MIMO I: Spatial Diversity

COS 463: Wireless Networks

Lecture 16

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[Parts adapted from D. Halperin *et al.,* T. Rappaport]

What is MIMO, and why?

- ***Multiple-Input, Multiple-Output (MIMO)*** communications
  - Send/receive > 1 signal on different transmit and receive antennas

- We've already seen **frequency, time, spatial** multiplexing in 463:
  - MIMO is a more powerful way to **multiplex** wireless medium **in space**
  - **Transforms multipath** propagation from **impediment** to **advantage**

Many Uses of MIMO

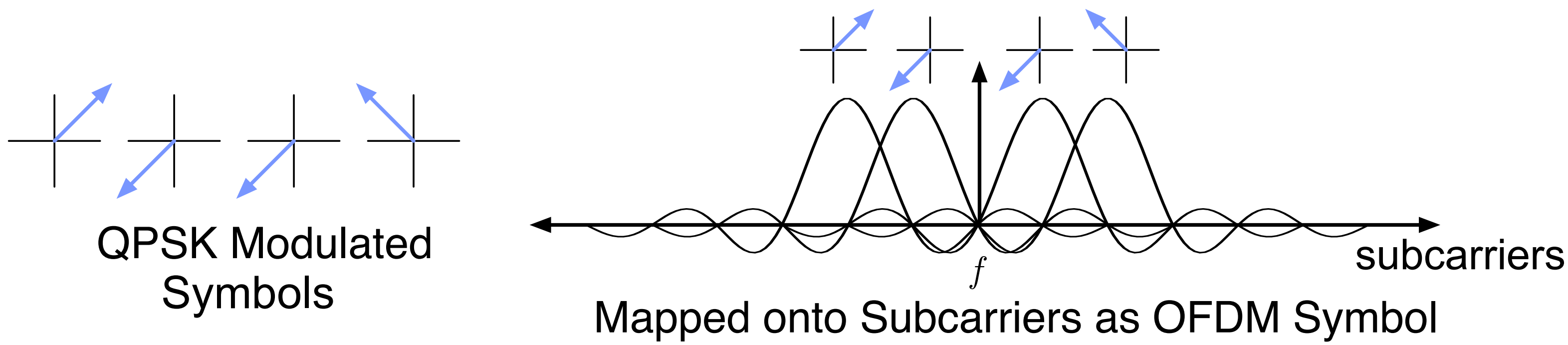
- At least three different ways to **leverage space:**

1. **Spatial diversity:** Send or receive redundant streams of information in parallel along multiple spatial paths
   - **Increase reliability and range** (unlikely all paths are degraded simultaneously)

2. **Spatial multiplexing:** Send independent streams of information in parallel along multiple spatial paths
   - **Increases rate,** if we can avoid interference

3. **Interference alignment:** **"Align"** two streams of interference at receiver, resulting in impact of **just one** interference stream

MIMO-OFDM



- Multipath fading: different effects on different frequencies
  - OFDM: Orthogonal Frequency Domain Multiplexing
  - Different subcarriers **are independent** of each other

- Channel model for OFDM: $y = h \cdot x + w$
  - **A single complex number** ***h*** captures the **effect of the channel** on data in a particular subcarrier

- **For MIMO: Think about each subcarrier,** **independent** of other subcarriers

Plan

1. **Today: Diversity in Space**
   - Receive Diversity
   - Transmit Diversity

2. Next time: Multiplexing in Space

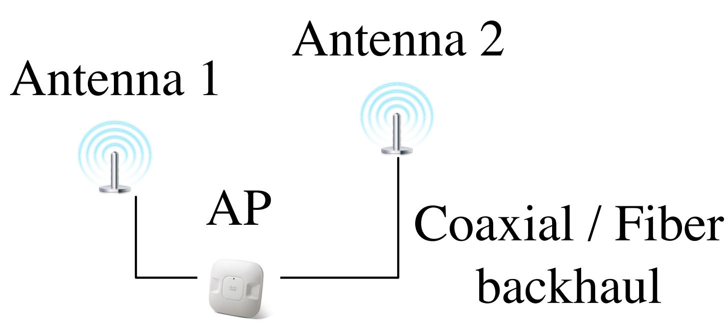
3. Next time: Interference Alignment

Path Diversity: Motivation

1. **Multi-Antenna** Access Points (APs), especially 802.11n,ac:

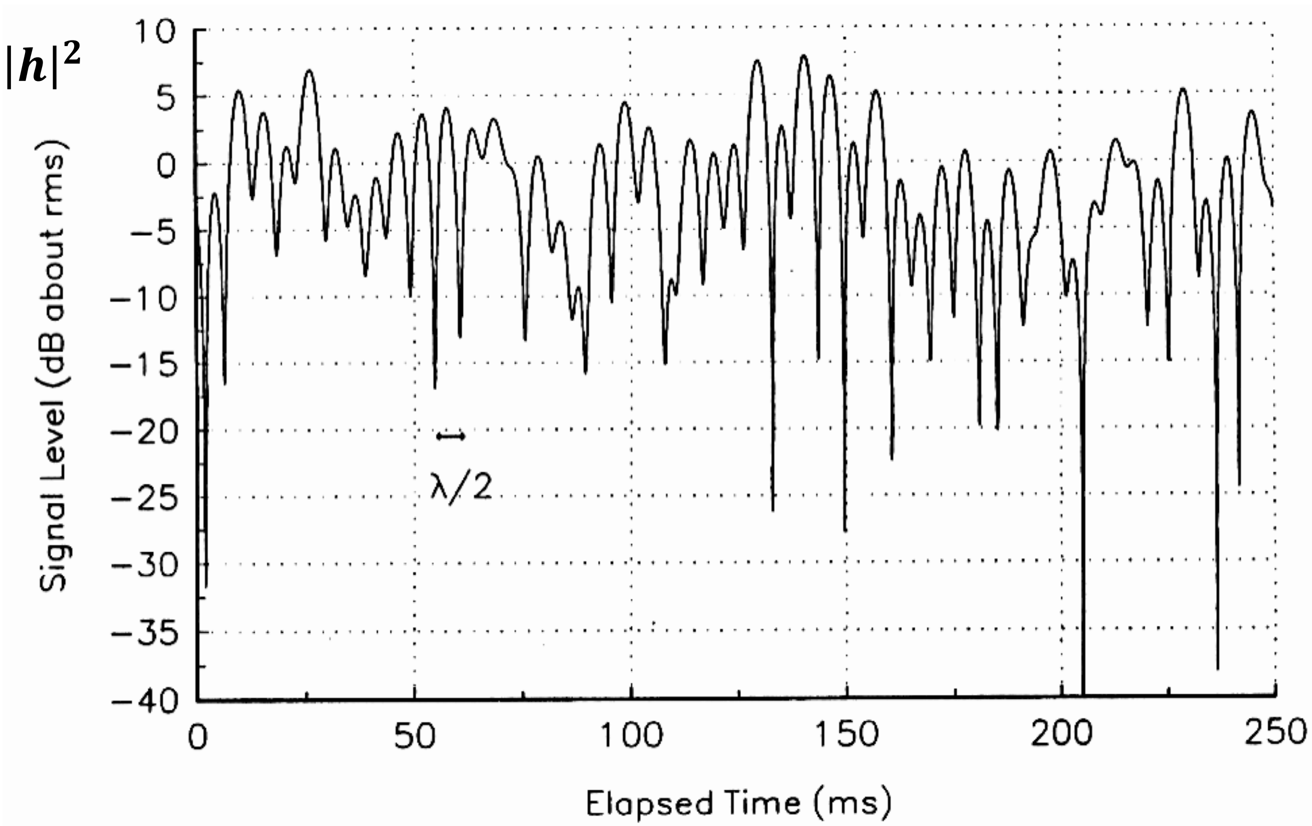
2. **Multiple APs** cooperating with each other:

3. **Distributed Antenna** systems, separating antenna from AP:



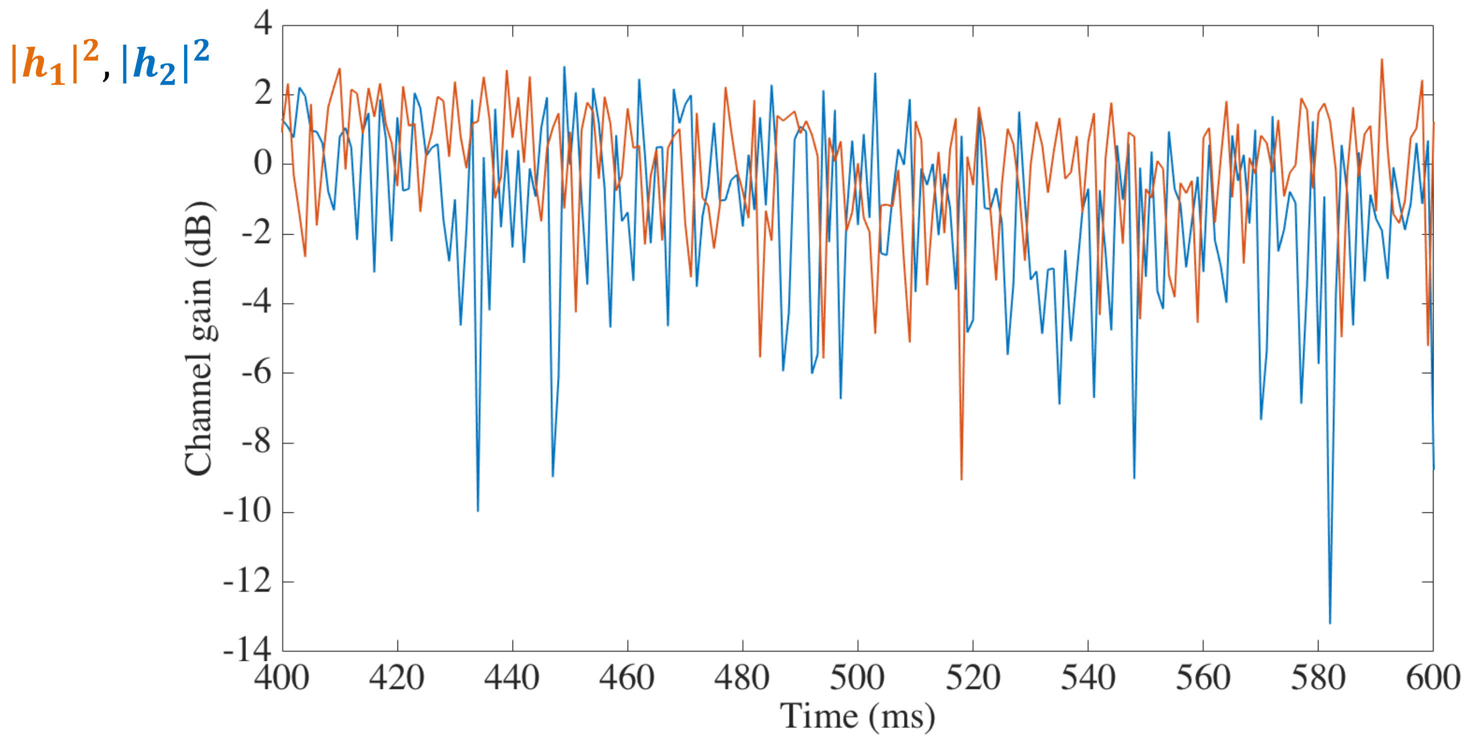
Review: Fast Fading

- Typical outdoor **multipath propagation environment**, channel ***h***
- **On one link each subcarrier's power level** experiences **Rayleigh fading:**

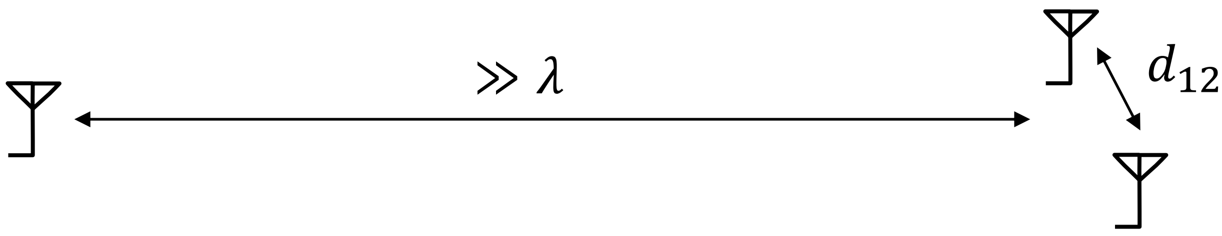


Uncorrelated Rayleigh Fading

- Suppose **two antennas,** separated by distance $\boldsymbol{d_{12}}$
- **Channels** from **each** to a **distant third antenna** ($\boldsymbol{h_{13}}$, $\boldsymbol{h_{23}}$) can be ***uncorrelated***
  - Fading happens at different times with **no bias for a simultaneous fade**



When is Fading Uncorrelated, and Why?



- **Channels** from **each antenna** (h_{13}, h_{23}) to a **third** antenna
  - Channels are **uncorrelated** when $d_{12} > \approx \mathbf{1.5\lambda}$
  - Channels **correlated, fade together** when $d_{12} < \approx \boldsymbol{\lambda}$

- This ***correlation distance*** depends on the **radio environment** around the **pair of antennas**
  - Increases, *e.g.*, atop cellular phone tower

Plan

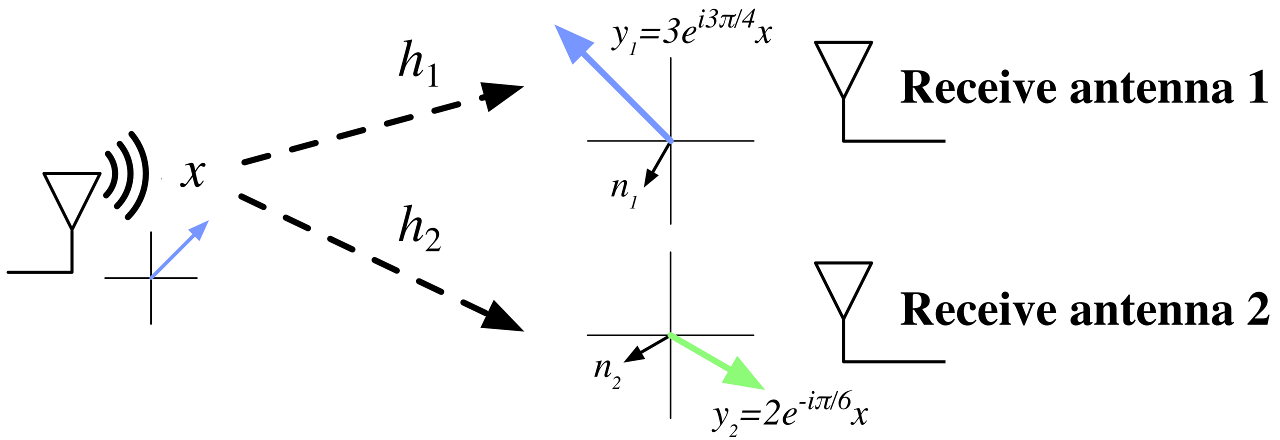
1. **Today: Diversity in Space**
   - **Receive Diversity**
     - Selection Diversity
     - Maximal Ratio Combining
   - Transmit Diversity

2. Next time: Multiplexing in Space

3. Next time: Interference Alignment

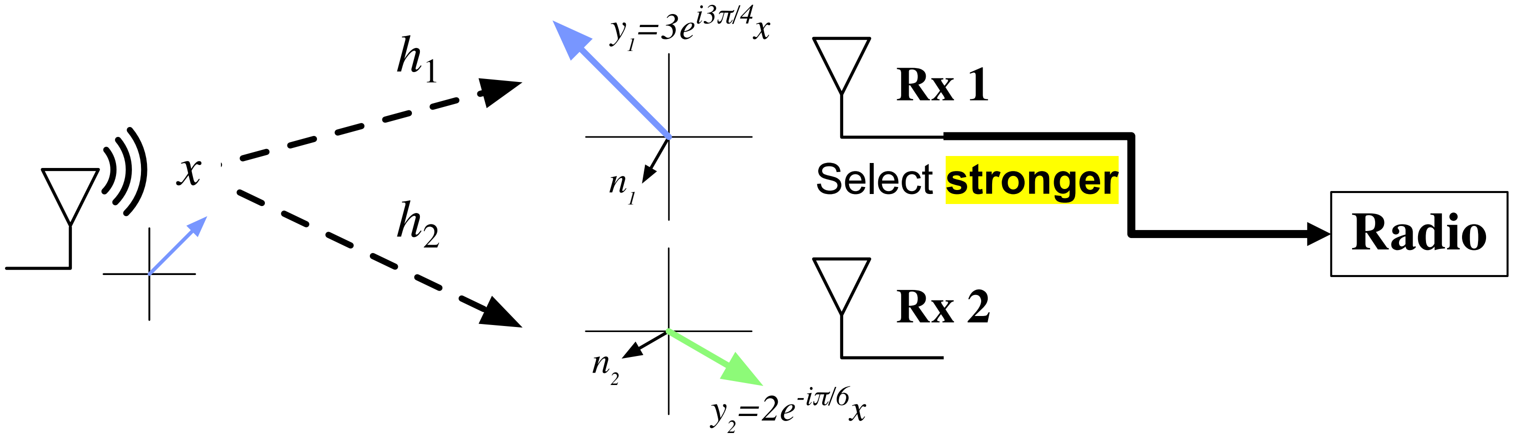
Channel Model for Receive Diversity

- **One transmit antenna** sends a symbol to **two receive antennas**
  - ***Receive diversity***, or ***Single-Input, Multi-Output (SIMO)***



- Each receive antenna **gets own copy** of transmitted signal via:
  - **A different path**
  - A (likely) **different channel**

Selection Diversity

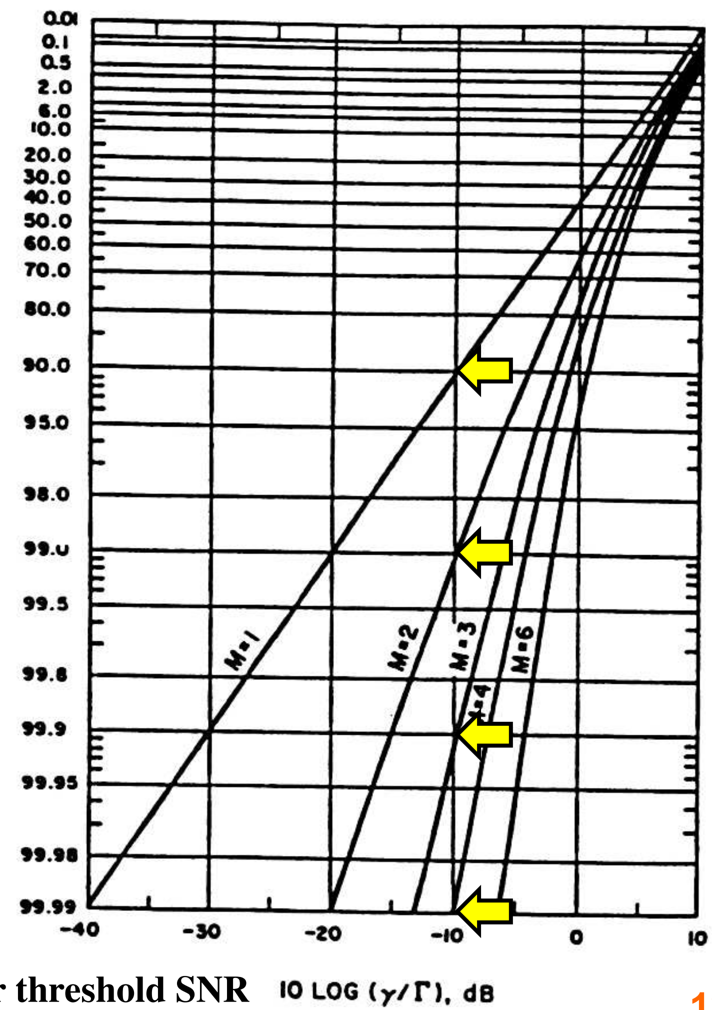


- **Two** receive antennas **share one receiving radio** via a switch
- **Receiver selects antenna with stronger signal** to connect to the radio
  - **Helps reliability** (both unlikely bad)
  - **Wastes received signal from other antenna(s)**

Selection Diversity:
Performance Improvement in Uncorrelated Rayleigh Fading

- In general, might have ***M*** **receive antennas** (average SNR Γ)
  - γ_i: SNR of the ***i*** **th** receive antenna
- Probability **selected SNR is less than** some threshold *γ (**outage**):*
  - $\Pr[\gamma_1, \cdots, \gamma_M \leq \gamma] = (\Pr[\gamma_i \leq \gamma])^M$
- **One more "9" of reliability** per additional selection branch

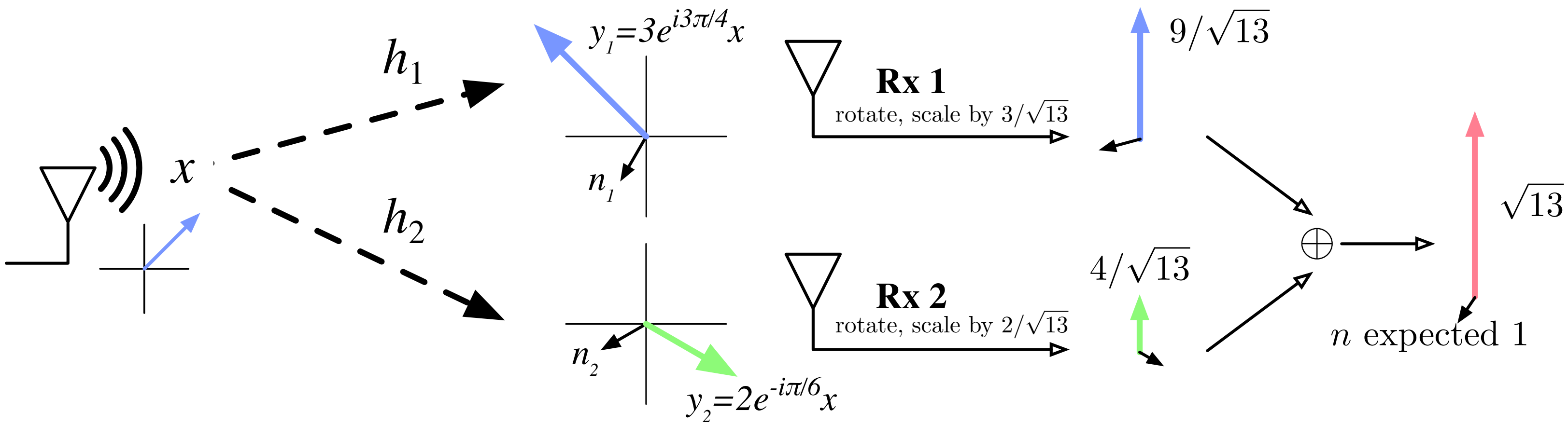
Probability (%) that Selected Antenna's SNR Exceeds Threshold γ



Higher probability (better) ↓

← **lower threshold SNR**

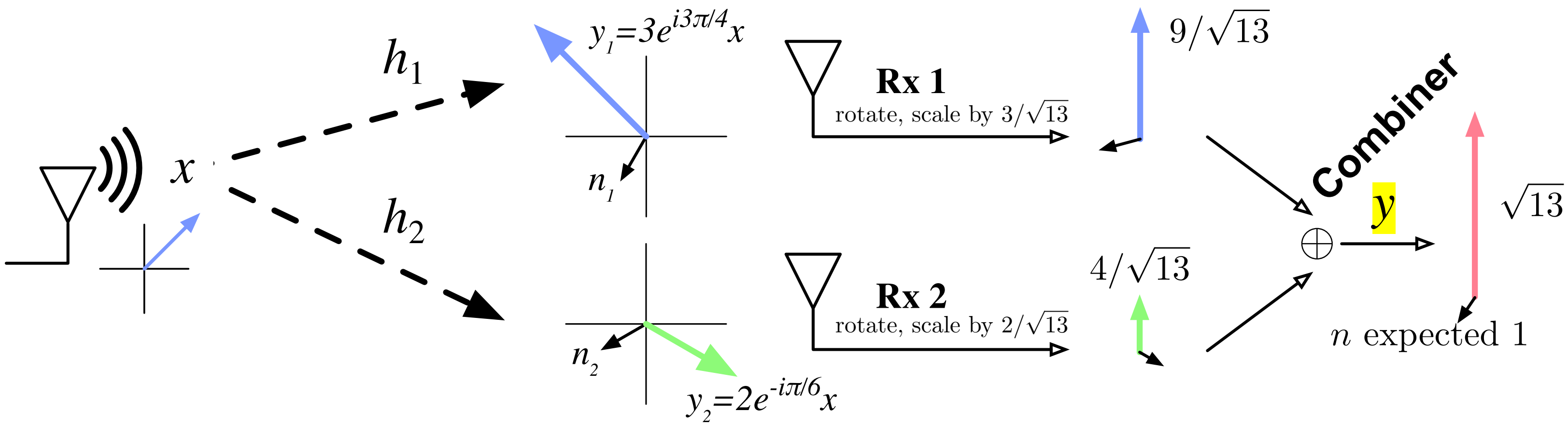
Leveraging All Receive Antennas



- Want to just add the two received signals together
  - But if we did the **signals would often cancel out**

- **Solution:** Receive ***M* radios, align signal phases, then add**
  - **Requires *M* receive radios,** in general

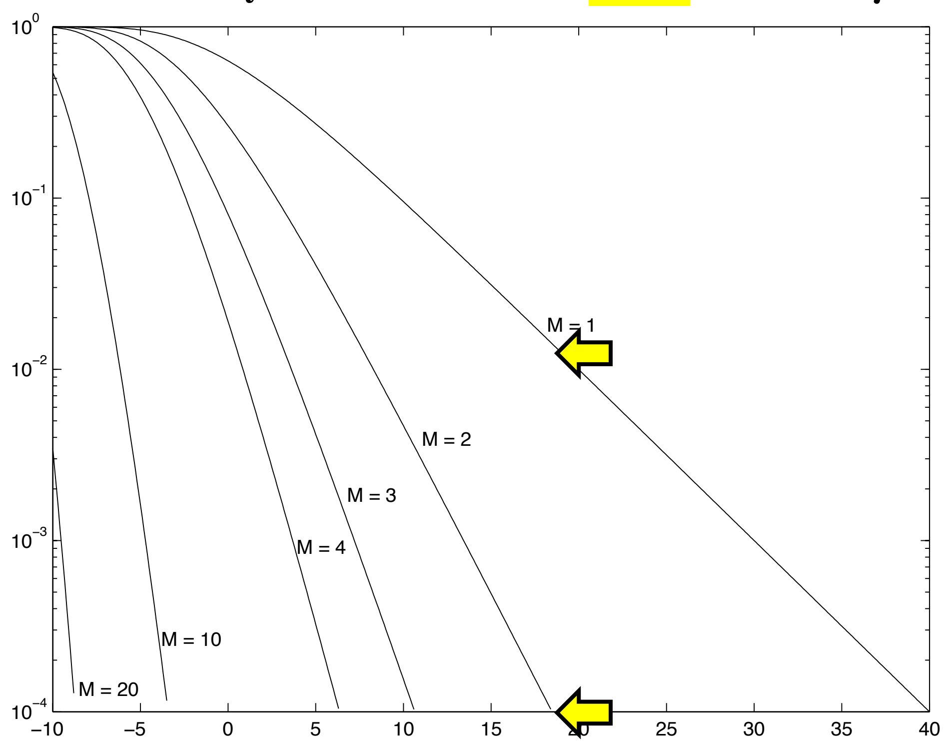
How to Choose Weights?



- Suppose phase of incoming signal on the *i* th branch is θ_i
- **To align { y_i } in phase,** let the **combiner** output $y = \sum_{i=1}^{M} a_i e^{-j\theta_i}$
  – How to choose **amplitudes a_i?**
- **Idea:** Put **more weight** into branches with **high SNR: Let $a_i = \gamma_i$**
  – This is called ***Maximal Ratio Combining (MRC)***

MRC: Performance Improvement

Probability that MRC's SNR is Under Threshold γ

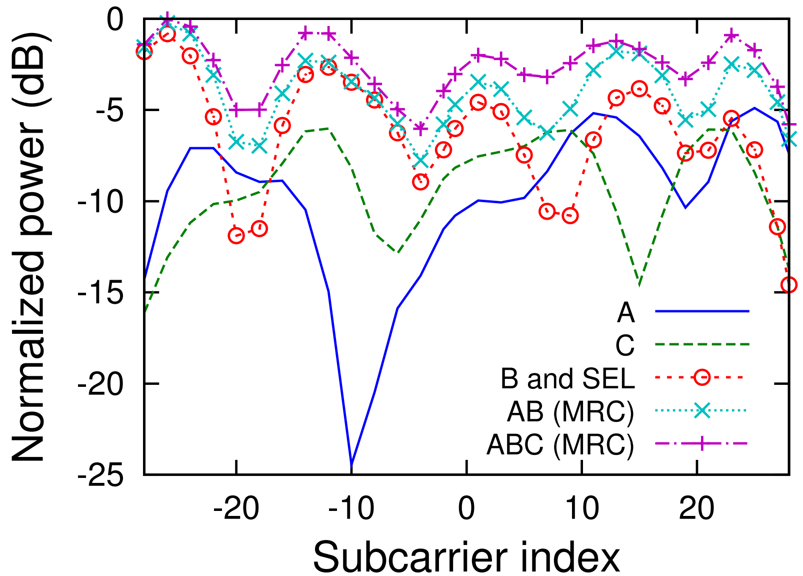


Lower probability (better) ↓

$10\log_{10}(\Gamma/\gamma)$ **Lower threshold SNR →**

- **Two "9"s of reliability improvement** between **one** (*i.e.*, no MRC) and **two** MRC branches

Selection Diversity, in Frequency



- Antennas A and C experience **different fades on different subcarriers**
- Selection Combining ("SEL") **improves** but certain subcarriers **still experience fading**
- MRC **increases power and flattens nulls, leading to fewer bit errors**

MRC's Capacity Increase

- MRC with M branches increases SNR
  - **Increased** Shannon capacity

- **Sub-linear** (logarithmic) **capacity increase** in *M:*
  - $C_{MRC} = BW \cdot \log(1 + M \cdot SNR)$ bits/second/Hz

Plan

1. **Today: Diversity in Space**
    - Receive Diversity
    - **Transmit Diversity**
        - **Transmit beamforming**
        - Introduction to Space-Time Coding: Alamouti's Scheme
2. Next time: Multiplexing in Space
3. Next time: Interference Alignment

Transmit Diversity: Motivation

- More **space, power, processing capability** available **at the transmitter?**
  - Yes, likely! *e.g.* Cell tower, Wi-Fi AP sending **downlink traffic** to mobile

- **But, a possible requirement: Transmitter** may **need to know radio channel** before transmission commences
  - *cf.* receive diversity: receiver knows channel from **preamble**

- **So, a tension:** Want to **separate** transmit antennas for path diversity
  - Need to move **timely** radio channel measurements between **locations**
    - Introduces **overhead and design complexity**

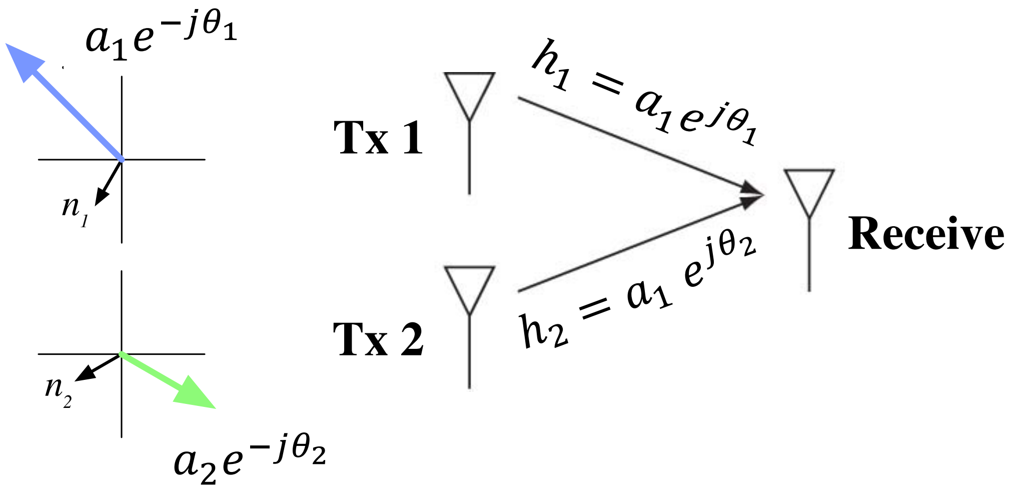
Transmit Beamforming: Motivation

- Suppose **transmitter knows** the channel to receivers

- Transmitters align their signals so that **constructive interference occurs** at the single receive antenna
  - Align **before transmission, not after reception** (receive beamforming)

Transmit Beamforming

- Leverage channel reciprocity, receive beamforming **"in reverse"**
- Send one **data symbol** x from two antennas



- At each transmit antenna, **multiply** (***pre-code***) x by the **complex conjugate of the respective channel** to the receive antenna

Plan

1. **Today: Diversity in Space**
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   - **Transmit Diversity**
     - Transmit beamforming
     - **Introduction to Space-Time Coding: Alamouti's Scheme**
2. Next time: Multiplexing in Space
3. Next time: Interference Alignment

Alamouti Scheme: Motivation

- Suppose **transmitters don't know channel** to receiver: what to do?

1. **Naïve ("blind") beamforming** (just send same signals)
   - Signals would **often cancel out**

2. **Repetition in time**
   - Each antenna **takes turns** transmitting **same symbol**
     - **Receiver combines coherently**
   - Uses ***M*** symbol times
     - Increases diversity ("SNR" term in Shannon capacity)
     - **Cuts Shannon rate** by **1/*M* factor**

Alamouti Scheme

- **Scope:** A **two-antenna transmit diversity** system (*M* = 2)
- Sends **two symbols**, $\mathbf{s}_1$ and $\mathbf{s}_2$, in **two symbol time** periods:

Symbol Time Period	1	2
Antenna 1:	Send s_1	Send $-s_2^*$
Antenna 2:	Send s_2	Send s_1^*

- Then, **by superposition** the **receiver hears:**

Symbol Time Period	1	2
Receiver hears:	$y_1 = h_1 s_1 + h_2 s_2$	$y_2 = -h_1 s_2^* + h_2 s_1^*$

Alamouti Receiver Processing

Symbol Time Period	1	2
Receiver hears:	$\text{y}[1] = h_1 s_1 + h_2 s_2$	$y[2] = -h_1 s_2^* + h_2 s_1^*$ $y^*[2] = h_2^* s_1 - h_1^* s_2$

$$\begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

- Rewrite (receiver has channel information):

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \propto \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix}$$

- So receiver **can solve for transmitted symbols**
- But, what's happening **in terms of the physical wireless channel?**

Intuition for Alamouti Receiver Processing

- Start with the inverted channel matrix:

$$\begin{bmatrix} \boldsymbol{s_1} \\ s_2 \end{bmatrix} \propto \begin{bmatrix} \boldsymbol{h_1^*} & \boldsymbol{h_2} \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix}$$

- Consider the **computation for s_1:**
  - **Rotate** $y[1]$ **by** $-\boldsymbol{\theta_1}$
  - **Rotate** $y^*[2]$ **by** $\boldsymbol{\theta_2}$
  - **Sum** the result

Alamouti: Impact of Phase Rotations

- Consider the **computation for s_1:**
  - **Rotate** $y[1]$ **by** $-\boldsymbol{\theta_1}$
  - **Rotate** $y^*[2]$ **by** $\boldsymbol{\theta_2}$
  - **Sum** the result

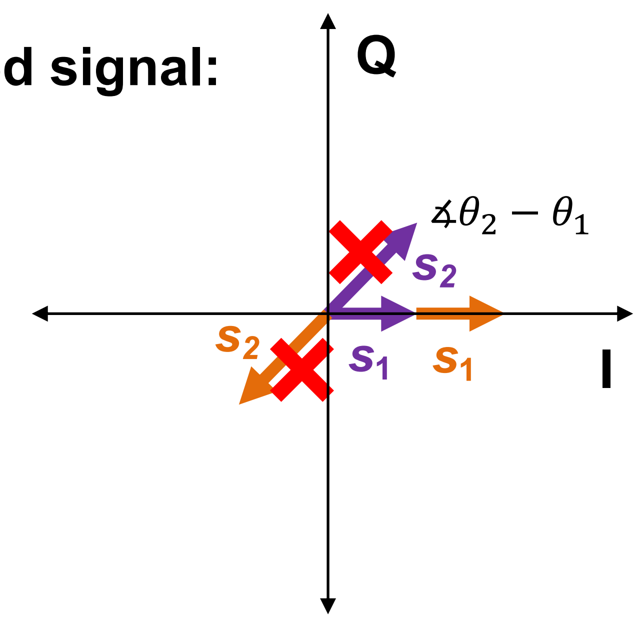
Symbol Time Period	**1**	**2**
Receiver hears:	$\mathrm{y}[1] = h_1 s_1 + h_2 s_2$	$y^*[2] = h_2^* s_1 - h_1^* s_2$
	$\downarrow \quad \downarrow$	$\downarrow \quad \downarrow$
Phase after rotation:	$\mathbf{0} \quad \boldsymbol{\theta_2 - \theta_1}$	$\mathbf{0} \quad \boldsymbol{\theta_2 - \theta_1}$

Alamouti: Receiver-Side Picture

Symbol Time Period	**1**	**2**
Receiver hears:	$y[1] = h_1 s_1 + h_2 s_2$	$y^*[2] = h_2^* s_1 - h_1^* s_2$
	$\downarrow \quad \downarrow$	$\downarrow \quad \downarrow$
Phase after rotation:	$\mathbf{0} \quad \boldsymbol{\theta_2 - \theta_1}$	$\mathbf{0} \quad \boldsymbol{\theta_2 - \theta_1}$

- Receiver then **sums all terms above:**

Received signal:



Alamouti: Interpretation

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \propto \underbrace{\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix}$$

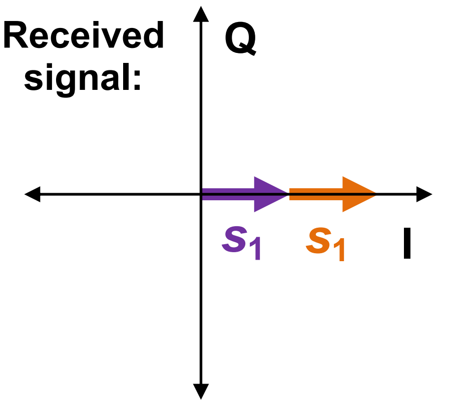
- Two **new signal *dimensions*:**

1. Multiply two received symbols by the **top column of H**
   - Name this dimension $[h_1^* \quad h_2]^T$
   - $\boldsymbol{s_1}$ arrives along this dimension **(only!)**

2. Multiply two received symbols by the **lower column of H**
   - Name this dimension $[h_2^* \quad -h_1]^T$
   - $\boldsymbol{s_2}$ arrives along this dimension **(only!)**

Alamouti: Performance

- Two dimensions: $[h_1^* \quad h_2]^T$, $[h_2^* \quad -h_1]^T$



- Send **half power** on each antenna
  - For both symbols, $SNR = \frac{|h|_1^2 + |h|_2^2}{2\sigma^2}$

- Rate gain from **enhanced SNR,** maintains **one symbol/symbol time**
  - **But not two symbols/symbol time: no true spatial multiplexing, yet**

Multi-Antenna Diversity: Summary

- Leverage path diversity
  - Decrease probability of “falling into” to deep Rayleigh fade on a single link

- Defined **new “dimensions”** of independent communication channels based on space
  - **Segue to spatial multiplexing** next time

Thursday Topic:
MIMO II: Spatial Multiplexing