MIMO II: Physical Channel Modeling, Spatial Multiplexing

COS 463: Wireless Networks

Lecture 17

Kyle Jamieson

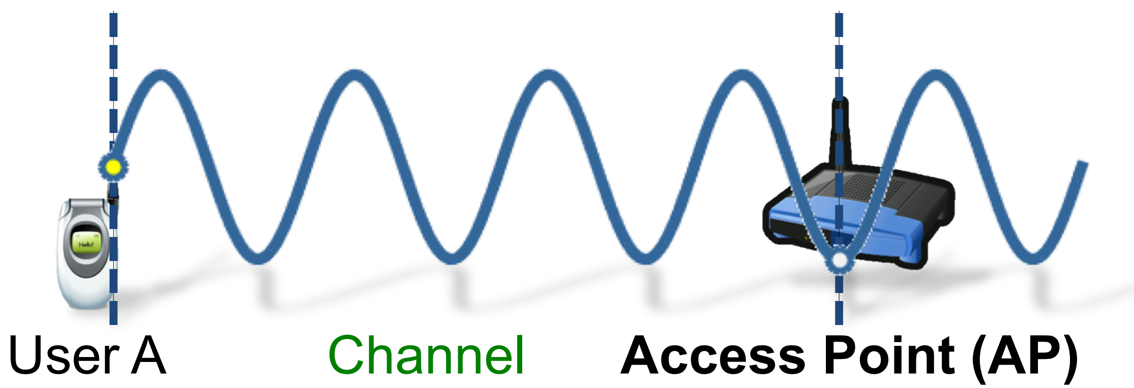
Today

1. **Graphical intuition in the I-Q plane**

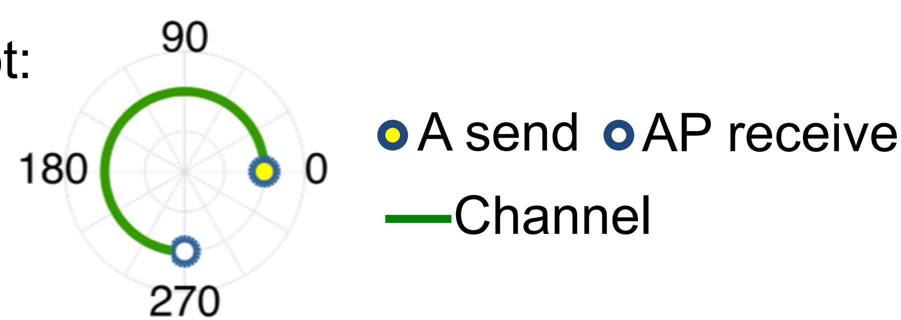
2. Physical modeling of the SIMO channel

3. Physical modeling of the MIMO channel

The problem of wireless interference

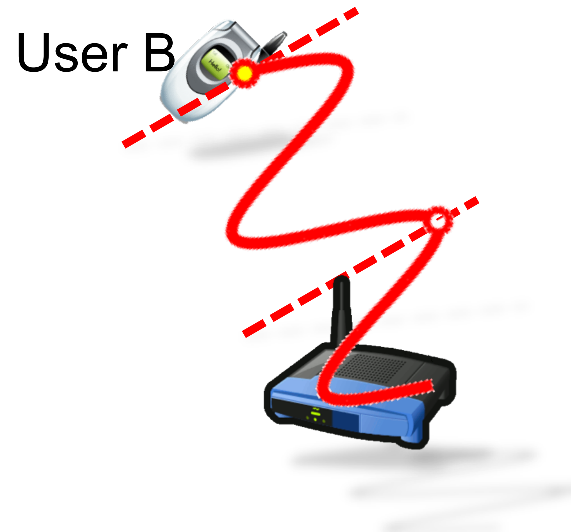


I-Q plot:

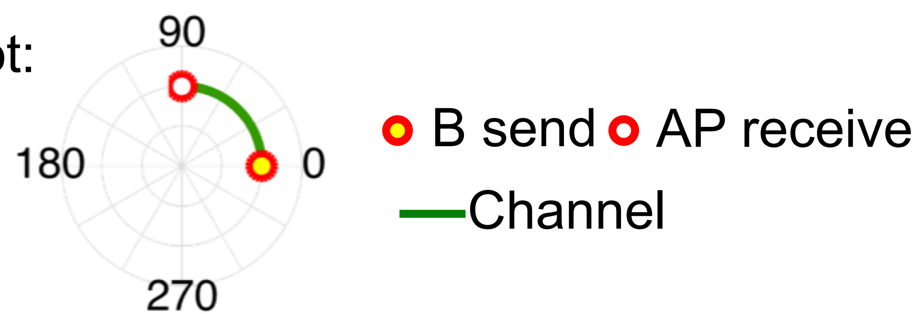


AP can estimate the channel, so can decode User A's signal ( )

The problem of wireless interference

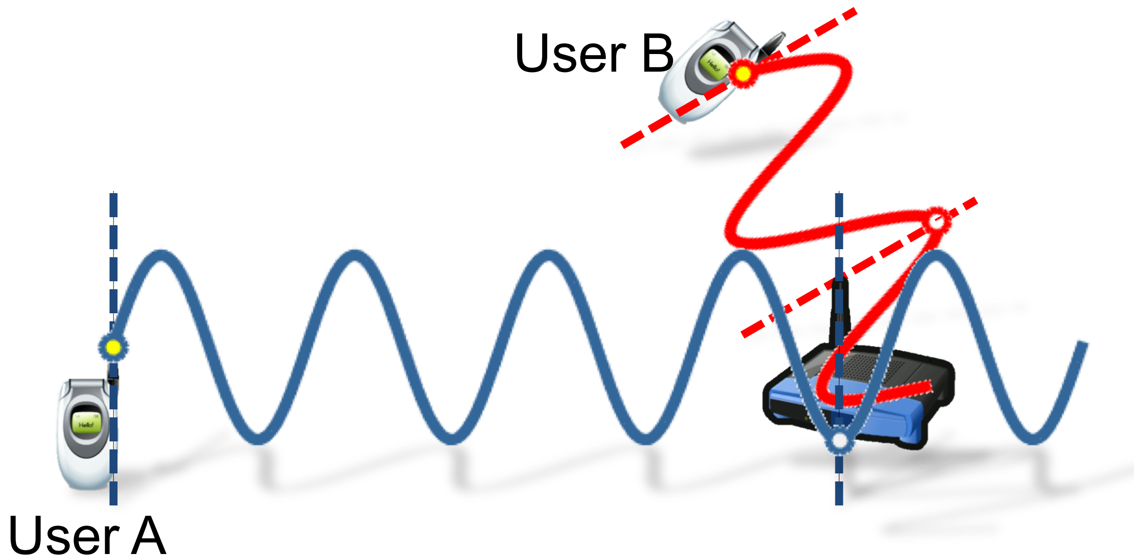


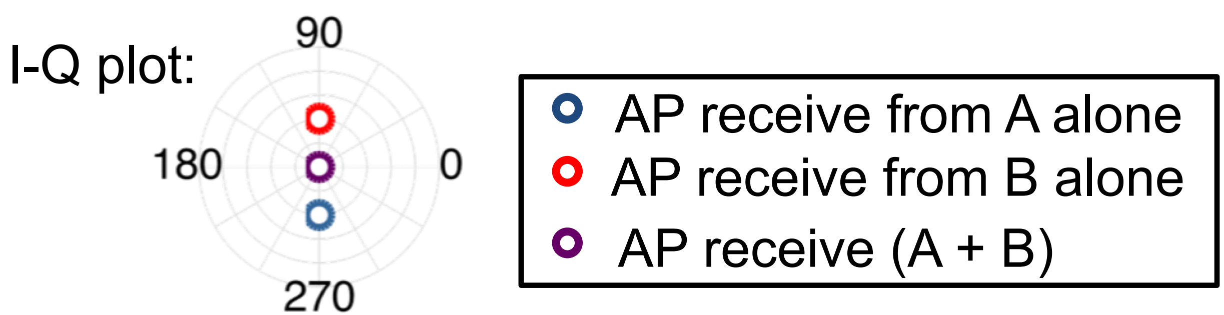
I-Q plot:



AP can estimate the channel, so can decode User B's signal ( )

The problem of wireless interference

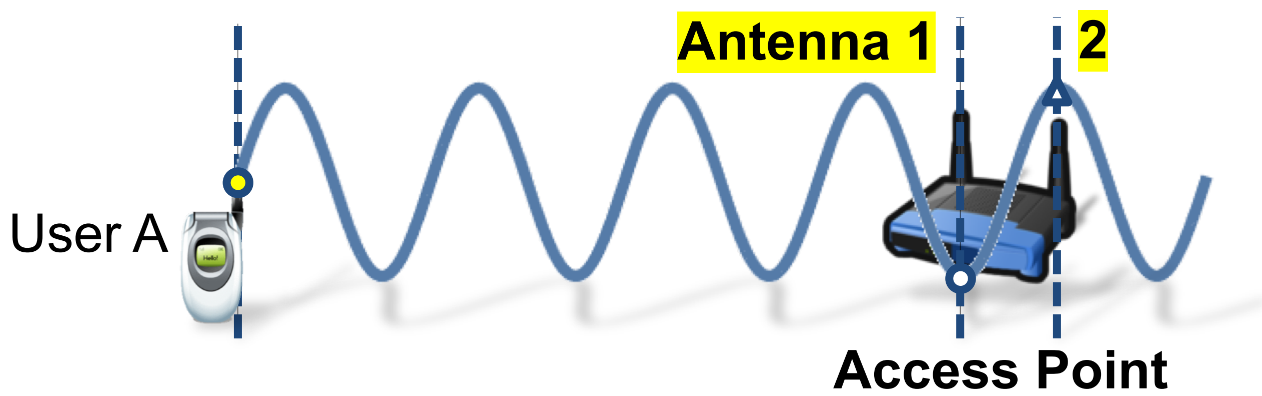


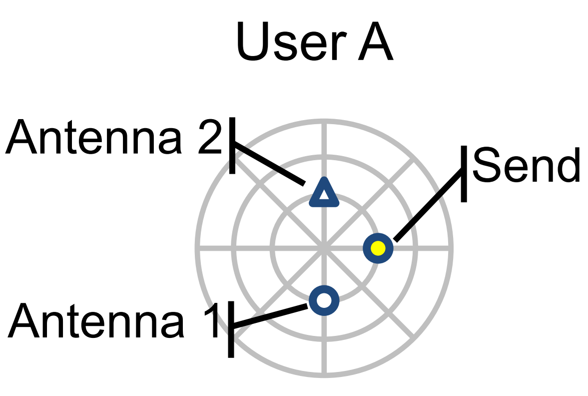


One received signal (), two sent (,), so **AP can't decode**

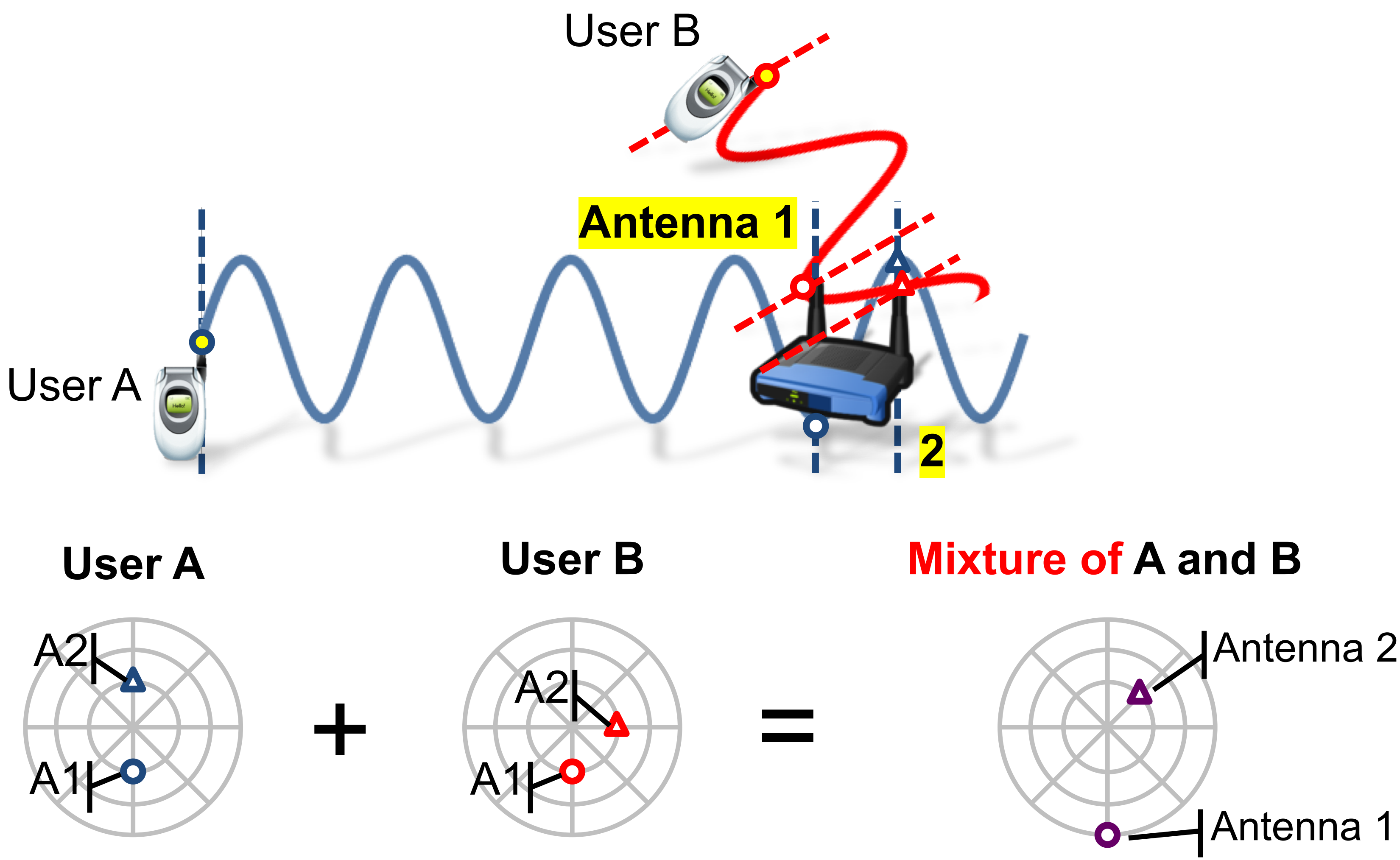
Leveraging Multiple Antennas

- Now, the AP hears **two** received signals, one on each antenna:





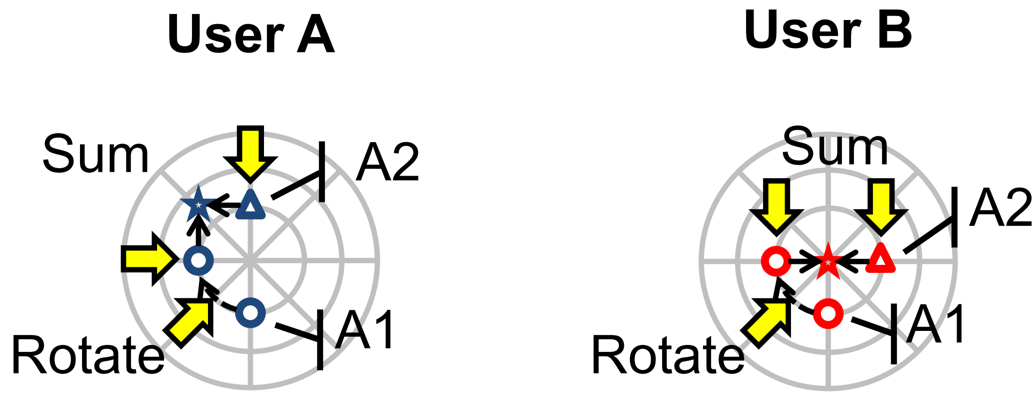
Leveraging Multiple Antennas



Intuition: Zero-Forcing Receiver

- MIMO ***zero-forcing receiver***

1. **Rotate** one antenna's signal (o)
2. **Sum** the two antennas' signals together (o + Δ)



Zero-forcing **cancels** B, **revealing** A

Can re-run to cancel A, revealing B

Spatial Multiplexing: More "Streams"

- Send **multiple streams of information** **over each of the spatial paths** between sender and receiver
  - This is called ***spatial multiplexing***

- **Potential for increased capacity by a factor of *N*** (minimum number of send or receive antennas):

$$C = BN \log\left(1 + \mathrm{SNR}\right) \mathrm{bits/s/Hz}$$

- **Potential for** a **multiplicative rate speed-up**

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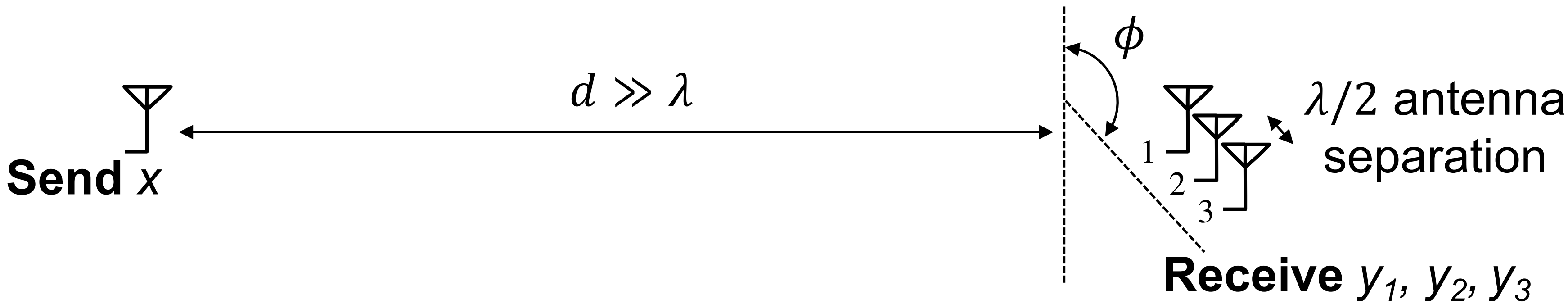
Physical Modeling of Multi-Antenna Channels

- Gain intuition as to how the RF channel (ambient environment) impacts capacity
- Many **physical antenna arrangement geometries** possible
- Limit discussion today to ***linear*** **antenna arrays, half-wavelength antenna spacing**
  - Details vary with more sophisticated antenna arrangements, but concepts do not

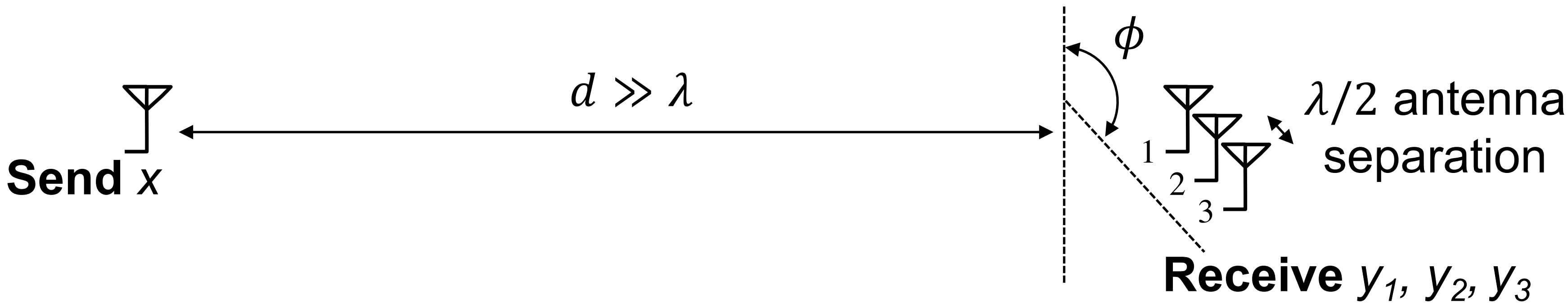
Line-of-Sight SIMO Channel: A Second Look



- **Vector notation** for the system: $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \vec{y} = \vec{h}x + \vec{w}$

- **Wireless channel** is now a **three-tuple vector:** $\vec{h} = \begin{bmatrix} ae^{j2\pi d_1} \\ ae^{j2\pi d_2} \\ ae^{j2\pi d_3} \end{bmatrix}$

Line-of-Sight SIMO Channel: A Second Look



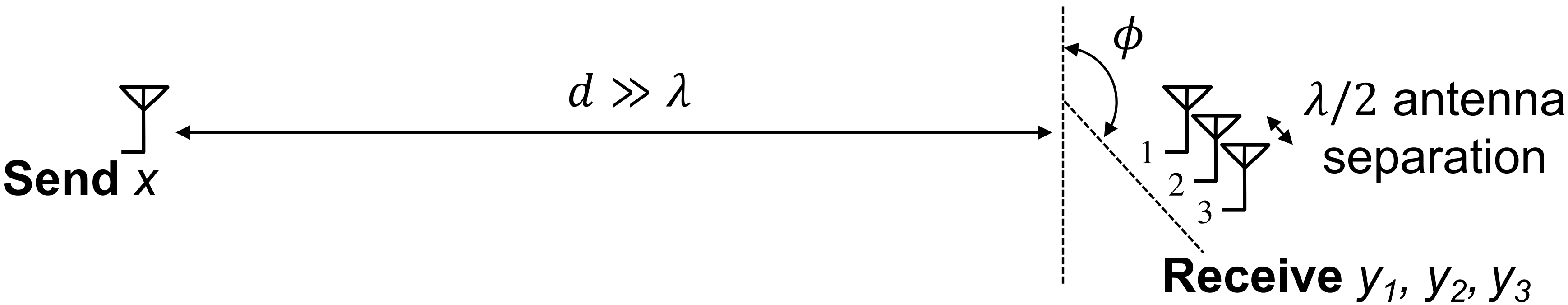
- **Wireless channel** is now a **three-tuple vector:** $\vec{h} = \begin{bmatrix} ae^{j2\pi d_1/\lambda} \\ ae^{j2\pi d_2/\lambda} \\ ae^{j2\pi d_3/\lambda} \end{bmatrix}$

- **Antenna separations:**
  - Assume $d_1 = d$
  - $d_2 \approx d + \frac{1}{2}\lambda \cos\phi$
  - $d_3 \approx d + \lambda \cos\phi$

- **Wireless channel:**

$$\vec{h} = ae^{j2\pi d/\lambda} \begin{bmatrix} 1 \\ e^{j\pi \cos\phi} \\ e^{j2\pi \cos\phi} \end{bmatrix}$$

Line-of-Sight SIMO Channel: Spatial Signature

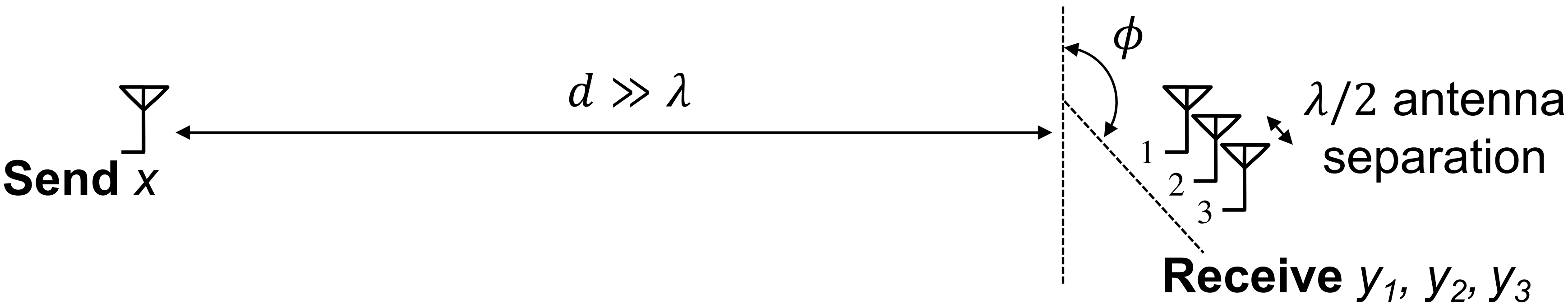


- The wireless channel **decomposes into two components:**

$$\vec{h} = \underbrace{a e^{j2\pi d/\lambda}}_{\text{Path component}} \underbrace{\begin{bmatrix} 1 \\ e^{j\pi \cos\phi} \\ e^{j2\pi \cos\phi} \end{bmatrix}}_{\textit{Spatial Signature}}$$

- The **angle of arrival** of the sender's signal **at the receive array** **determines the spatial signature**

Line-of-Sight SIMO Channel: Maximal Ratio Combining (Review)



- Maximal ratio combining **"projects" the received signals $\vec{y}$ onto the receive spatial signature:**

$$\tilde{y} = \vec{h}^* \cdot \vec{y}$$

- **Reverses the phases** in the spatial signature to **align** each antenna's component of the above sum
  - **SNR improvement** but **no multiplexing**

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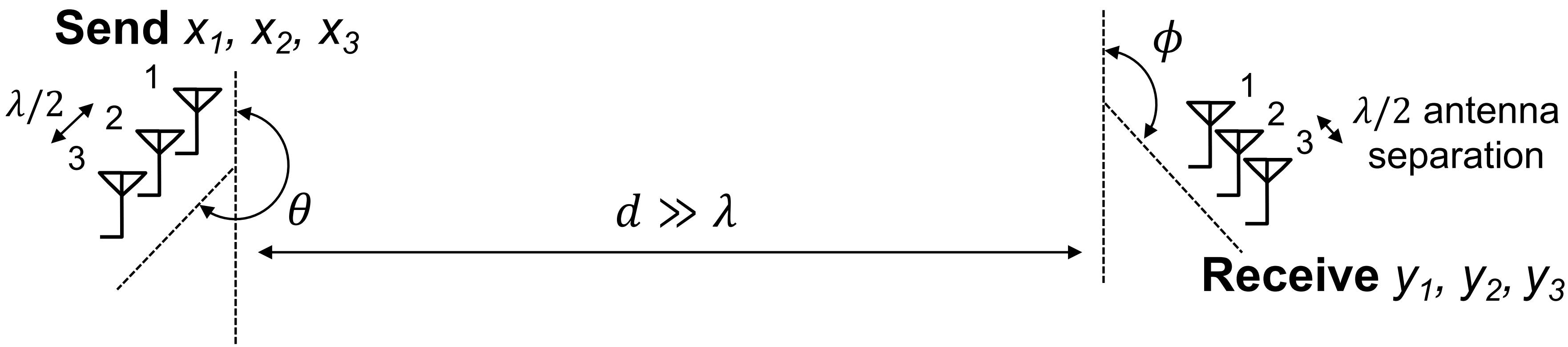
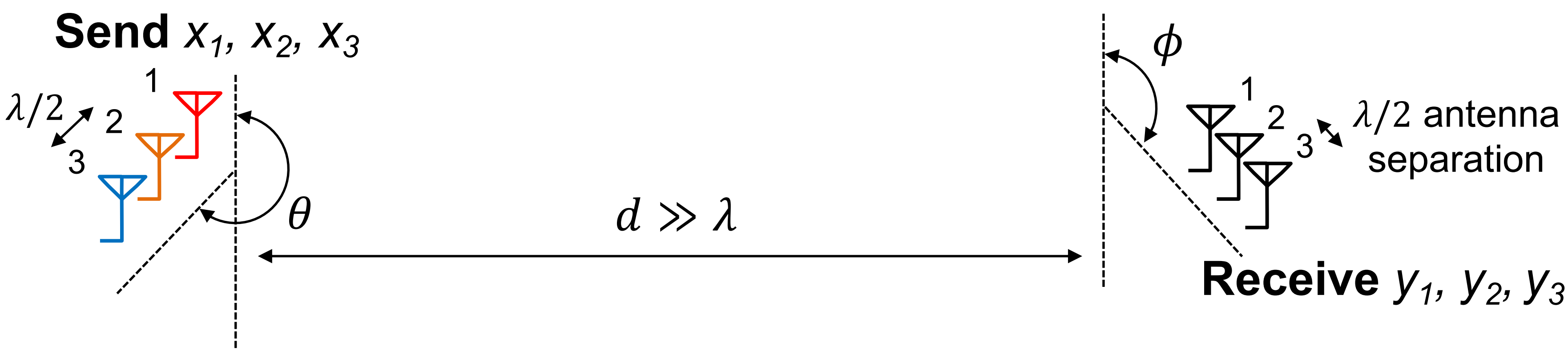
3. **Physical modeling of the MIMO channel**
   - **Line-of-Sight MIMO Channel**
   - Geographically-Separated Transmit Antennas
   - Geographically-Separated Receive Antennas
   - MIMO Link in Multipath

The Line-of-Sight MIMO Channel



- Want to transmit **three symbols** per symbol time: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
- $\boldsymbol{h_{kl}}$: channel between *k* th receive and *l* th transmit antenna
- $\vec{y} = \mathrm{H}\vec{x}$, where $\mathrm{H} = \begin{bmatrix} h_{11} & h_{12} & h_{11} \\ h_{21} & h_{22} & h_{11} \\ h_{31} & h_{32} & h_{11} \end{bmatrix}$ is the MIMO ***channel matrix***

The Line-of-Sight MIMO Channel: Channel Matrix



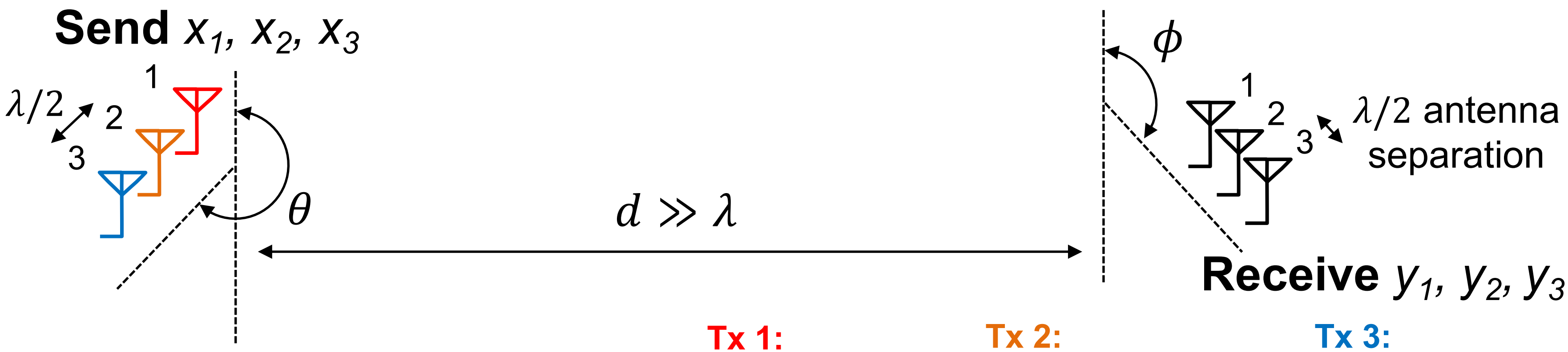
- h_{kl}: channel between k th receive and l th transmit antenna

- Suppose as before, $d_{11} = d$
  - Then $d_{kl} = d + \frac{1}{2}(k-1)\cos\phi + \frac{1}{2}(l-1)\cos\theta$

- **Channel matrix** $\mathbf{H} = ae^{j2\pi d/\lambda}$

$$\begin{array}{ccc} \text{Tx 1:} & \text{Tx 2:} & \text{Tx 3:} \\ \end{array}$$
$$\mathbf{H} = ae^{j2\pi d/\lambda}\begin{bmatrix} 1 & e^{j\pi\cos\theta} & e^{j\pi\, 2\cos\theta} \\ e^{j\pi\cos\phi} & e^{j\pi(\cos\phi+\cos\theta)} & e^{j\pi(\cos\phi+2\cos\theta)} \\ e^{j2\pi\cos\phi} & e^{j\pi(2\cos\phi+\cos\theta)} & e^{j\pi(2\cos\phi+2\cos\theta)} \end{bmatrix}$$

The Line-of-Sight MIMO Channel: Identical Spatial Signatures



- **Channel matrix** $\mathbf{H} = ae^{j2\pi d/\lambda}$

$$
\begin{array}{ccc}
\textbf{Tx 1:} & \textbf{Tx 2:} & \textbf{Tx 3:} \\
\end{array}
$$

$$
\mathbf{H} = ae^{j2\pi d/\lambda}\begin{bmatrix} 1 & e^{j\pi \cos\theta} & e^{j\pi\, 2\cos\theta} \\ e^{j\pi \cos\phi} & e^{j\pi\,(\cos\phi + \cos\theta)} & e^{j\pi\,(\cos\phi + 2\cos\theta)} \\ e^{j2\pi \cos\phi} & e^{j\pi\,(2\cos\phi + \cos\theta)} & e^{j\pi\,(2\cos\phi + 2\cos\theta)} \end{bmatrix}
$$

- **Transmit antenna 2's** channel and **spatial signature:**

$$
\begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = ae^{j2\pi\left(\frac{d}{\lambda} + \cos\theta\right)} \begin{bmatrix} 1 \\ e^{j\pi\cos\phi} \\ e^{j2\pi\cos\phi} \end{bmatrix}
$$

The Line-of-Sight MIMO Channel: Takeaways

- **Spatial signature: How to phase-shift** received signals to align them

- **Spatial signature** of **Transmit antenna 1**
  - Equals spatial signature of **Tx antenna 2**, **Tx antenna 3**

- So any **receiver attempt** to align signal from **Transmit antenna 1**
  - **Also aligns transmit antennas 2 and 3**

Spatial mux fail

- Result is **interference between x_1, x_2, x_3**
  - **Can send same single symbol x** on all transmit antennas
    - Results in **same power gain as MRC**

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   - Geographically-Separated Receive Antennas
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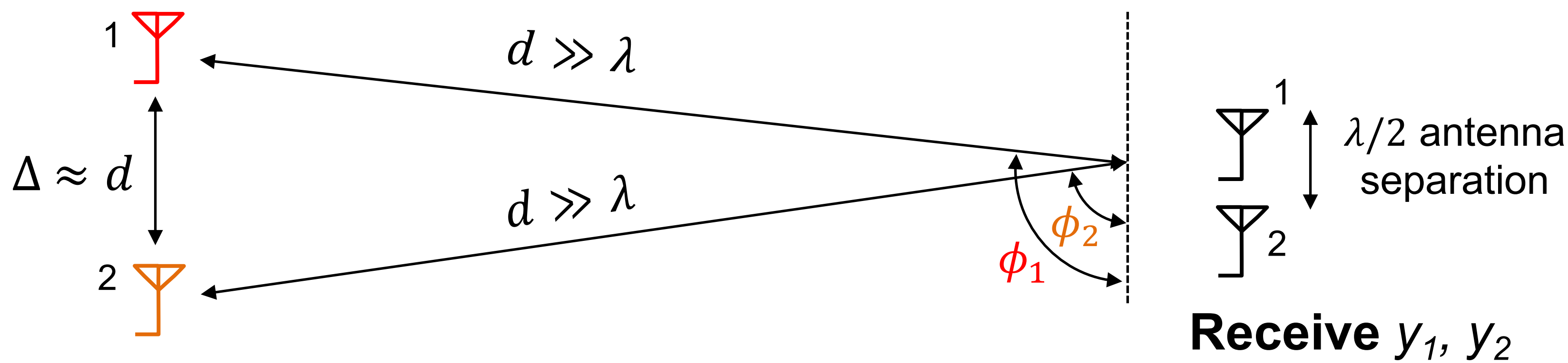
Geographically-Separated Transmit Antennas: *Space-Division Multiple Access (SDMA)*

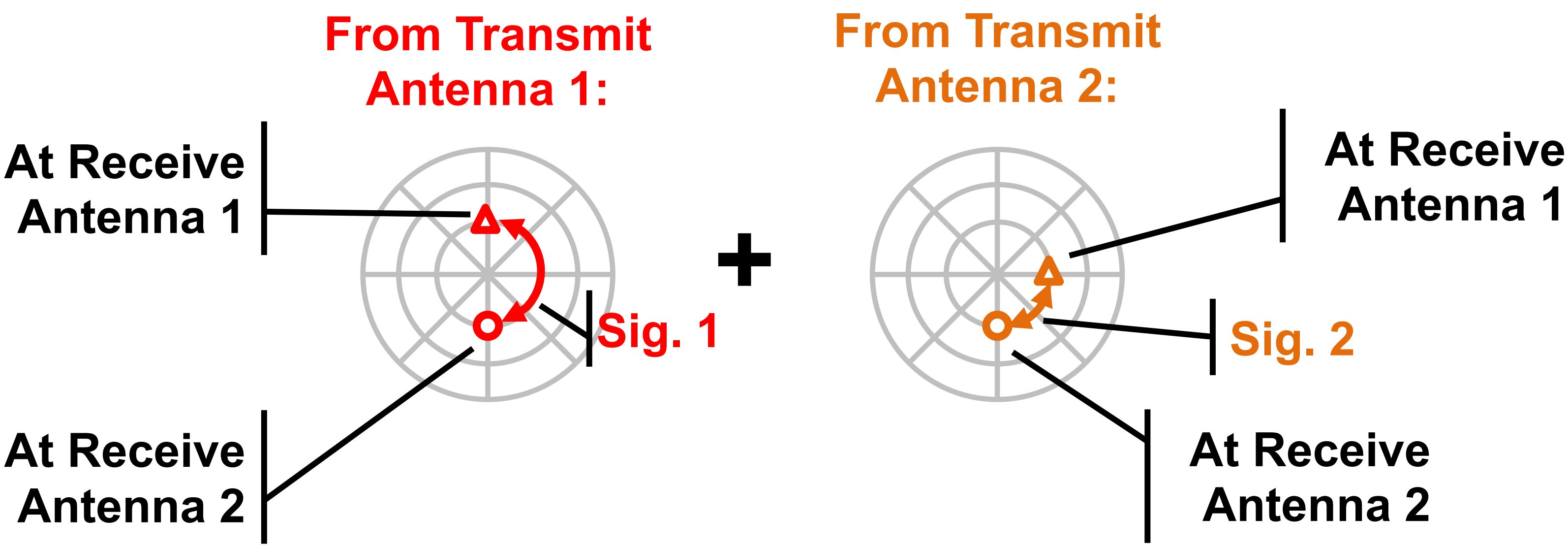


- **Channel matrix** $\mathbf{H} = a e^{j2\pi d/\lambda} \begin{array}{cc} \textbf{Tx 1:} & \textbf{Tx 2:} \\ \left[\begin{matrix} 1 \\ e^{j\pi\cos\phi_1} \end{matrix}\right. & \left.\begin{matrix} 1 \\ e^{j\pi\cos\phi_2} \end{matrix}\right] \\ \underbrace{\hphantom{e^{j\pi\cos\phi_1}}}_{\textbf{Sig. 1}} & \underbrace{\hphantom{e^{j\pi\cos\phi_2}}}_{\textbf{Sig. 2}} \end{array}$
- **Different spatial signatures** for Transmit Antenna **1, 2**

Spatial Signature = Series of Phase Differences

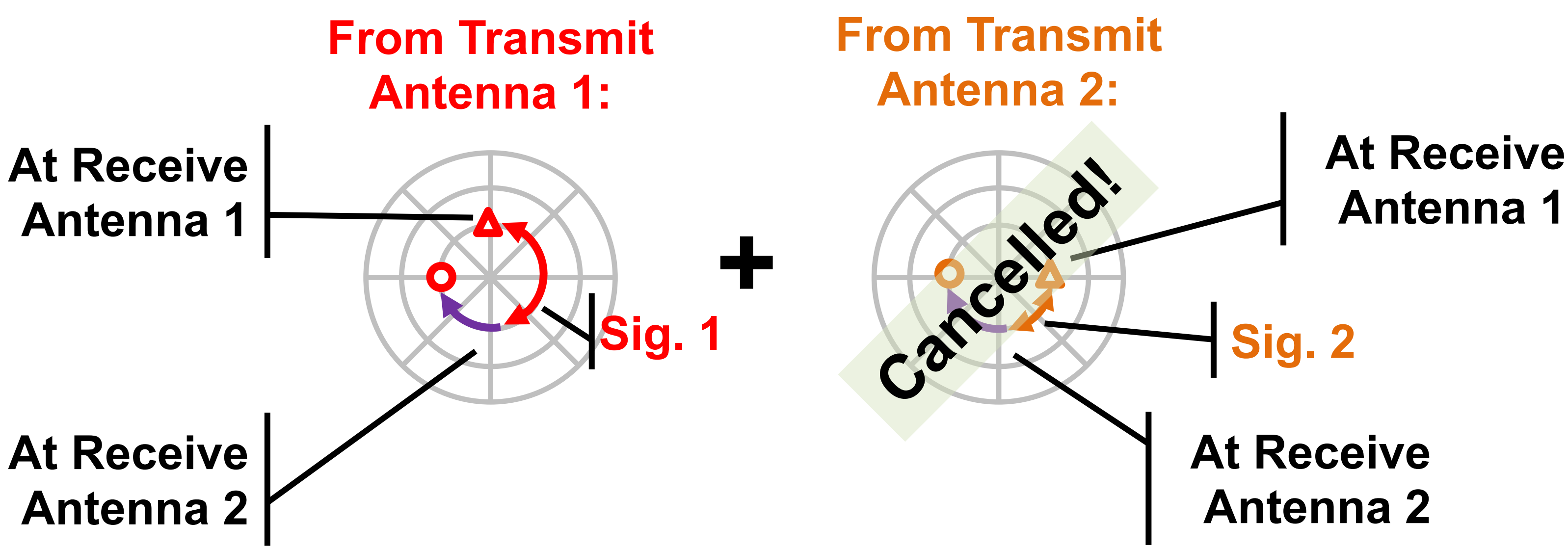
- **Channel matrix** $\mathbf{H} = ae^{j2\pi d/\lambda}\begin{array}{cc} \text{Tx 1:} & \text{Tx 2:} \\ \left[\begin{matrix} 1 \\ e^{j\pi\cos\phi_1} \end{matrix}\right. & \left.\begin{matrix} 1 \\ e^{j\pi\cos\phi_2} \end{matrix}\right] \end{array}$

  $\underbrace{\qquad}_{\text{Sig. 1, } h_{\phi_1}}$ $\underbrace{\qquad}_{\text{Sig. 2, } h_{\phi_2}}$



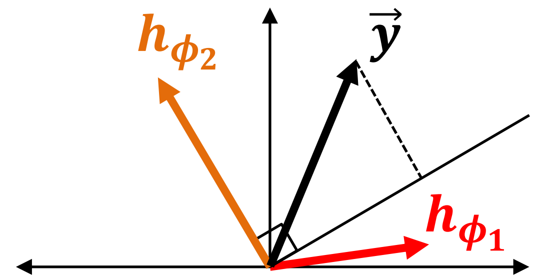
The Zero-Forcing Receiver
(via Spatial Signatures)

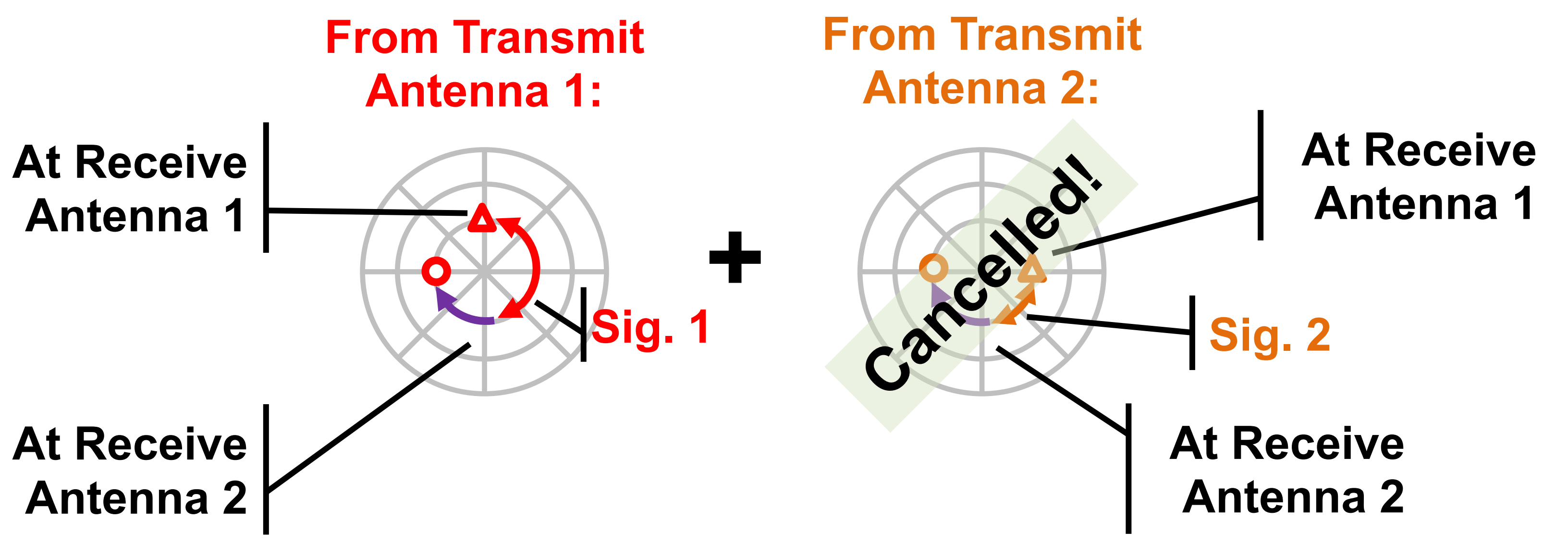
- Suppose want to receive from **Transmit Antenna 1**
  - (Recall:) **Rotate Receive Antenna 2's** signal so that **Signature 2** **cancels itself**



The Zero-Forcing Receiver
(via Spatial Signatures)

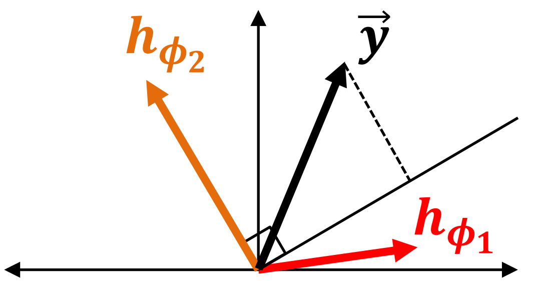
- One **spatial signature** = One **direction**
- **Zero forcing** **Antenna 2** is **projection**
  - Onto **subspace ⊥ to** h_{ϕ_2}



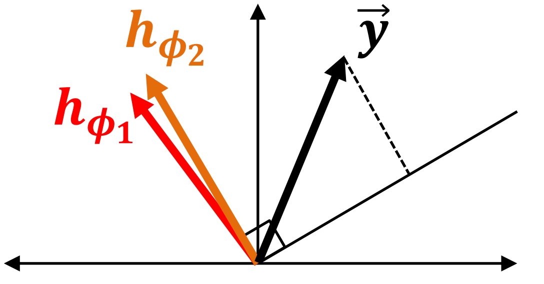


MIMO Separability: Discussion

- **Transmit antenna** separation →
  - **Spatial signature** separation →
    - **Better projection,**
    - **Better performance**



- **MIMO antenna array without multipath**
  - No transmit antenna separation
  - No spatial signature separation
    - Cancel **Tx Ant 2:** cancels **Tx Ant 1**
  - **No spatial multiplexing**



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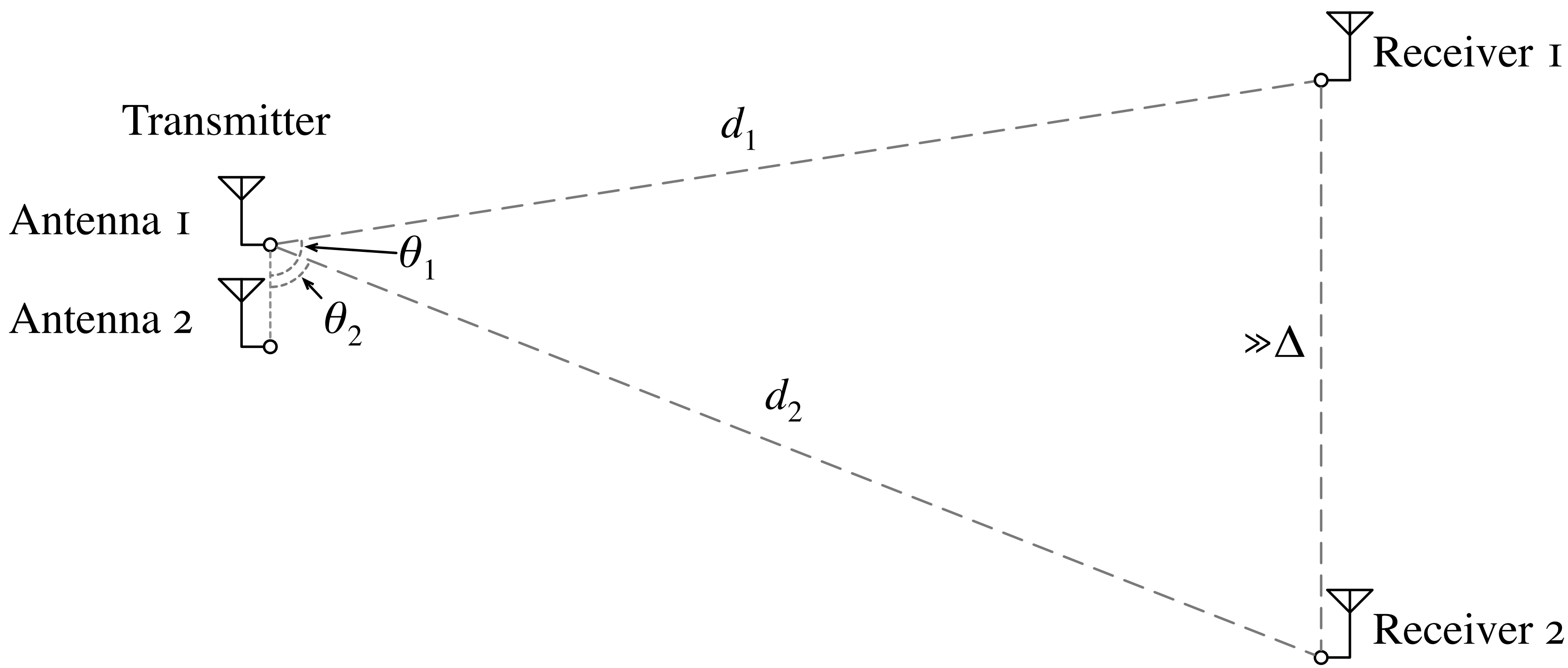
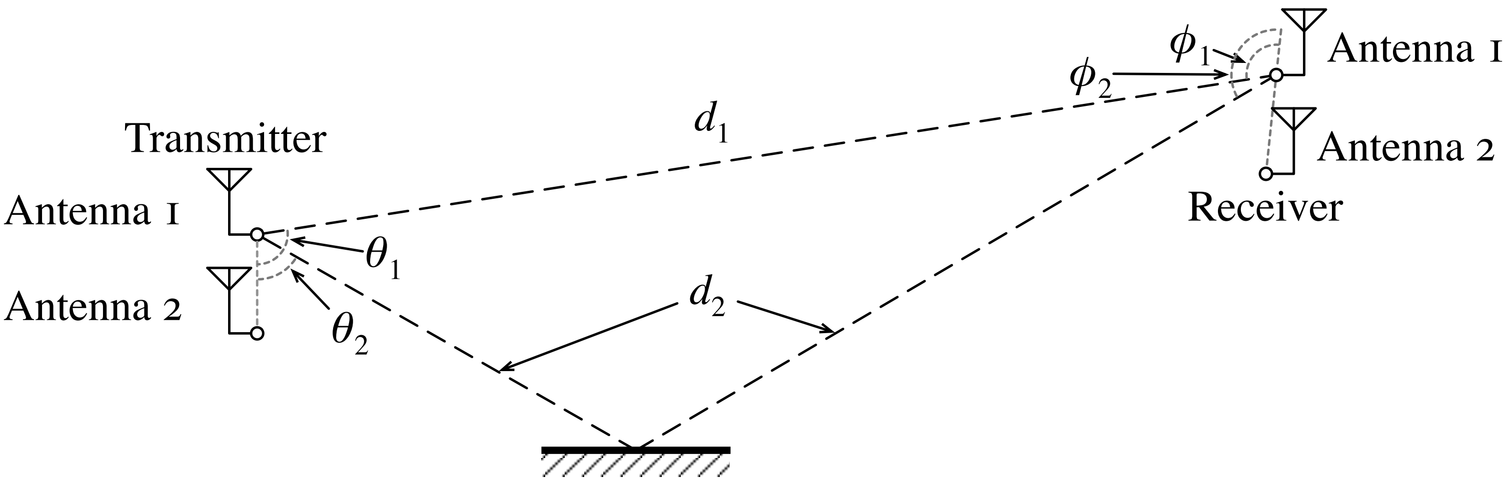
Geographically-Separated Receive Antennas: SDMA Downlink



- **Different spatial signatures** for Receive Antennas **1, 2**
  - **Rows, instead of columns** in the MIMO matrix

MIMO in Multipath

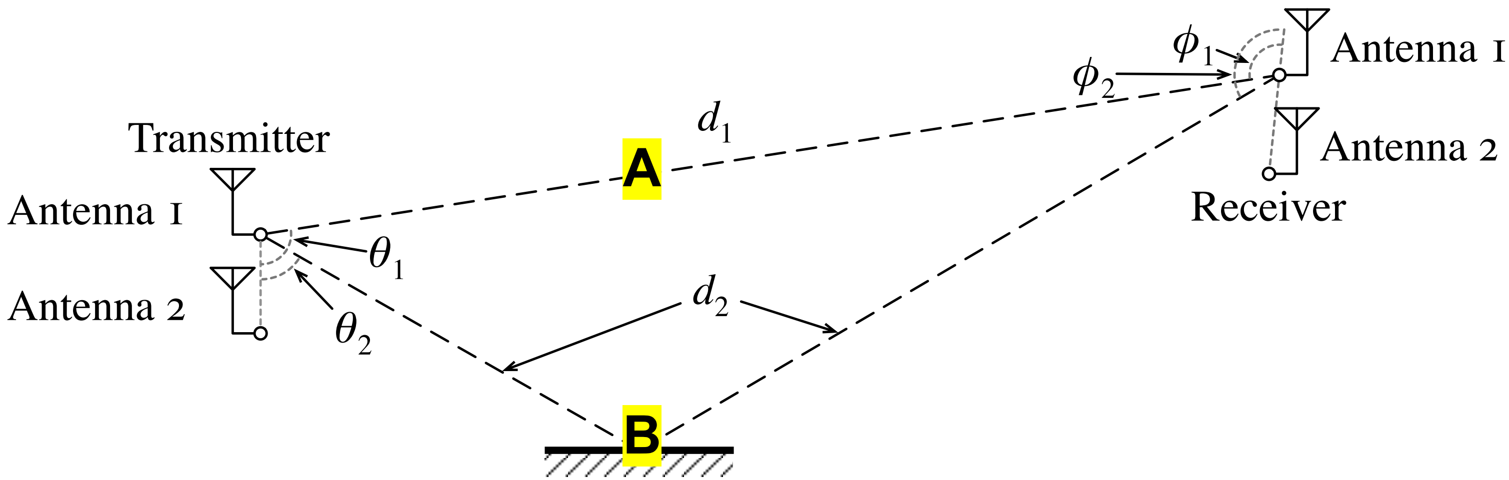


Tx antenna 1: **Tx antenna 2:**

$$\mathbf{H} = \begin{bmatrix} a_1 e^{j2\pi d_1/\lambda} + a_2 e^{j2\pi d_2/\lambda} & a_1 e^{j2\pi\left(\frac{d_1}{\lambda}+\cos\theta_1\right)} + a_2 e^{j2\pi\left(\frac{d_2}{\lambda}+\cos\theta_2\right)} \\ a_1 e^{j2\pi\left(\frac{d_1}{\lambda}+\cos\phi_1\right)} + a_2 e^{j2\pi\left(\frac{d_2}{\lambda}+\cos\phi_2\right)} & a_1 e^{j2\pi\left(\frac{d_1}{\lambda}+\cos\theta_1+\cos\phi_1\right)} + a_2 e^{j2\pi\left(\frac{d_2}{\lambda}+\cos\theta_2+\cos\phi_2\right)} \end{bmatrix}$$

- **Neither column** is a multiple of the other
  - So **H** has **two different transmit antenna spatial signatures**

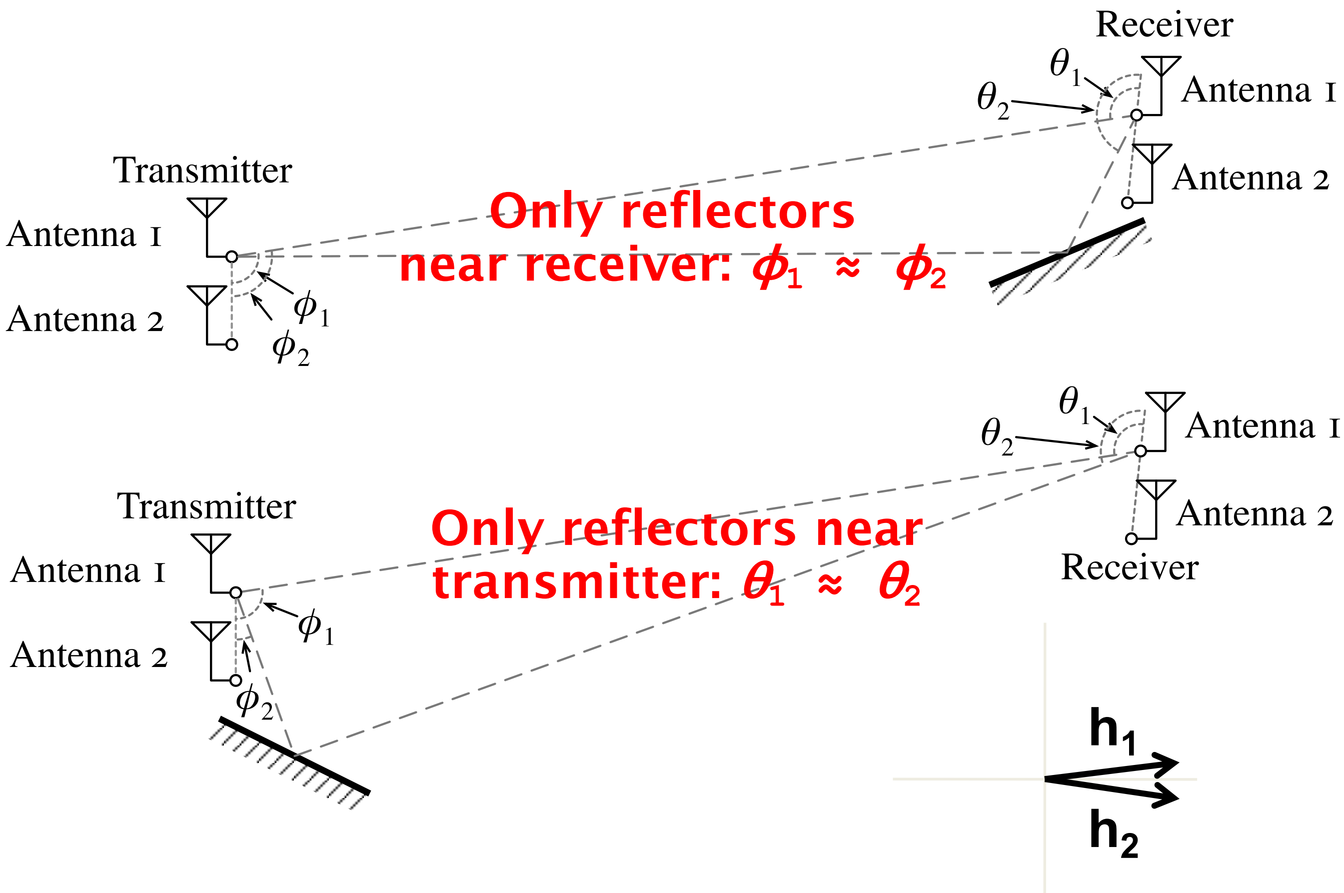
Different Spatial Signatures: Intuition



- **Channel matrix H** has **two different spatial signatures**

- Imagine perfect **signal "relays" A, B**
  - **This H is the product of:**
    - Geographically-separated **receive** antenna channel
    - Geographically-separated **transmit** antenna channel

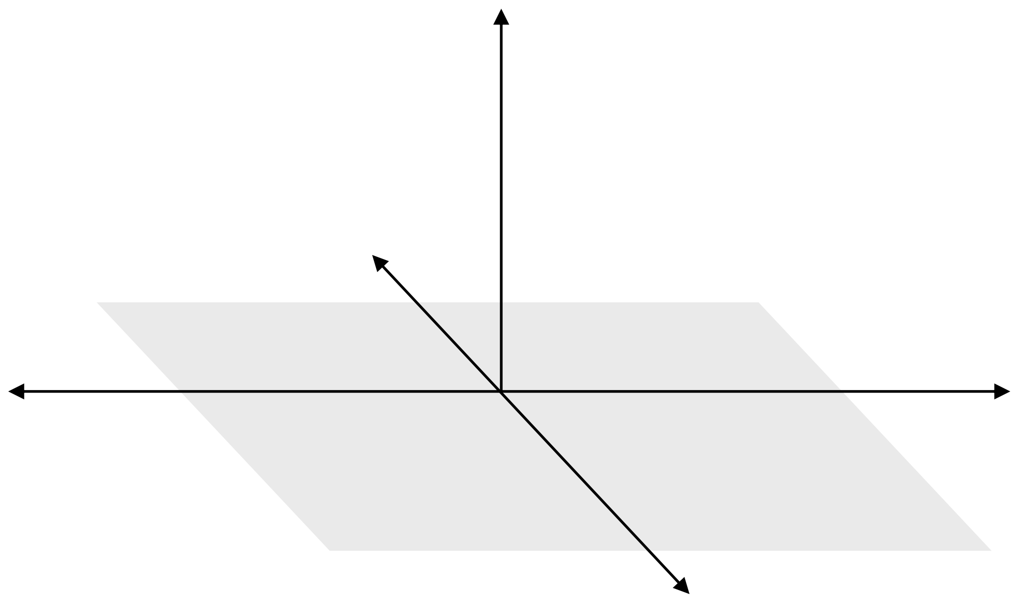
“Poorly-Conditioned” MIMO channels



When channel is poorly conditioned, spatial signatures are closer aligned

How Many Streams are Possible?

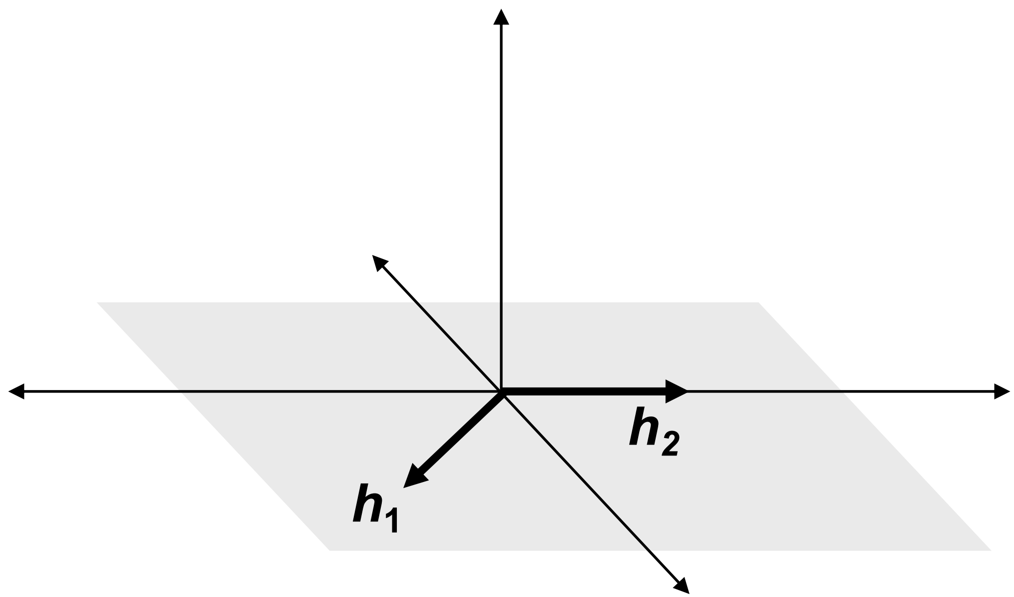
- **Received signals** live in an n_r-dimensional vector space
  - *e.g.* $\boldsymbol{n_r}$ **= 3** receive antennas → 3-D vector space:



- Cancel by **projection**. Therefore, **at most $\boldsymbol{n_r}$** streams possible

How Many Streams are Possible?

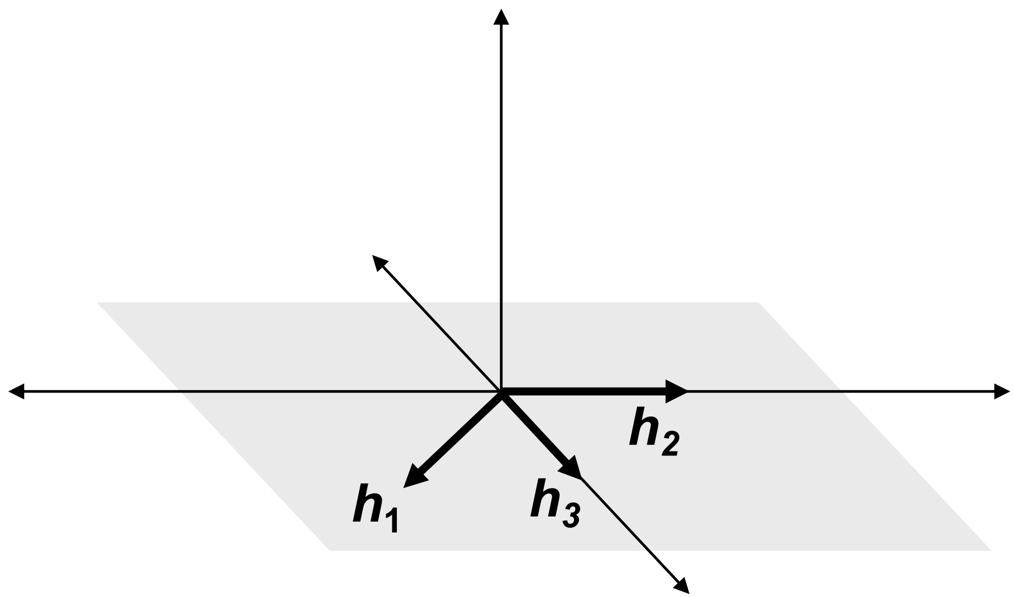
- **One** spatial signature **per transmit antenna**
  - *e.g.* $n_r = 3$ receive, $\boldsymbol{n_t} = \mathbf{2}$ transmit antennas:



- Therefore, **at most $\boldsymbol{n_t}$** streams possible

How Many Streams are Possible?

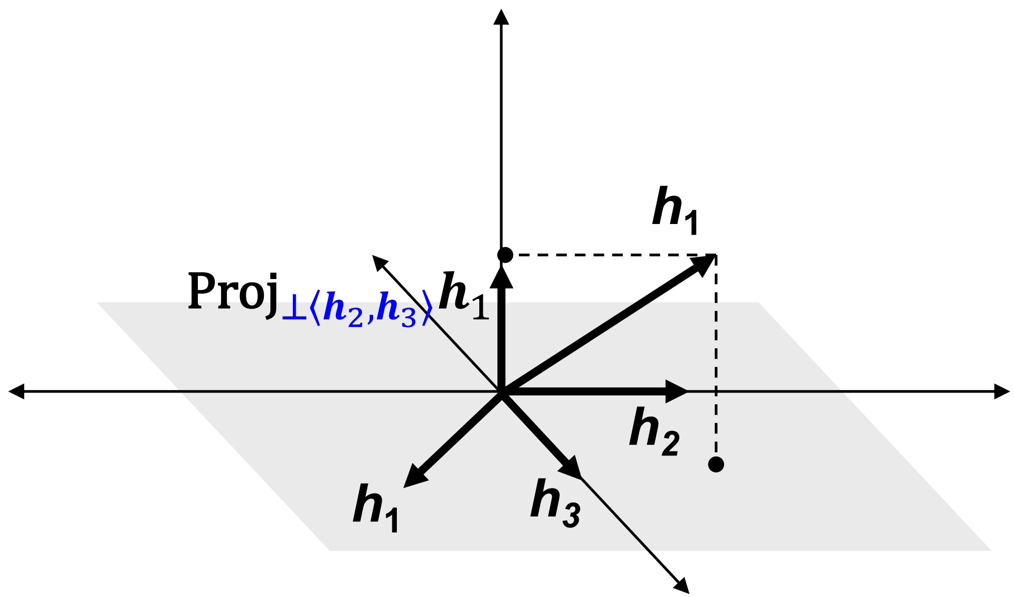
- Need enough strong physical paths in the wireless channel
  - *e.g.* $\boldsymbol{n_r} = 3$, $\boldsymbol{n_t} = \mathbf{3}$ **but two physical paths** confines $\{\boldsymbol{h_i}\}$ to a **plane**



- At most **# physical paths** possible streams

How Many Streams are Possible?

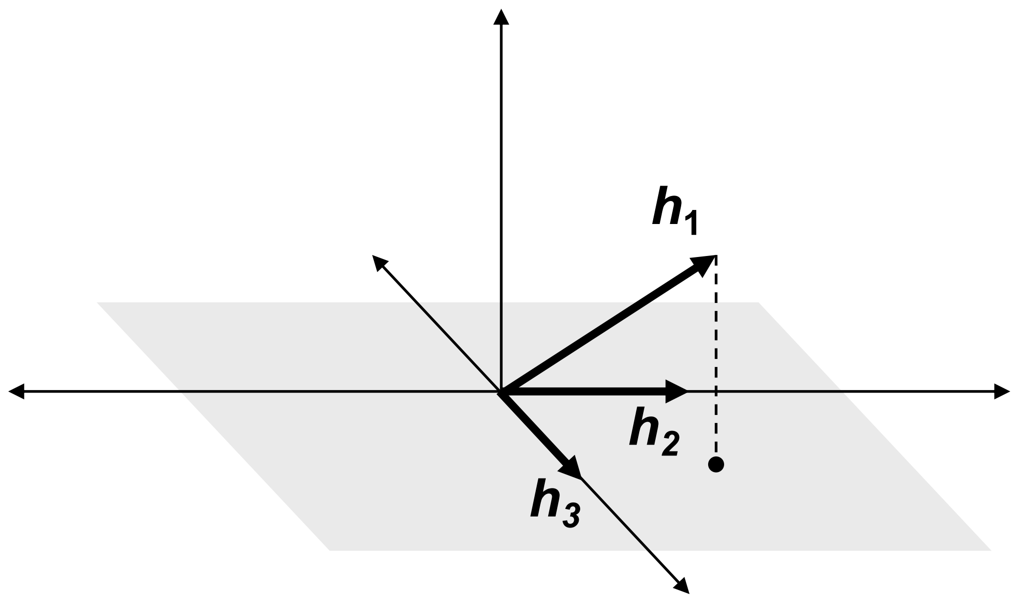
- Need enough strong physical paths in the wireless channel
  - *e.g.* $\boldsymbol{n_r}$ = 3, $\boldsymbol{n_t}$ **= 3** and **three physical paths**



- At most **# physical paths** possible streams

Degrees of Freedom

- The figure of merit that summarizes the number of streams possible is called the number of ***degrees of freedom*** **of H**



- Degrees of freedom = **min { n_t , n_r , # strong paths }**

Summary

- **Spatial multiplexing** requires **either:**
  - **Spatially-separated** receivers / transmitters (SDMA), **or**
  - Multiple antennas at **both** ends of a link (MIMO), **and**
    - Enough **physical channel** propagation paths
- **Degrees of freedom** quantify spatial multiplexing potential

Tuesday Topic:
MIMO III: Channel Capacity, Interference Alignment