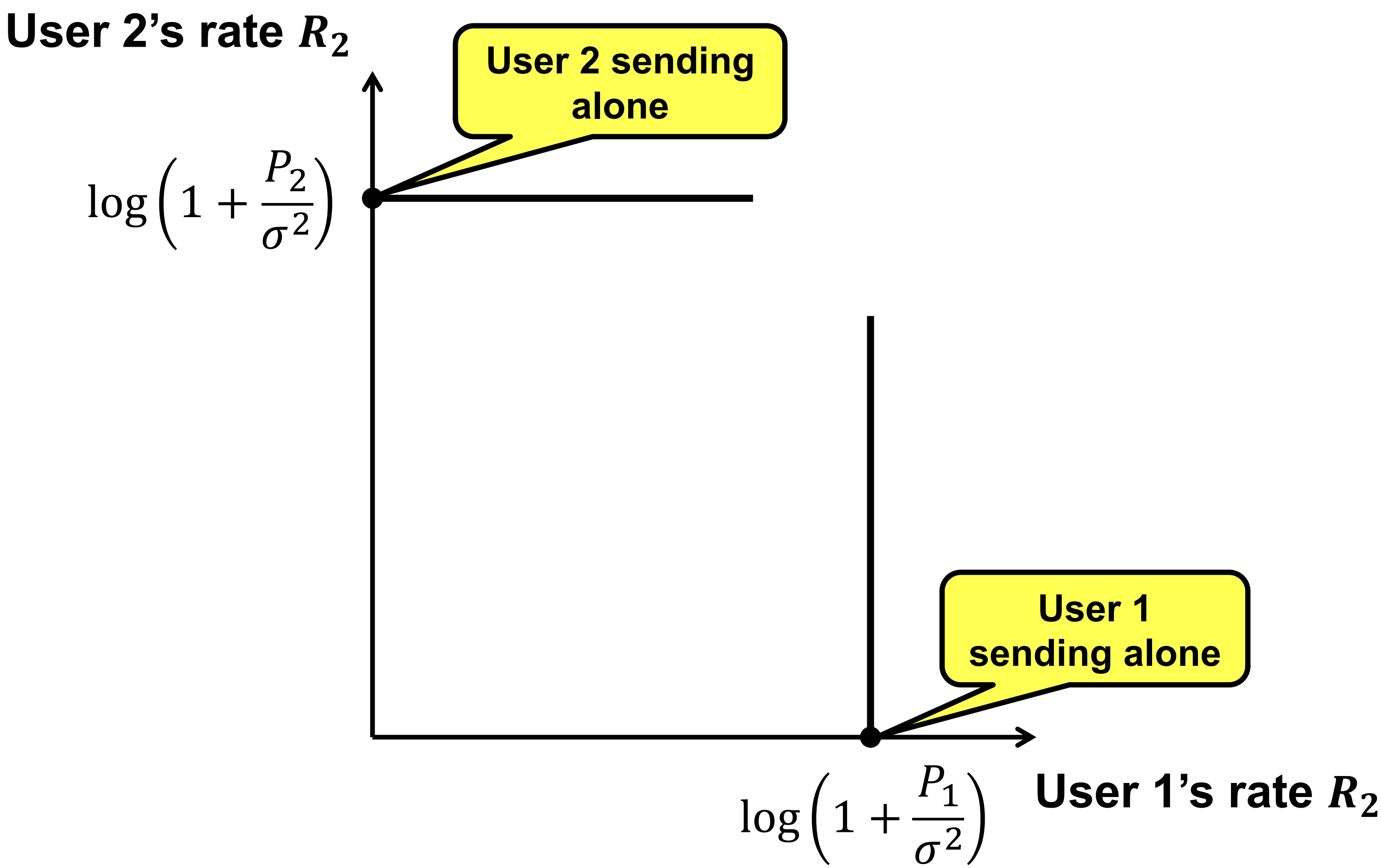
- **What are the fundamental limits of communication here?**

Extension of Capacity to Multiple Users

- **Single-channel** Shannon capacity is a **single rate** (bits/s/Hz)

- Generalizing for two users **capacity** becomes a ***region***:
  - Set of all pairs (R_1, R_2) such that **simultaneously,**
    - User 1 can reliably communicate at rate R_1 **and**
    - User 2 can reliably communicate at rate R_2
  - **Tradeoff between reliable communication rates:**
    - If **User 1** wants to **increase** its rate, **User 2** may need to **decrease** its rate

Two-User Interference Channel: Single-User Bounds



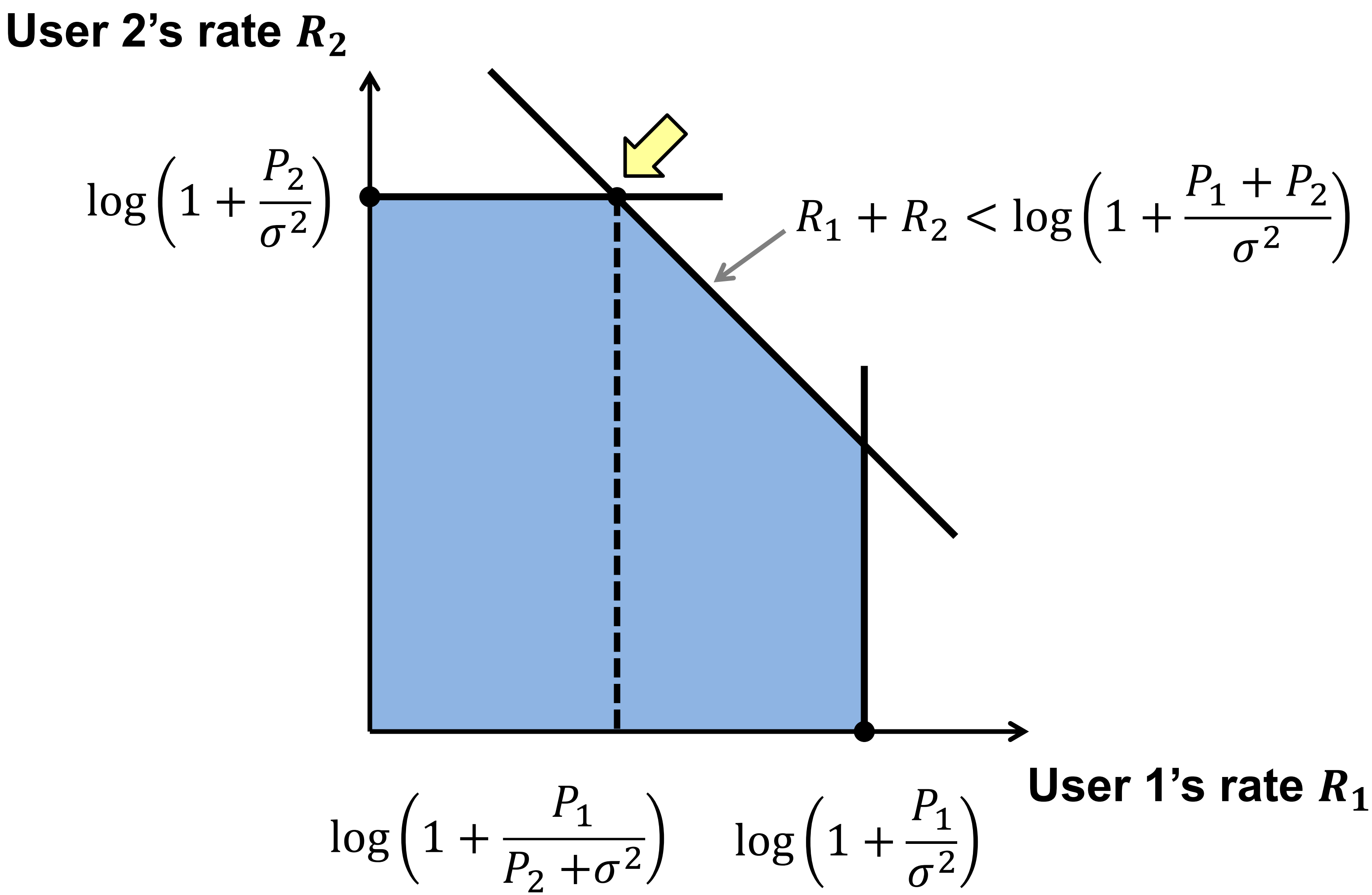
Interference Doesn't Help

- **Assumption:** User 1's data are completely independent from User 2's data, and vice-versa

- **Thought exercise:** Point-to-point link sending with power $P_1 + P_2$
  - Must outperform interfering link (otherwise interference helps)

- So therefore, $R_1 + R_2 < \log\left(1 + \frac{P_1 + P_2}{\sigma^2}\right)$

Two-User Interference Channel: Capacity Region



Successive Interference Cancellation (SIC)

- Receiver decodes information **from both senders** in three stages:

1. Decode data of user 1, **treating signal from user 2 as noise**

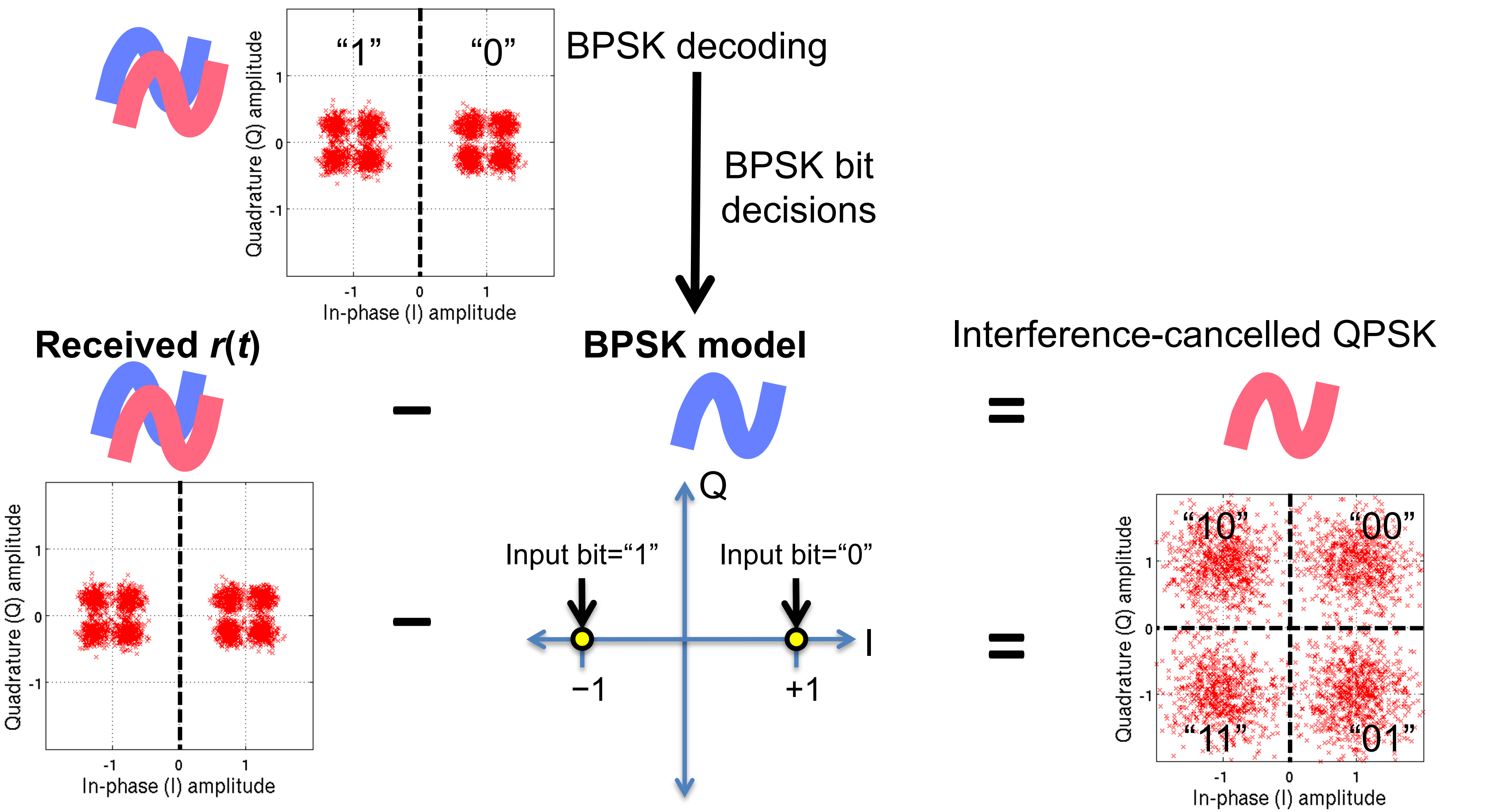
2. Reconstruct **user 1's** signal ($x'_1[m]$) from decoded data and **subtract from aggregate** received signal $y[m]$, **cancelling it:**

$$y'[m] = y[m] - x'_1[m]$$

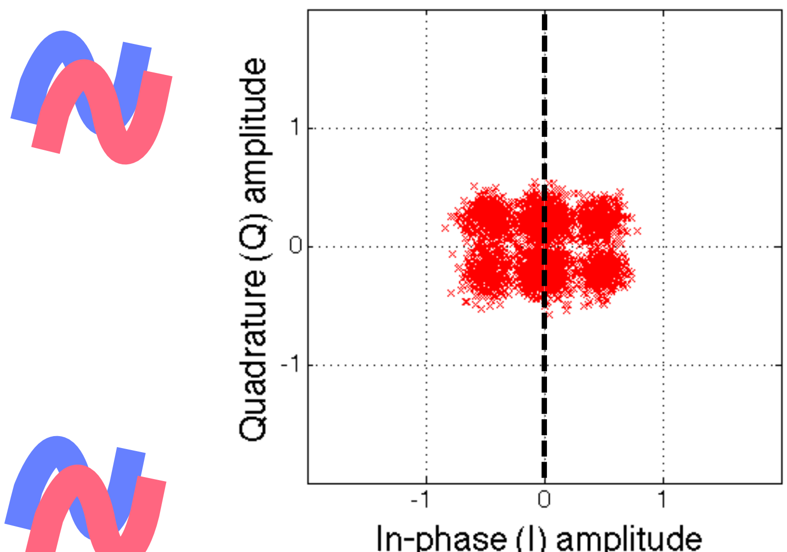
$$= x_2[m] + (x_1[m] - x'_1[m]) + w[m]$$

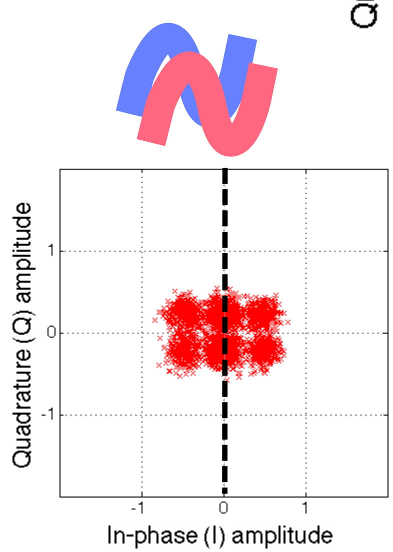
3. Decode **user 2's** signal from y'[m]

Decoding Strong BPSK with weak QPSK



Power Difference Helps Superposition Coding





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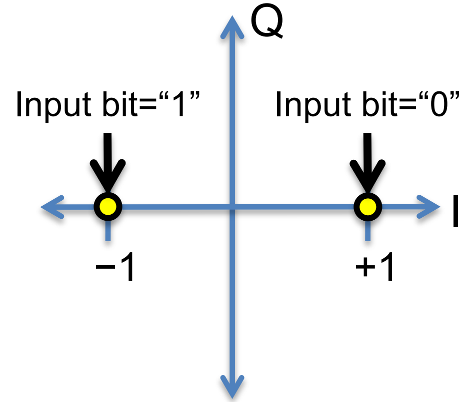
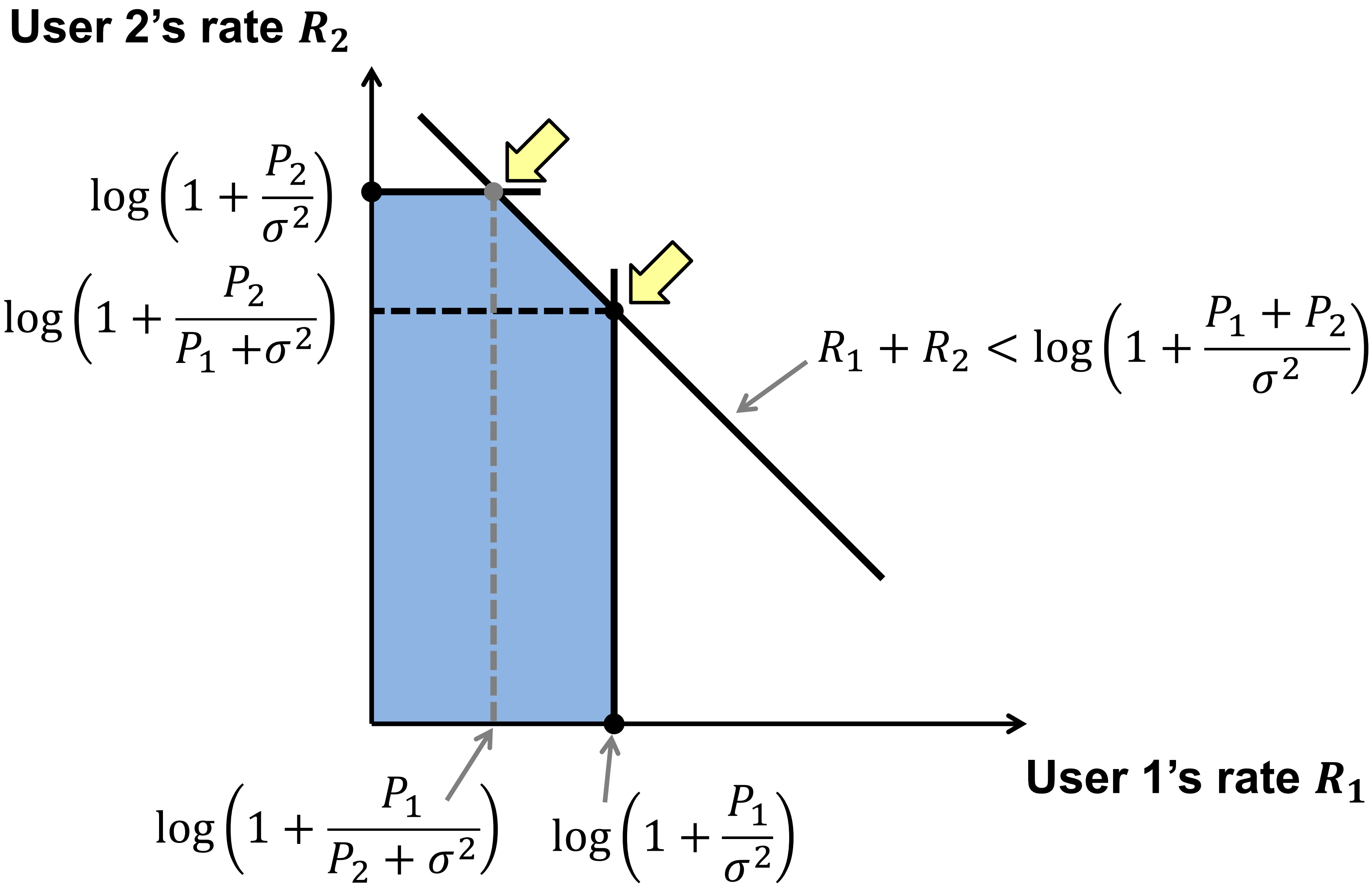
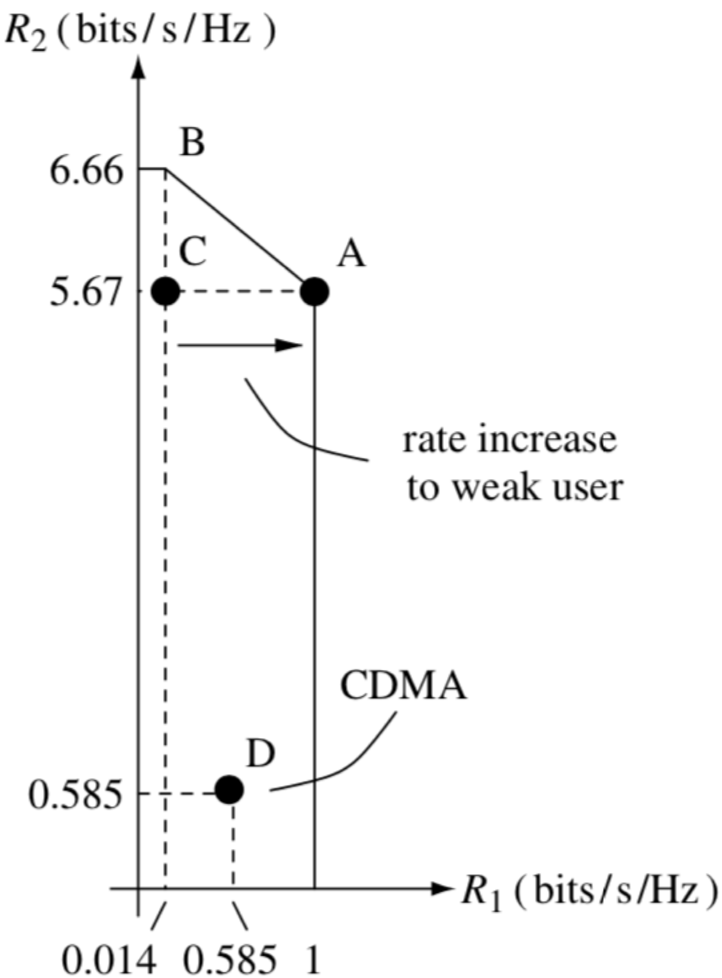


SIC: Choice of User Order



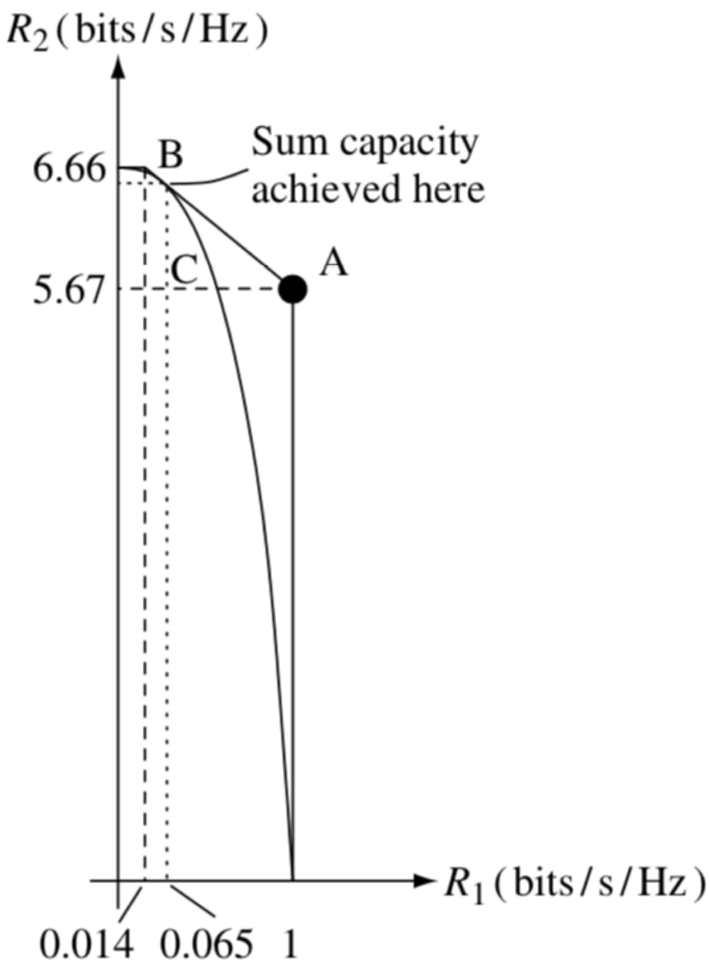
Comparison with CDMA



- **CDMA:** Every user decoded treating the other users as noise
  - Achieves **Point C**
    - **But, User 1 starves**

- **CDMA power control:** Reduce power of the strong user
  - **Achieves Point D**

Comparison with TDMA/FDMA + Power Control



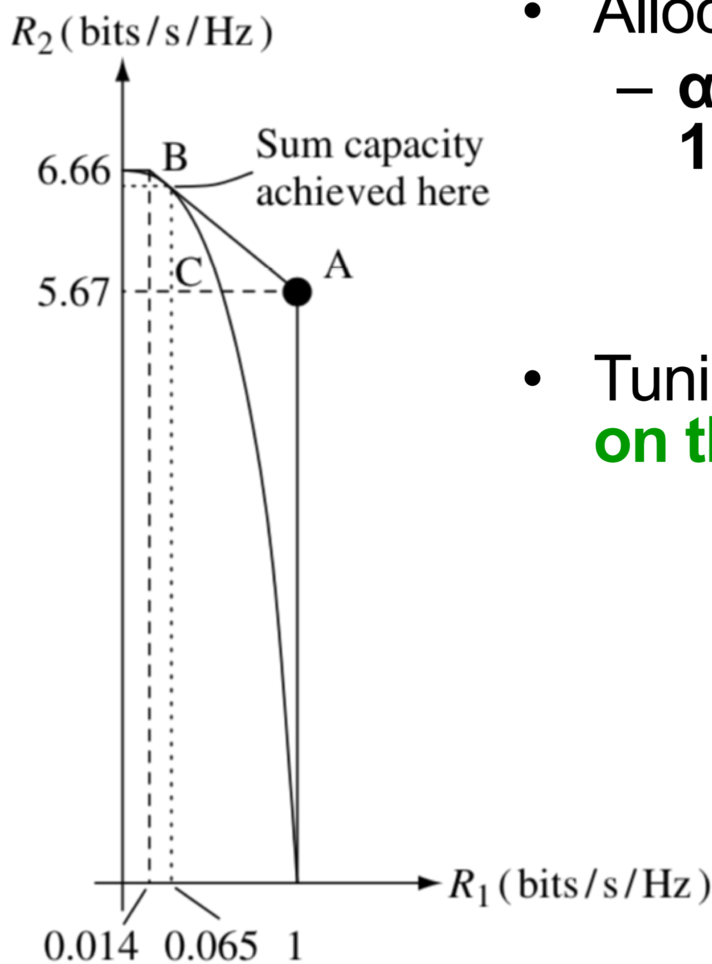
- Allocate **α** time- or frequency-**fraction** to **User 1; 1 − α** to **User 2**
- **Scale each user's power** according to allocated proportion
- User 1 maximum rate:

$$\alpha \log\left(1 + \frac{P_1}{\alpha\sigma^2}\right)$$

- User 2 maximum rate:

$$(1-\alpha)\log\left(1 + \frac{P_2}{(1-\alpha)\sigma^2}\right)$$

Comparison with TDMA/FDMA + Power Control



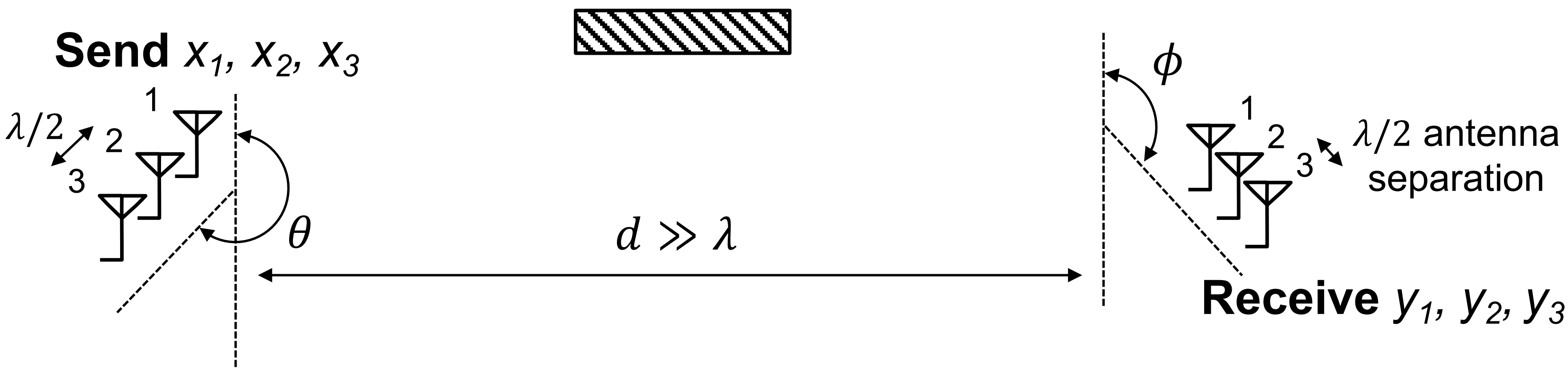
- Allocate as follows:
  - **α** time- or frequency-fraction to **User 1; 1 − α** to **User 2**

- Tuning α, the users **can achieve a point on the (optimal) A-B line**

Today

1. Interference Channel Capacity
2. **MIMO Channel Capacity**
3. Interference Alignment

Review: The MIMO Channel

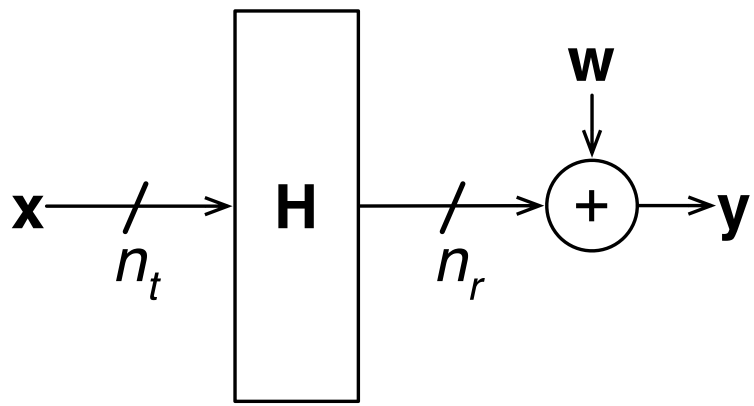


- Transmit **three symbols** per symbol time: $\vec{\boldsymbol{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
- Represent the MIMO channel as $\vec{y} = \mathbf{H}\vec{\boldsymbol{x}} + \vec{\boldsymbol{w}}$
  - $\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{11} \\ h_{21} & h_{22} & h_{11} \\ h_{31} & h_{32} & h_{11} \end{bmatrix}$ is the MIMO ***channel matrix,*** $\vec{\boldsymbol{w}}$ noise

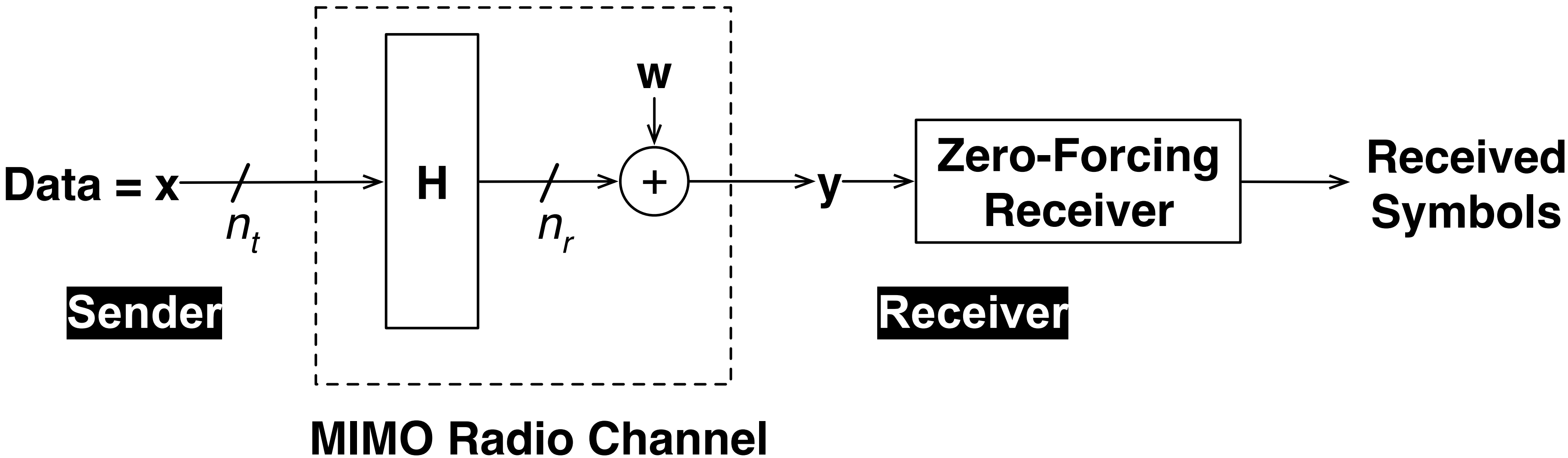
Recap: MIMO Radio Channel

- MIMO **link** with ***n_t* transmit**, ***n_r* receive** antennas

- MIMO radio **channel itself:**

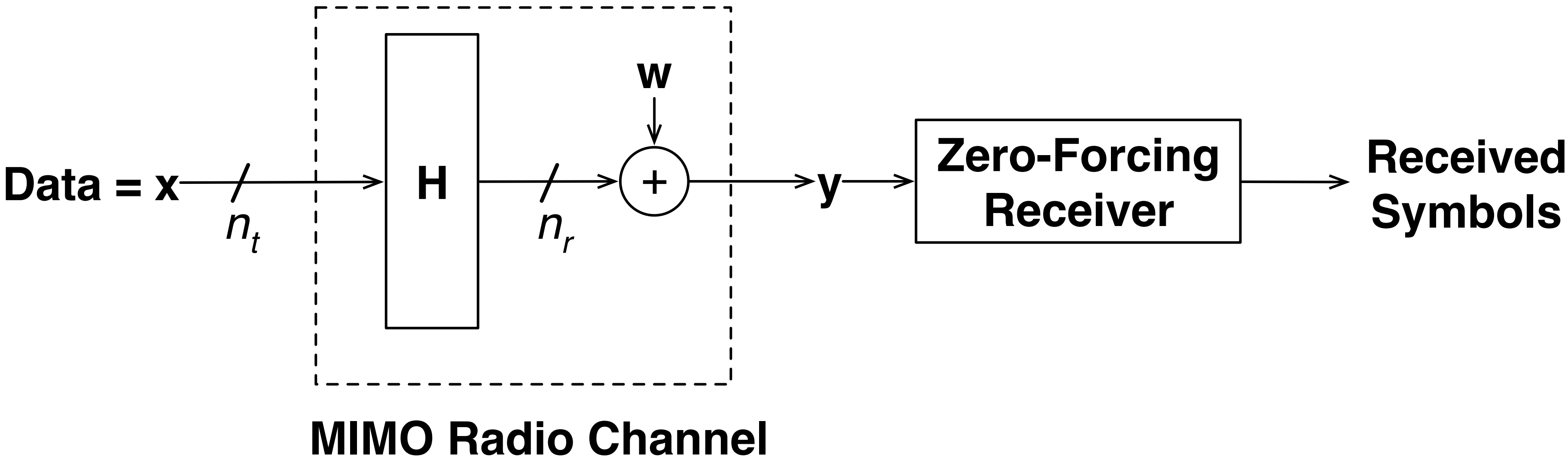


Recap: Zero-Forcing MIMO



- **Transmitter** does not know **H** (channel)
- Each symbol time:
  - Sends n_t symbols (**original Data**), one per transmit antenna
- Data arrives **mixed together** at receiver antennas **y**

Recap: Zero-Forcing MIMO



- **Receiver** knows **H** (channel)

- Each symbol time:
  - Receive n_r mixed-up signals ***y***
  - **For each** of the n_t transmitted symbols:
    - Zero-Forcing Receiver **nulls all but that symbol**

Today

1. Interference Channel Capacity

2. **MIMO Channel Capacity**
   - **Vector Space Intuition**
   - Eigenmode Forcing via Singular Value Decomposition

3. Interference Alignment

MIMO Channel Capacity: Motivation

- **The story so far:** **Copy** data into $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ each symbol time
  - Looked at when this performed **well, poorly**
    - **Answer: MIMO channel conditioning** ← **"Rich multipath environment"** around sender, receiver

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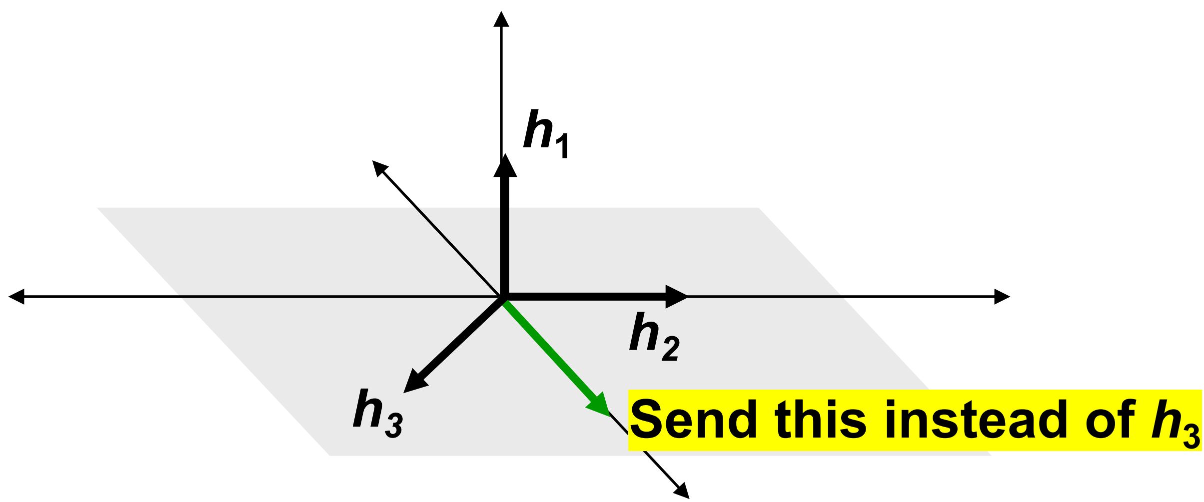
- **Today's first topic:** Is this the **best bits/seconds/Hz possible?**
  - *What's the capacity of a MIMO channel?*
    - Similar question: **Shannon capacity** of a single-input, single-output (**SISO**) **channel**

Where's the Room for Improvement?

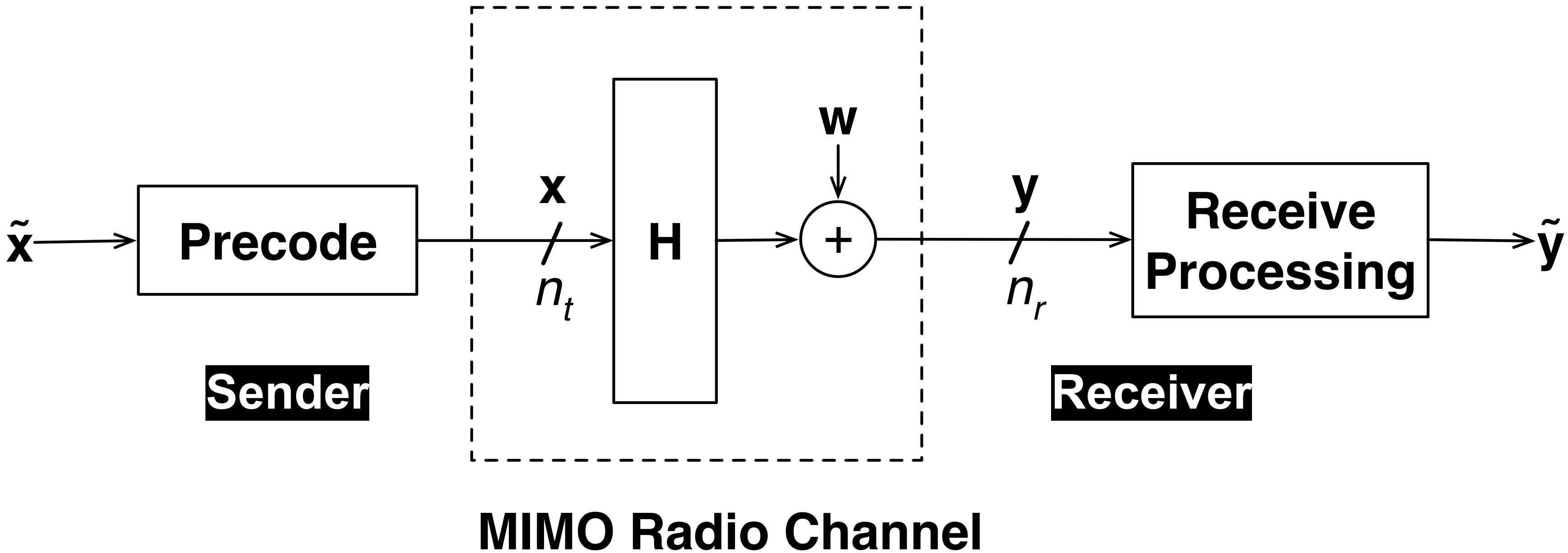
- Suppose the **transmitter knows** **H** (channel)
- Zero-forcing receiver heard $\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_3}$
  - **Power loss at receiver** (due to $\text{Proj}_\perp$) for $\boldsymbol{h_3}$



- **Idea:** Use transmit antennas 2 and 3 to send the ideal direction
  - **No longer** simply **one symbol, one transmit antenna**

How Might We Control Directions?

- Sender ***precodes*** data $\tilde{x}$ into **actual transmission in desired directions x**
- Receiver processing changes accordingly



What Kind of Precoding?

- Recall, we wanted to make **independent channels** on each **wireless channel path**

- Suppose **H** were diagonal:

$$\mathrm{H} = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{n_t} \end{bmatrix}$$

- Then the $\boldsymbol{y_k}$ channel output would only depend on $\boldsymbol{x_k}$
  - **Parallel, independent channels**

Today

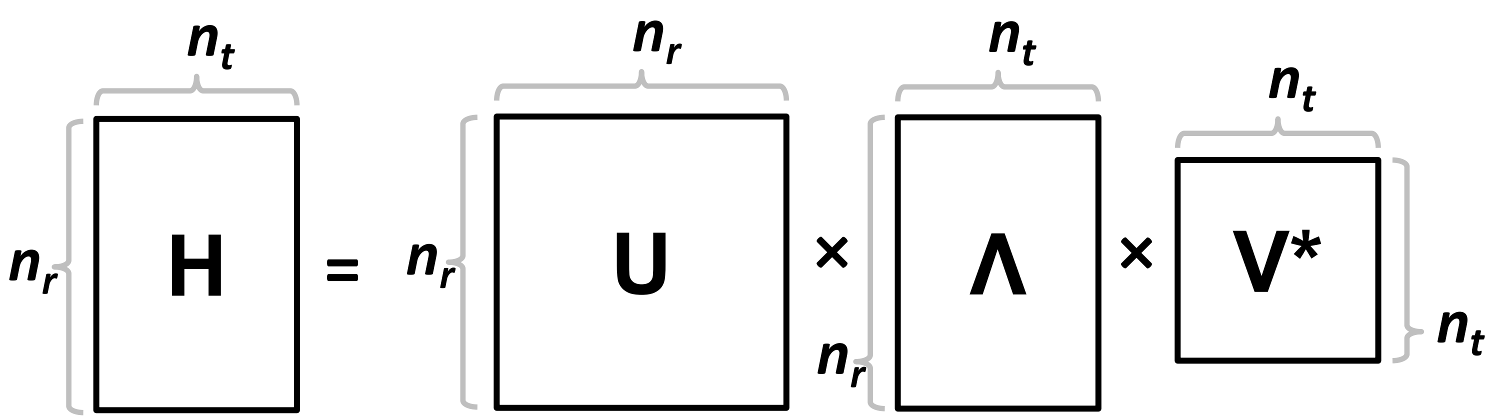
1. Interference Channel Capacity

2. **MIMO Channel Capacity**
   – Vector Space Intuition
   – **Eigenmode Transmission**

3. Interference Alignment

Singular Value Decomposition (SVD)

- The insight lies in a special way of “factoring” matrix H
- Any matrix **H** has an SVD: **H → UΛV***
  - **Λ** is a diagonal matrix (contains **zeroes off-diagonal**)
  - **U** and **V** are *unitary* (**UU* = U*U = VV* = V*V = I**)

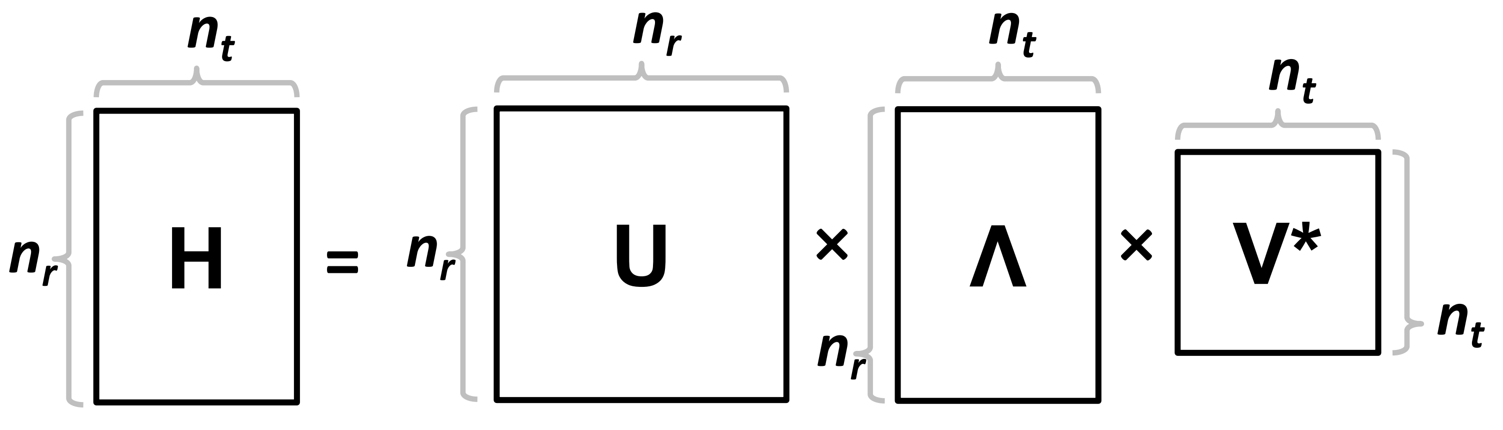


Interpreting the SVD Steps

- **Λ** matrix with the $m = \min(n_t, n_r)$ ***singular values*** $\lambda_1, \cdots, \lambda_m$
  - **One per** significant radio channel **path**

- **V* translates** to the **radio channel path coordinate system** where channels are decoupled

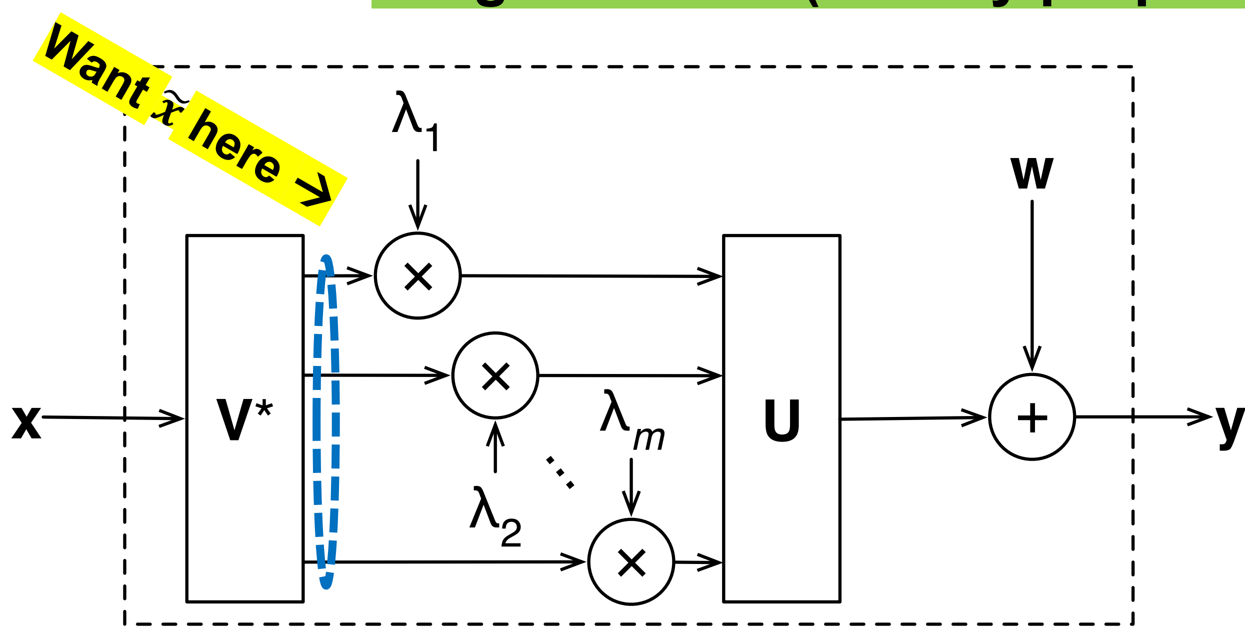
- **U translates back,** to antenna coordinate system (undoes the **V*** translation)



Leveraging the SVD in a Practical System

- **Alone, SVD does nothing** (just analyzes what **H** does)
- Want to put data into the **radio channel coordinate system**

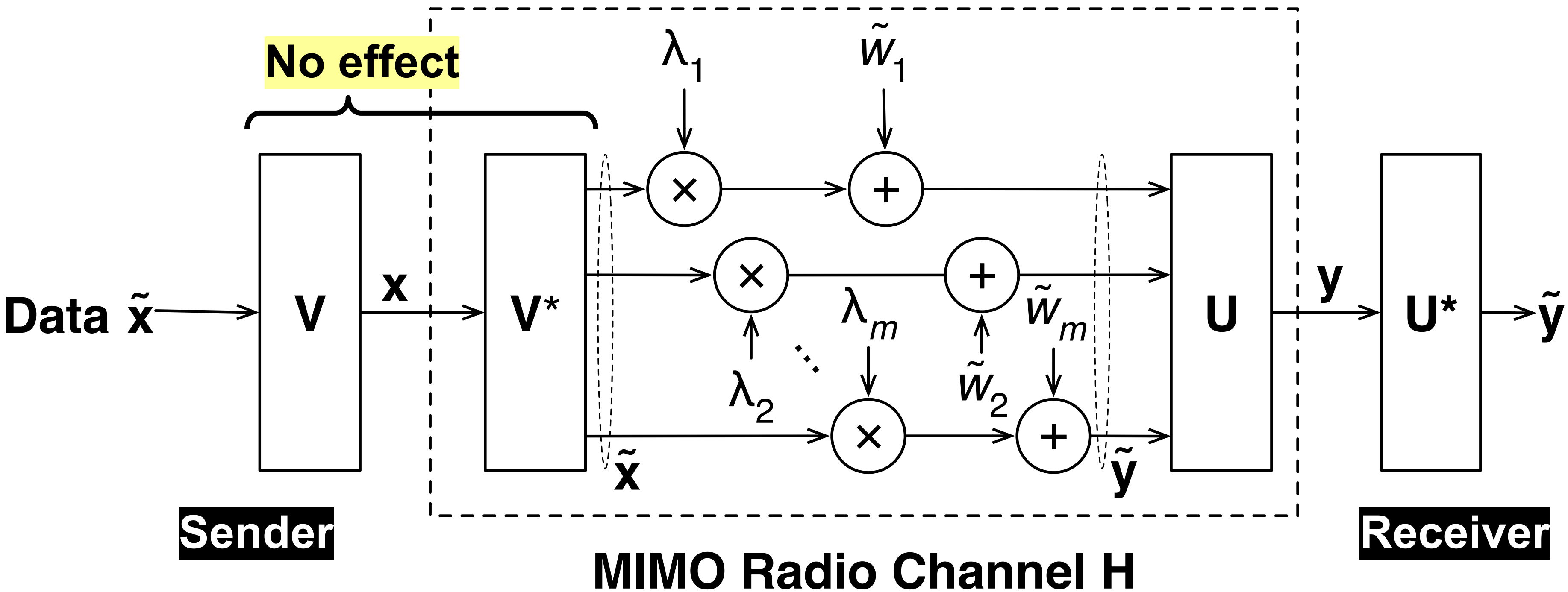
Insight: VV* = I (Unitary property)



MIMO Radio Channel H

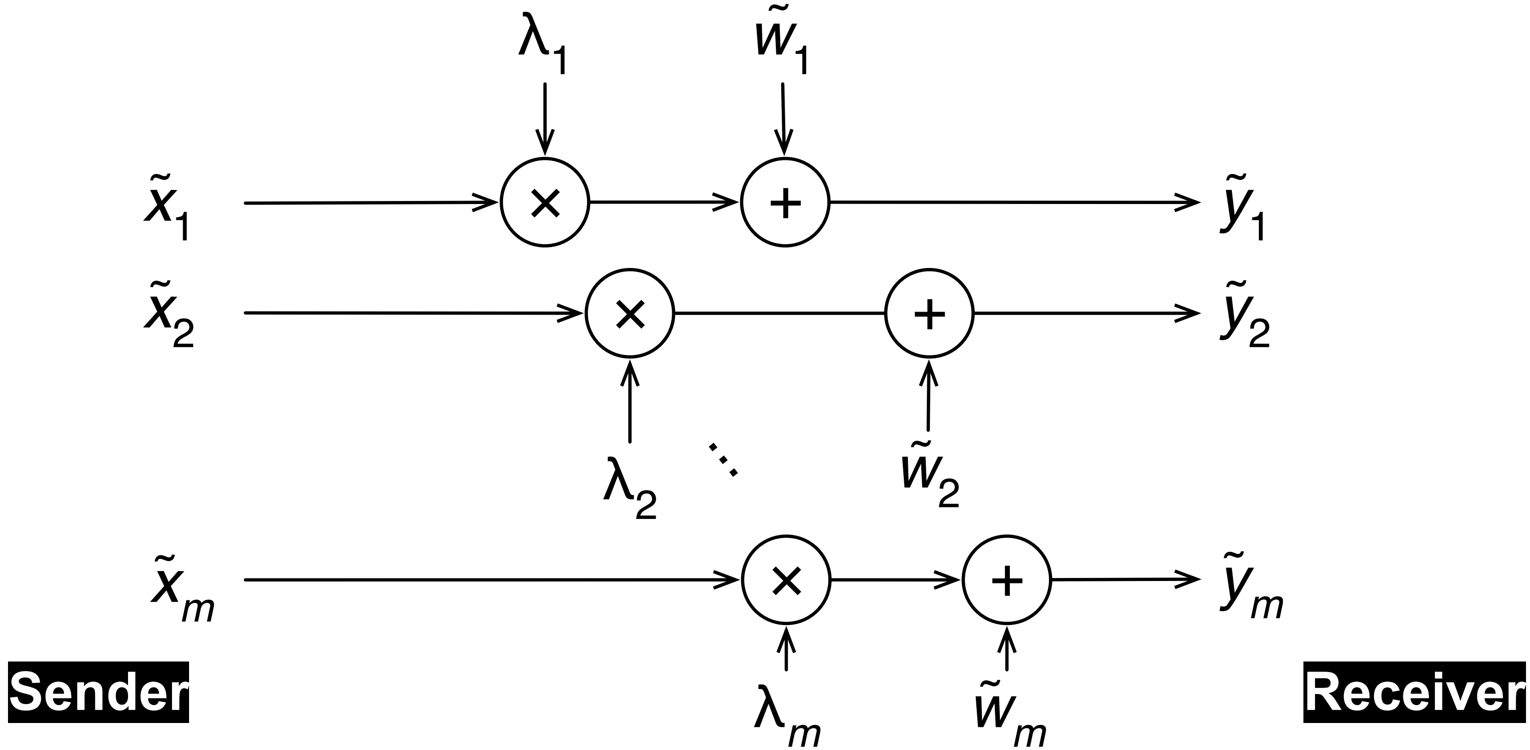
Leveraging the SVD in a Practical System

- Sender precodes with **V,** receiver “post-codes” with **U***
  - V is unitary, so **V*V** = **I** (same for U)
    - So data sees **independent channels**
- This is called ***MIMO eigenmode transmission***



A Model for Eigenmode Transmission

- **Performance model** for the eigenmode transmitter/receiver
- All channels **decoupled**, transmit power P_k → SNR on i th channel: $\frac{P_i \lambda_i^2}{\sigma^2}$



Performance: Uniform Power Division

- At high SNR (the common case in wireless LANs), with ***total transmit power P*** **evenly divided over spatial paths**

- Data rate = $\sum_{i=1}^{k} \log\left(1 + \frac{P\lambda_i^2}{kN_0}\right) \approx k \log(\text{SNR})$

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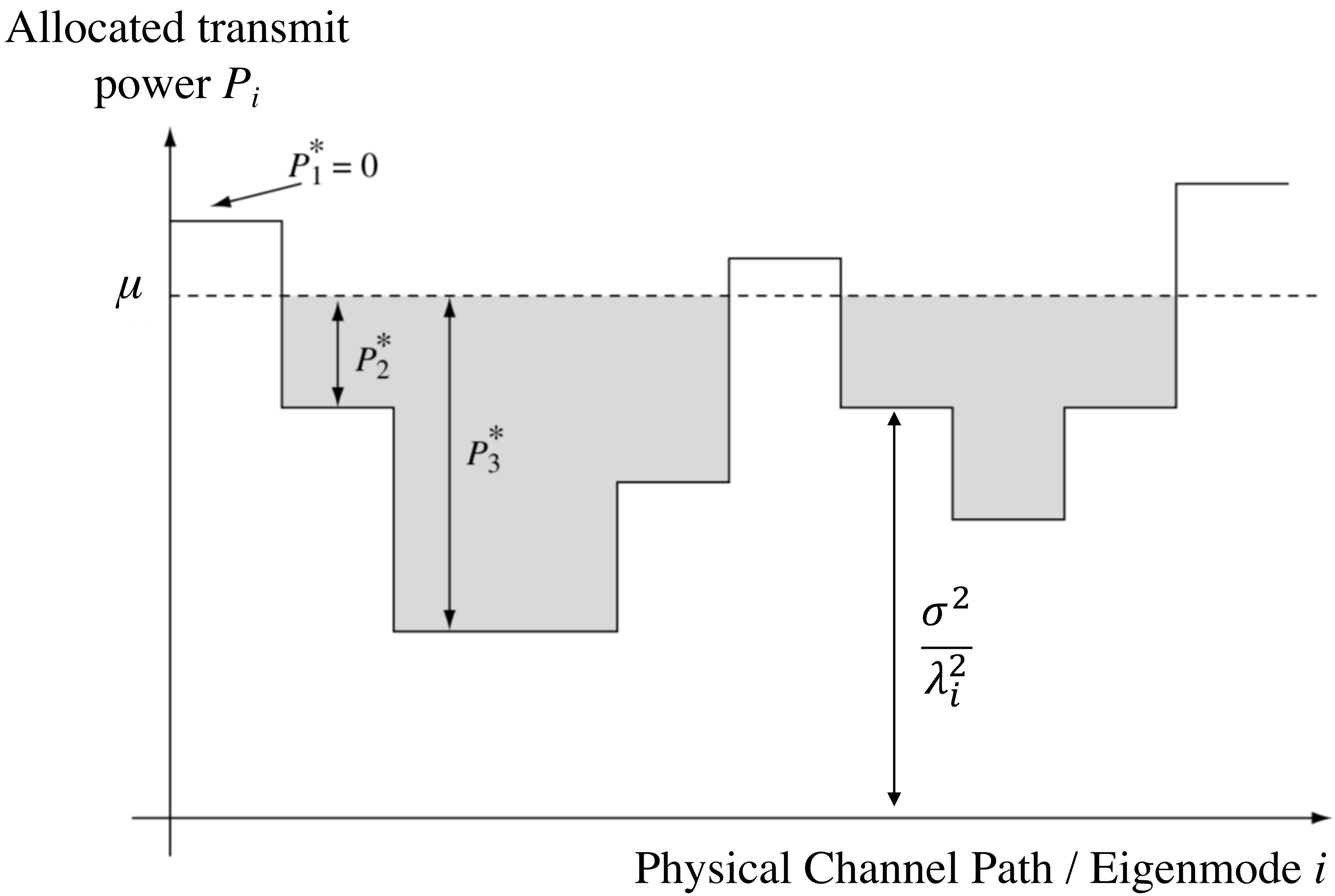
- *How can we do better?*
  - **Idea:** Allocate **different transmit powers** P_i to different radio channel paths *i*
  - Problem we've **seen before in 463** in **OFDM context**

Waterfilling for MIMO Power Allocation



MIMO Capacity: Takeaways

- **OFDM – MIMO analogy:**

  A **transformation** (**OFDM: FFT**, **MIMO: SVD**)
  renders **interfering channels** in
  (**OFDM: frequency**, **MIMO: space**) **independent**

- **MIMO Eigenmode transmission:**
  - Transmitter **sends directionally**, along spatial paths of the radio channel
  - Receiver **listens directionally**, along same spatial paths
  - Achieves the **MIMO channel capacity**

Today

1. Interference Channel Capacity

2. MIMO Channel Capacity

3. **Interference Alignment**

Interference Alignment (IA)

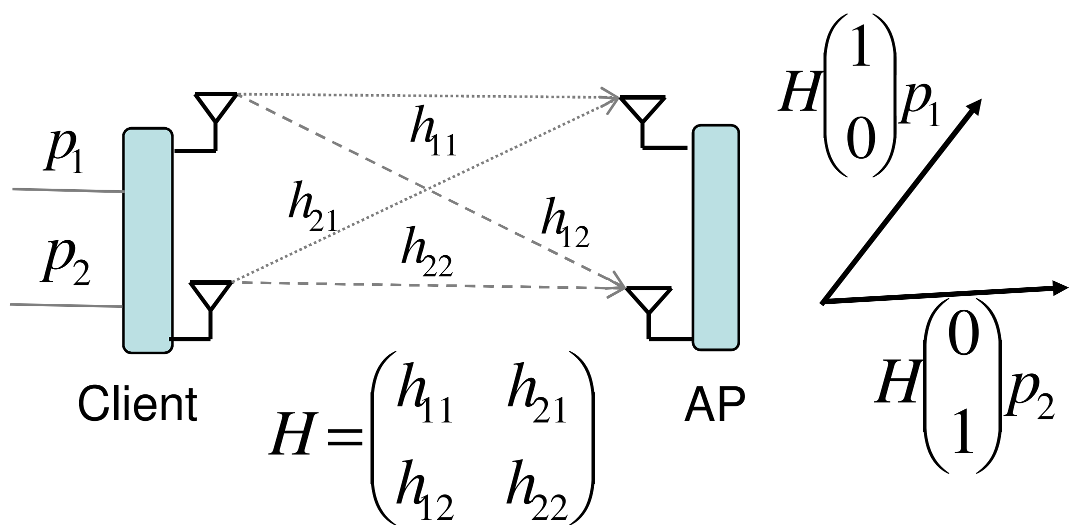
- **Number of concurrent MIMO streams** a client can send is **limited by the number of antennas**
  - Sending more streams results in **interference between streams**
  - Also limited by the amount of multipath in the environment

- **New Idea:** Use MIMO precoding techniques to **align interference** at receivers to advantage

- Requires APs **cooperating via a wired backhaul**
  - *e.g.* APs owned by one organization

MIMO channel representation

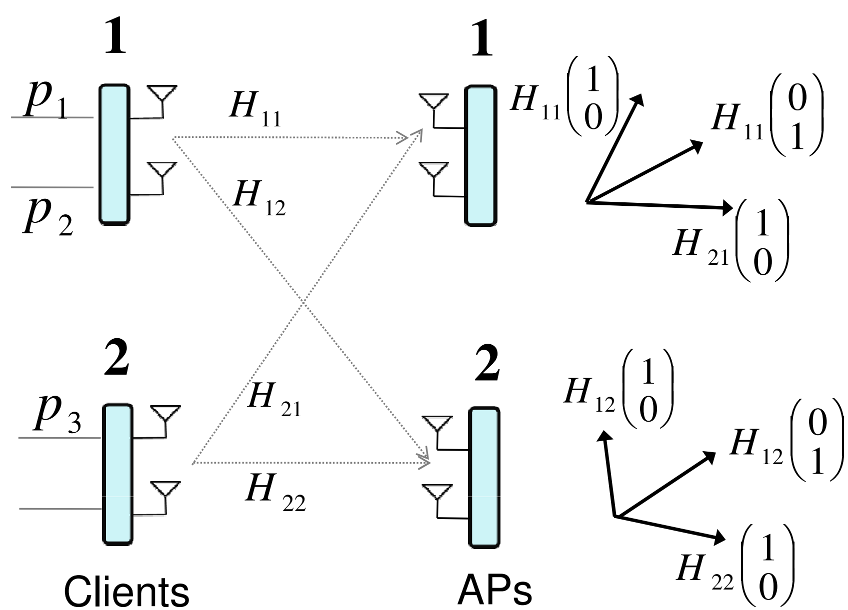
- As before, model channel from one antenna *i* to another *j* as one complex number h_{ij}
- Channel matrix **H** from a client to an AP is formed by $[h_{ij}]$



Uplink: Interference Between Networks

- **Client 1 has 2 packets** for AP 1; **Client 2 has a packet** for AP 2
  - Two-antenna APs, so each decoding in a 2-D space

- Three packets form three vectors in the 2-D space at each AP
  - **Therefore, the APs can't decode these 3 packets**

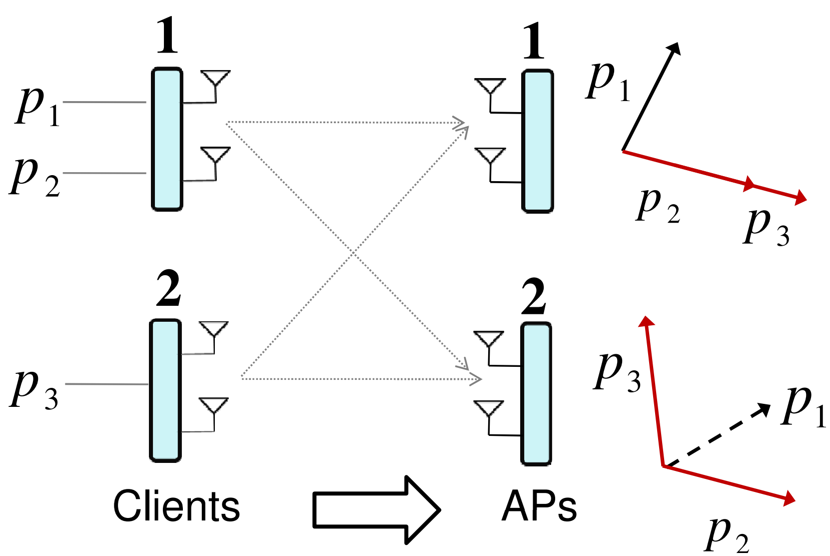


Interference alignment: Basic idea (1)

1. Clients transmit p_2 and p_3 **aligned** at access point (AP) **1**
   – They **add up** in their one direction

2. AP **1** **zero-forces to decode p_1**, sends it **over backhaul to AP 2**

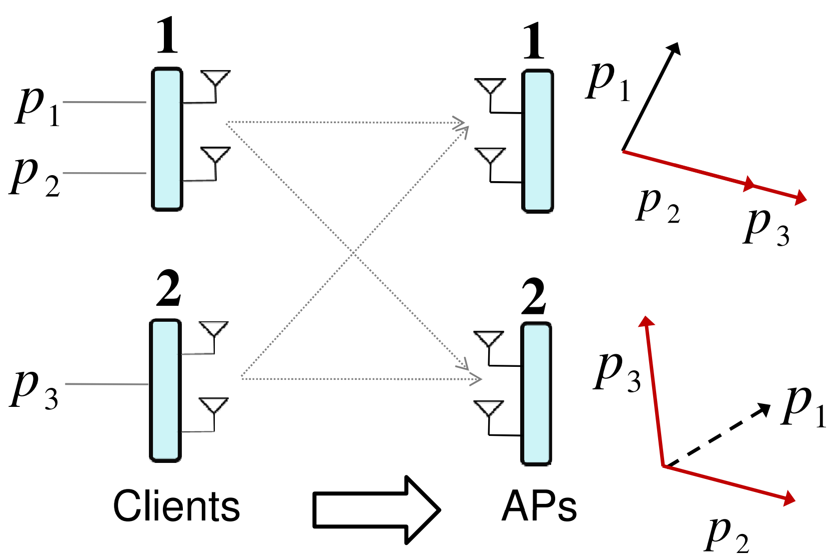
3. AP **2** subtracts p_1 from the signal it receives, **cancelling it**



Interference alignment: Basic idea (2)

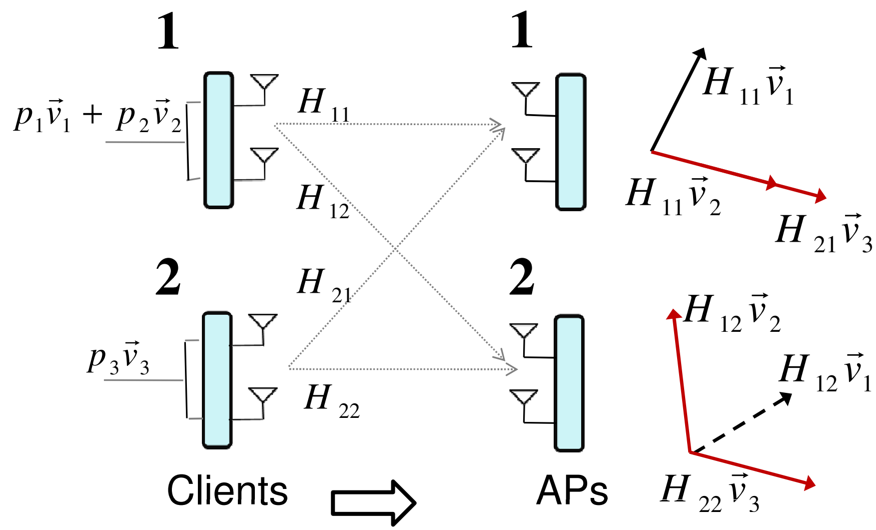
4. AP **2** uses zero-forcing receiver to **decode p_2, p_3**

5. AP **2** sends p_2 to AP **1** (or onward on behalf of client **1**)



Uplink: Sketching a Practical Protocol

- Transmit precoding: client multiplies packet by **vector** *v*
  - **Changes alignment at receiver**



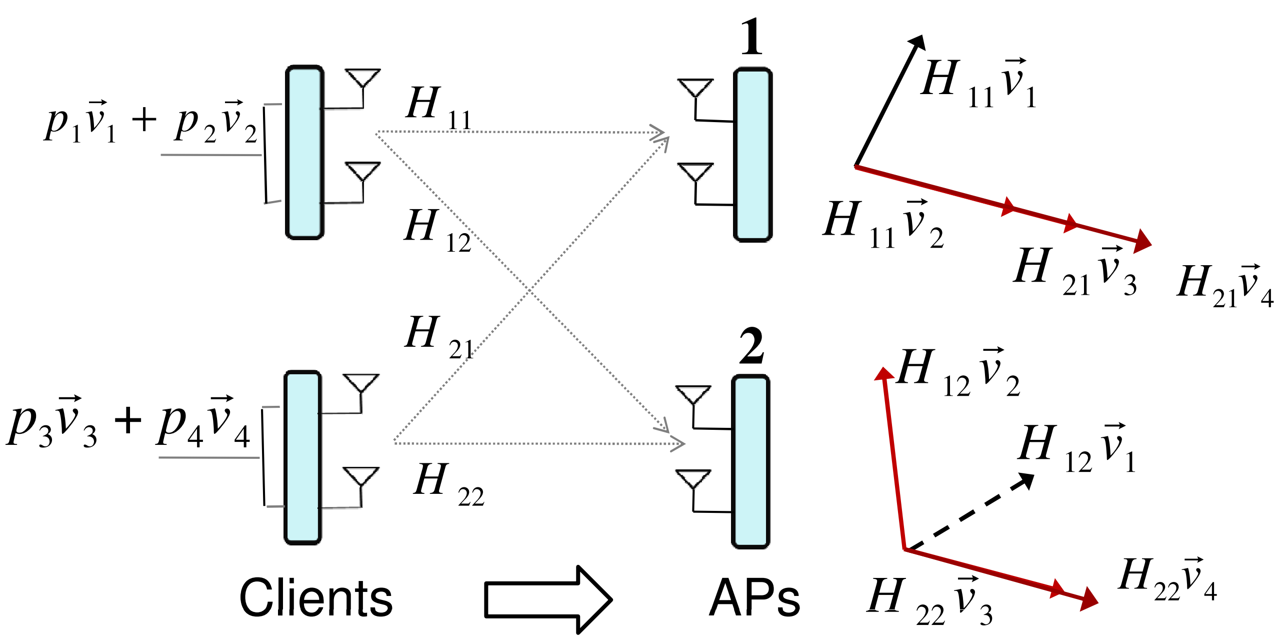
1. **Client 1** picks **random** precoding vectors v_1 and v_2

2. **Client 1** begins transmission

3. **Client 2** chooses v_3 so that $\mathbf{H}_{11}v_2 = \mathbf{H}_{21}v_3$
   – *How does client 2 know* $\mathbf{H}_{11}$ and $\mathbf{H}_{21}$?
     - Client 1 can **include in its packet header**

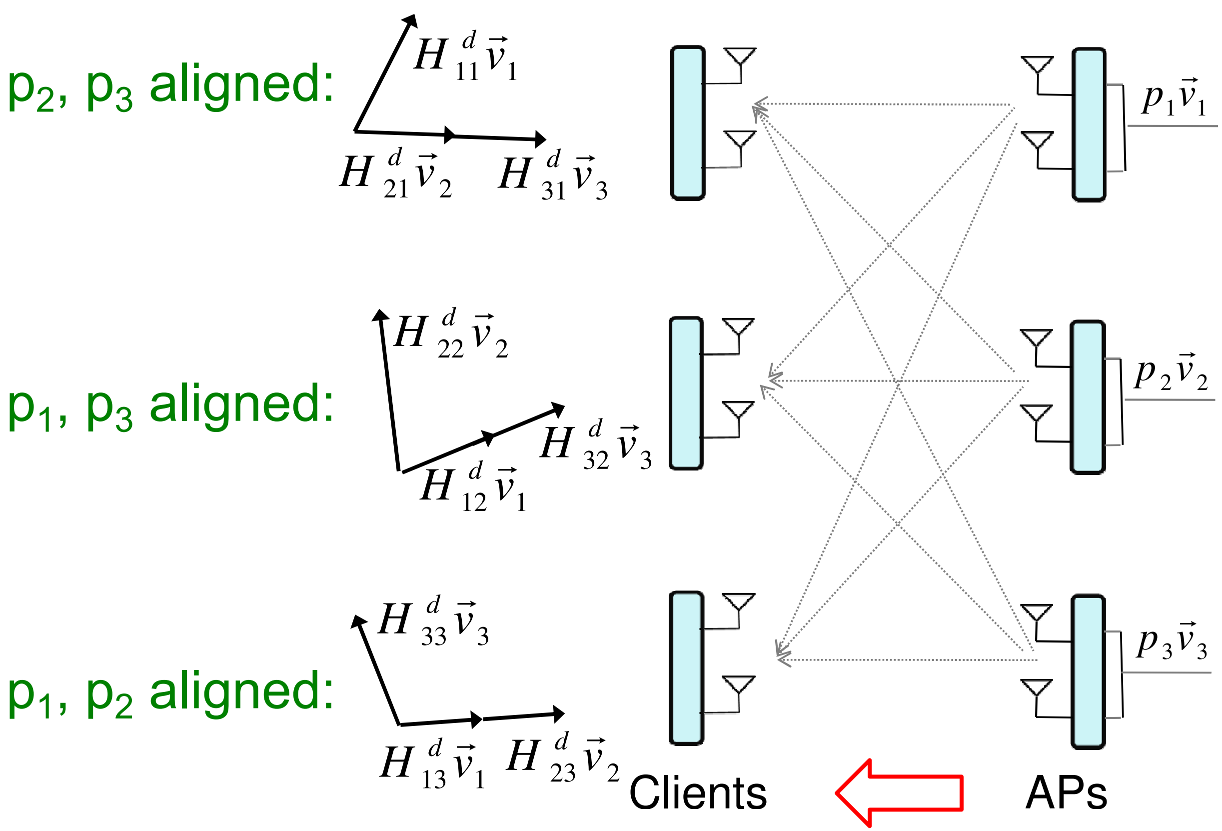
Uplink: Four Concurrent Packets?

- All packets but one (p_1) must align at AP **1**, so AP **1** can decode
- Subtract p_1 from the four packets at AP **2**, leaving three packets
- AP **2** can only decode two packets at a time (2-d space)
  - **Can't decode p_3 and p_4 at AP 2: Can only decode p_1 and p_2**



Downlink Interference Alignment

- Clients **can't exchange frames over backhaul**
- **Instead, align neighboring APs' interference** at each client



Thursday Topic:

Ripple II: Faster Communication through Physical Vibration

(Download & Pre-read paper!)