Receiver Designs for the Radio Channel

COS 463: Wireless Networks

Lecture 15

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[Parts adapted from C. Sodini, W. Ozan, J. Tan]

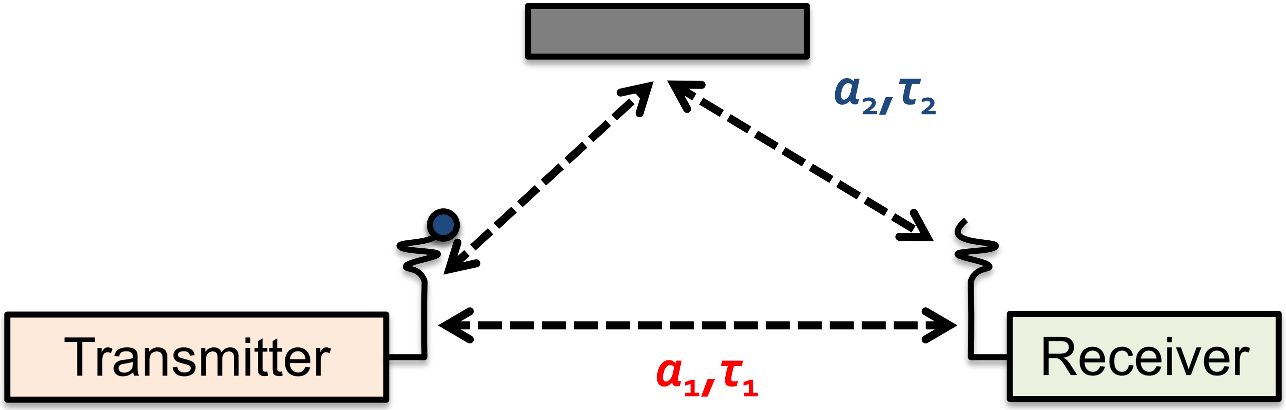
Today

1. **Delay Spread and Frequency-Selective Fading**

2. Time-Domain Equalization

3. Orthogonal Frequency Division Multiplexing

Last Time: Multipath causes Delay Spread

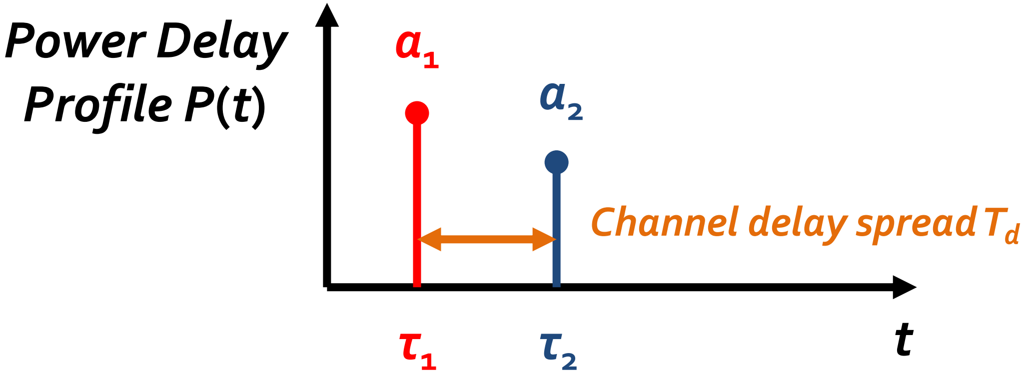


Transmitter

Receiver

a_2, τ_2

a_1, τ_1



Power Delay Profile P(t)

a_1

a_2

Channel delay spread T_d

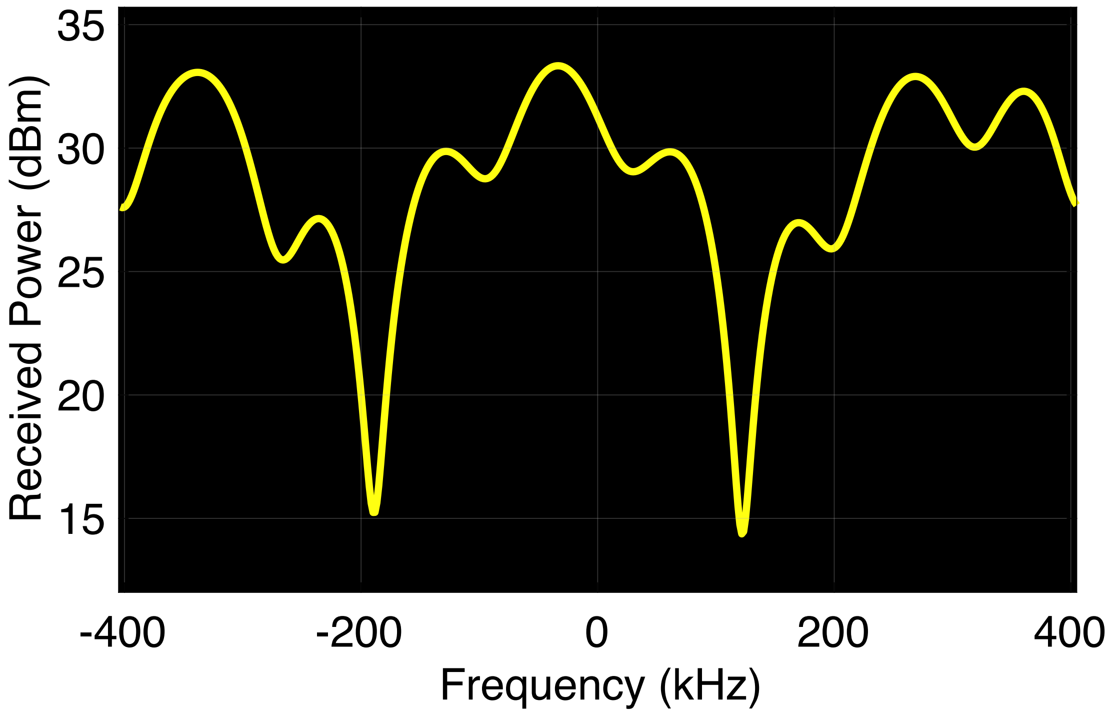
τ_1

τ_2

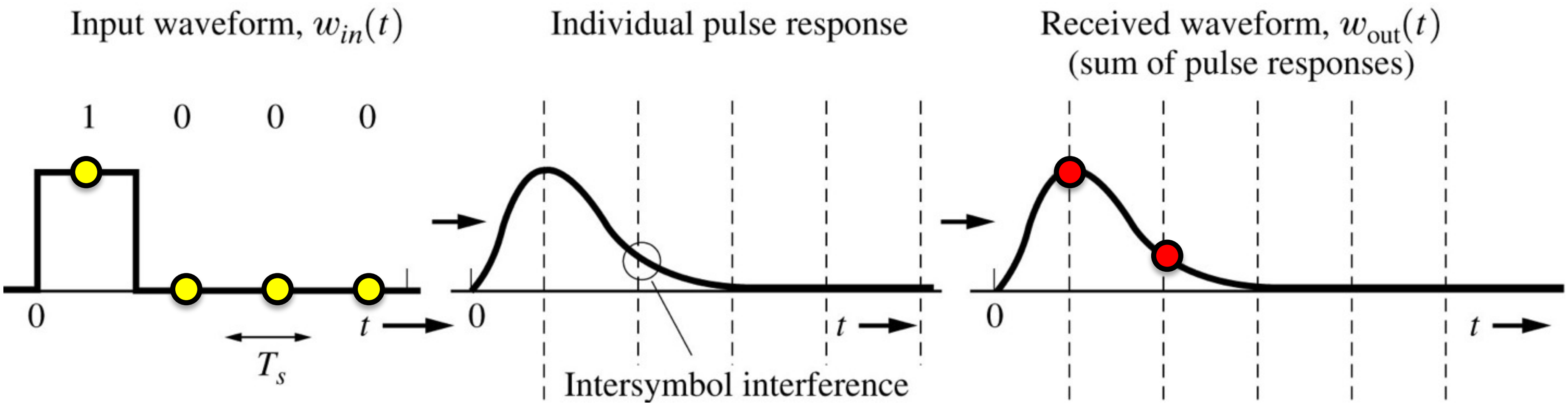
t

Last Time: Multipath causes Frequency *Selectivity*

- Interference between reflected and line-of-sight radio waves results in **frequency dependent fading**

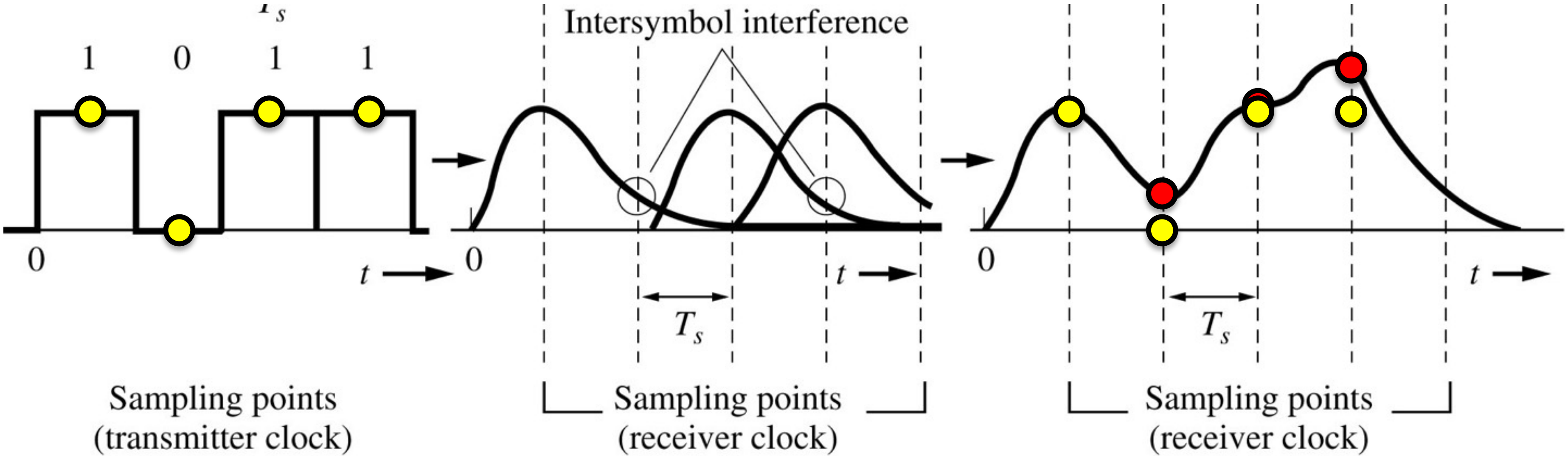


Problem: Inter-symbol interference (ISI)



- **Transmitted signal** 🟡
- **Received signal with ISI** 🔴

Problem: Inter-symbol interference (ISI)



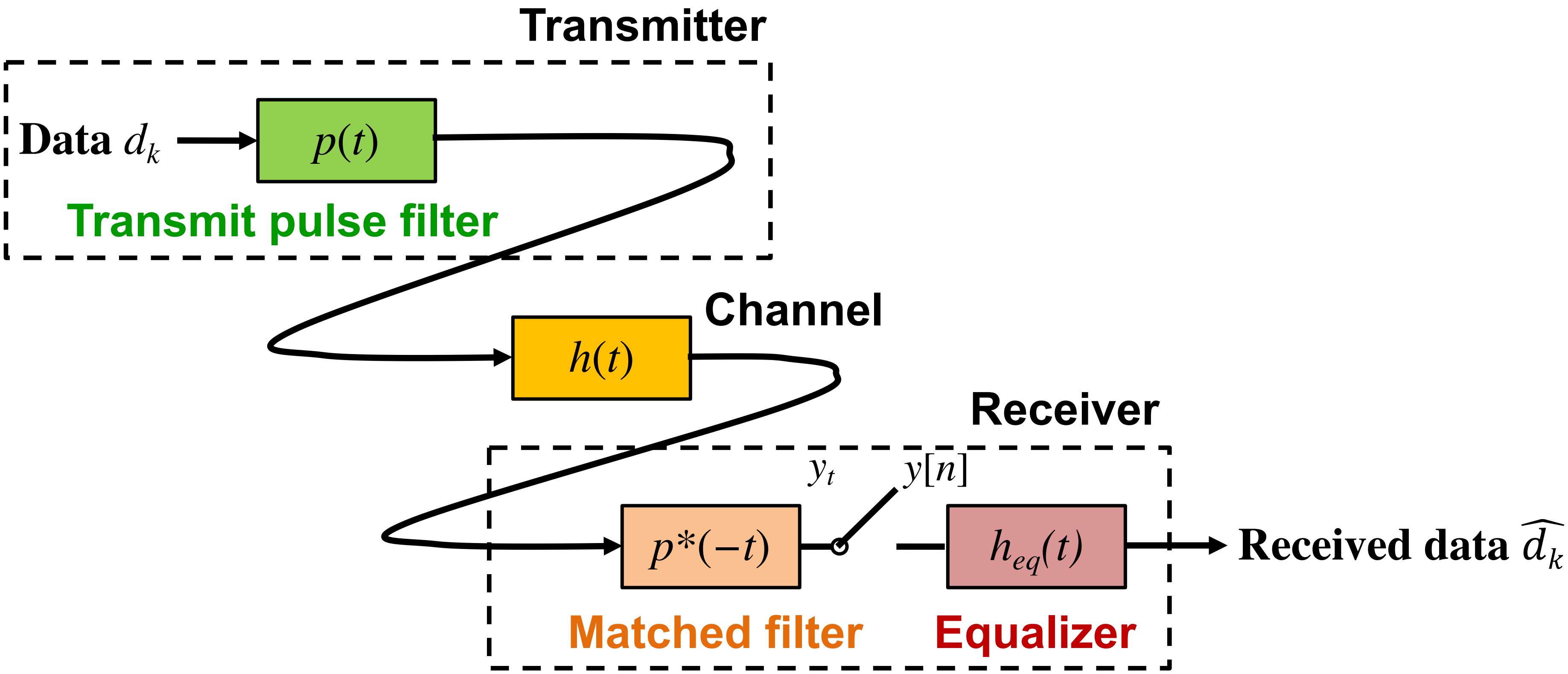
- Transmitted signal 🟡
- Received signal with ISI 🔴

- ISI at one symbol **depends on** the value of **other** symbols

Today

1. Delay Spread and Frequency-Selective Fading
2. **Time-Domain Equalization**
3. Orthogonal Frequency Division Multiplexing

Wideband System Design

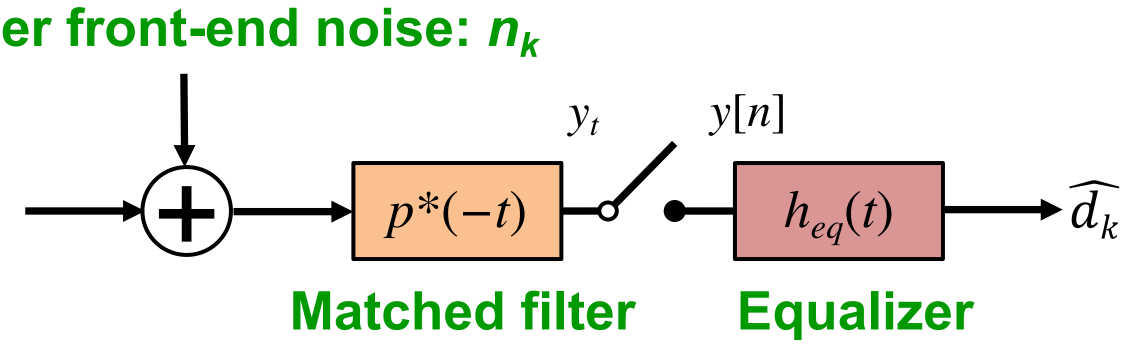


$$f(t) = (p * h)(t) * p^*(-t)$$

- ***Composite channel f*** between data and received signal *y*[*n*] is made up of the transmit pulse shape, radio channel, and matched filter

Zero-Forcing Equalizer

Receiver:

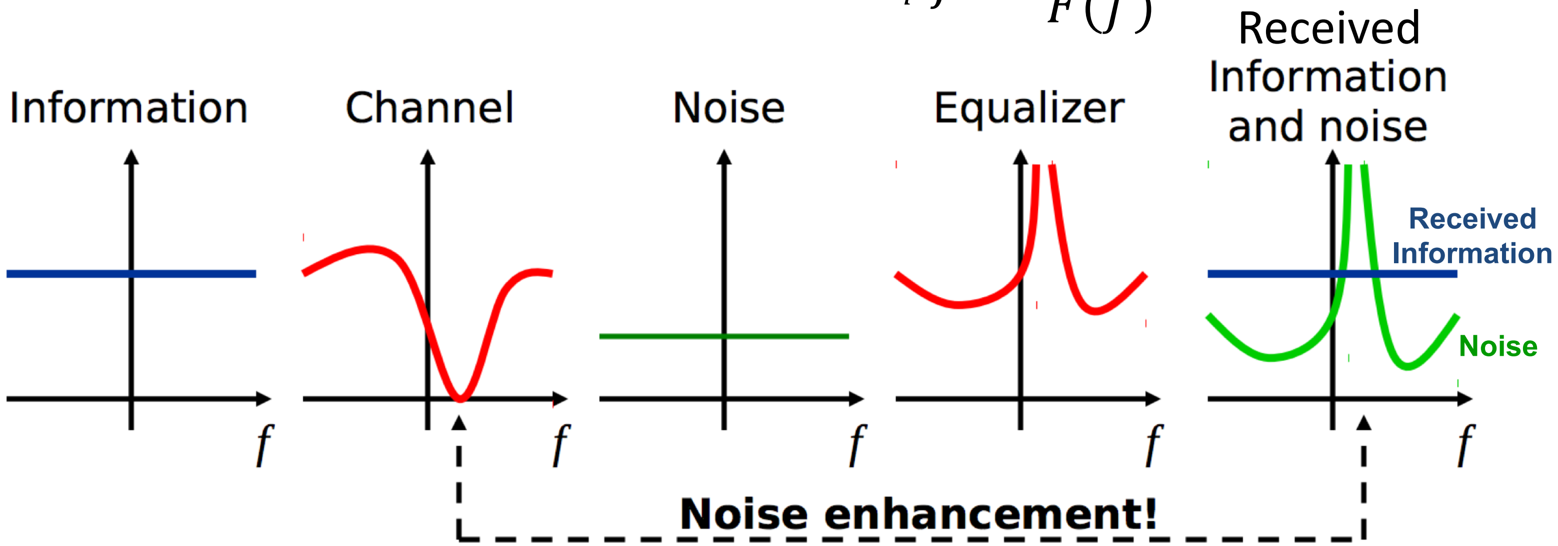


Receiver front-end noise: n_k

Matched filter

Equalizer

$$H_{eq}(f) = \frac{1}{F(f)}$$



Information

Channel

Noise

Equalizer

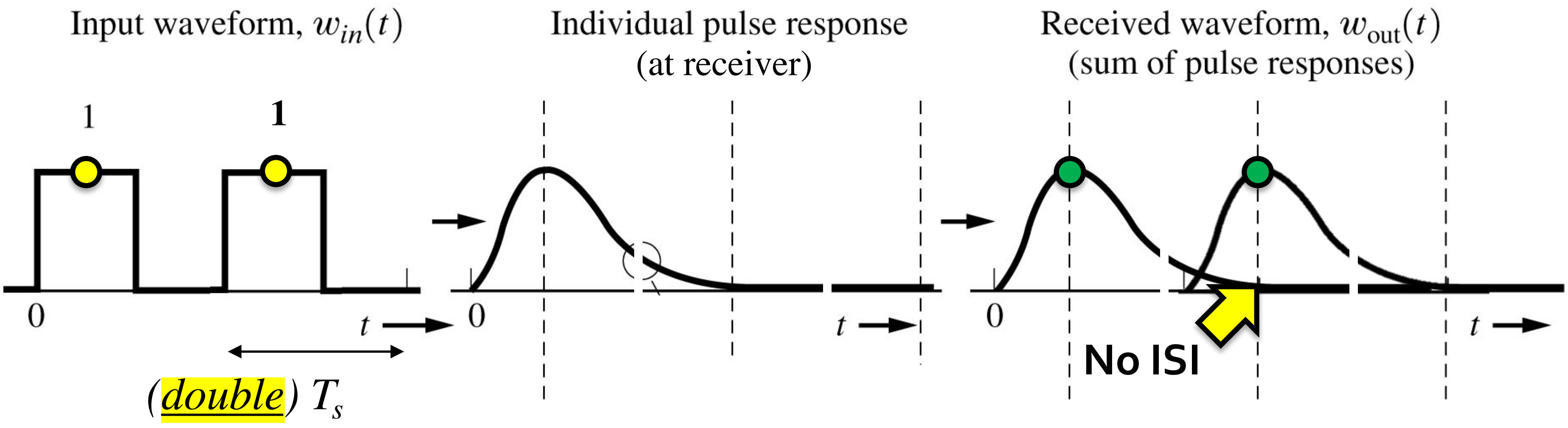
Received Information and noise

Noise enhancement!

Today

1. Delay Spread and Frequency-Selective Fading
2. Time-Domain Equalization
3. **Orthogonal Frequency Division Multiplexing**

Simple Solution: Increase symbol time

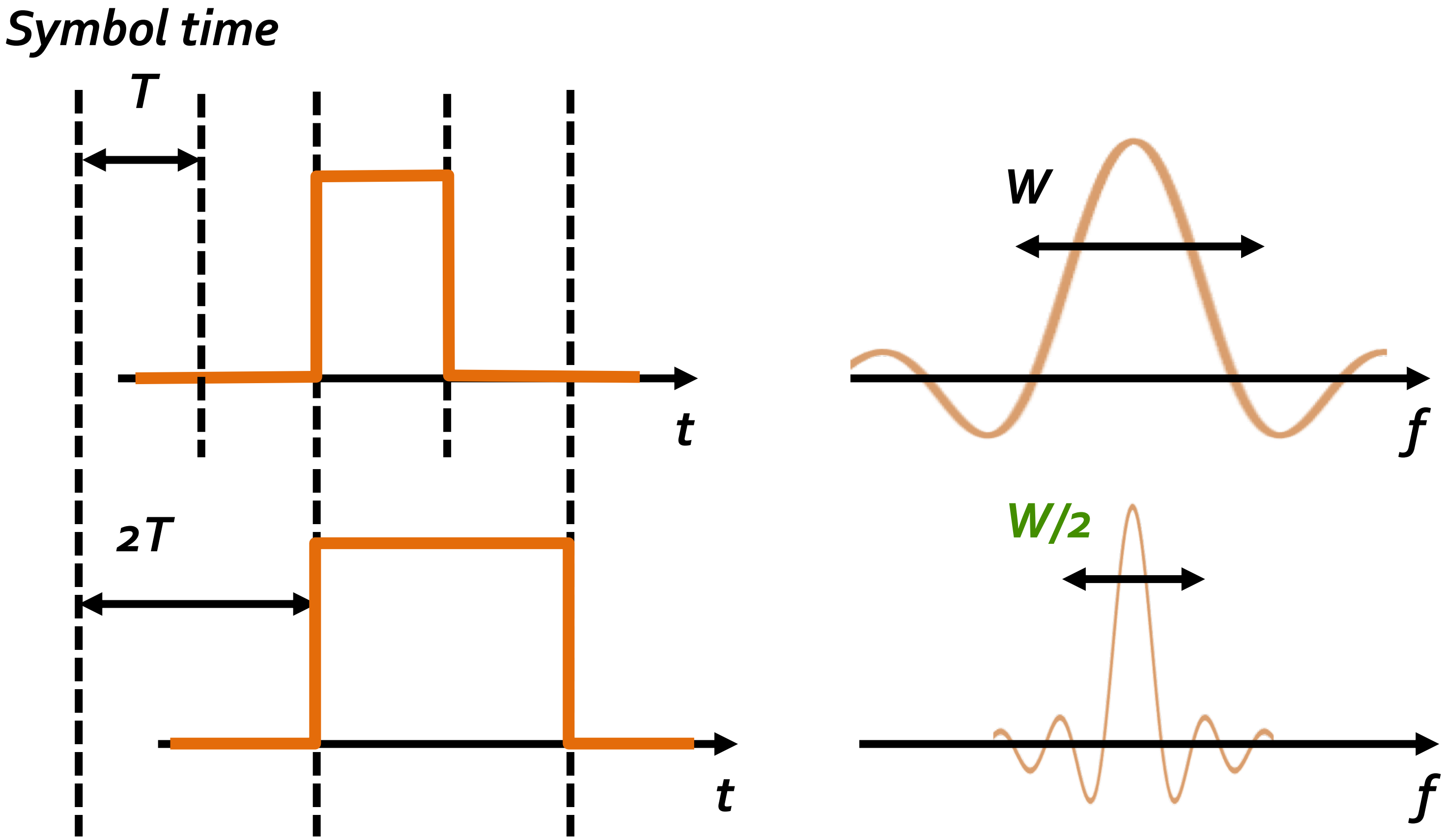


Transmitted signal ●

Received signal ●

- Choose symbol time $T_s \gg$ channel delay spread T_d

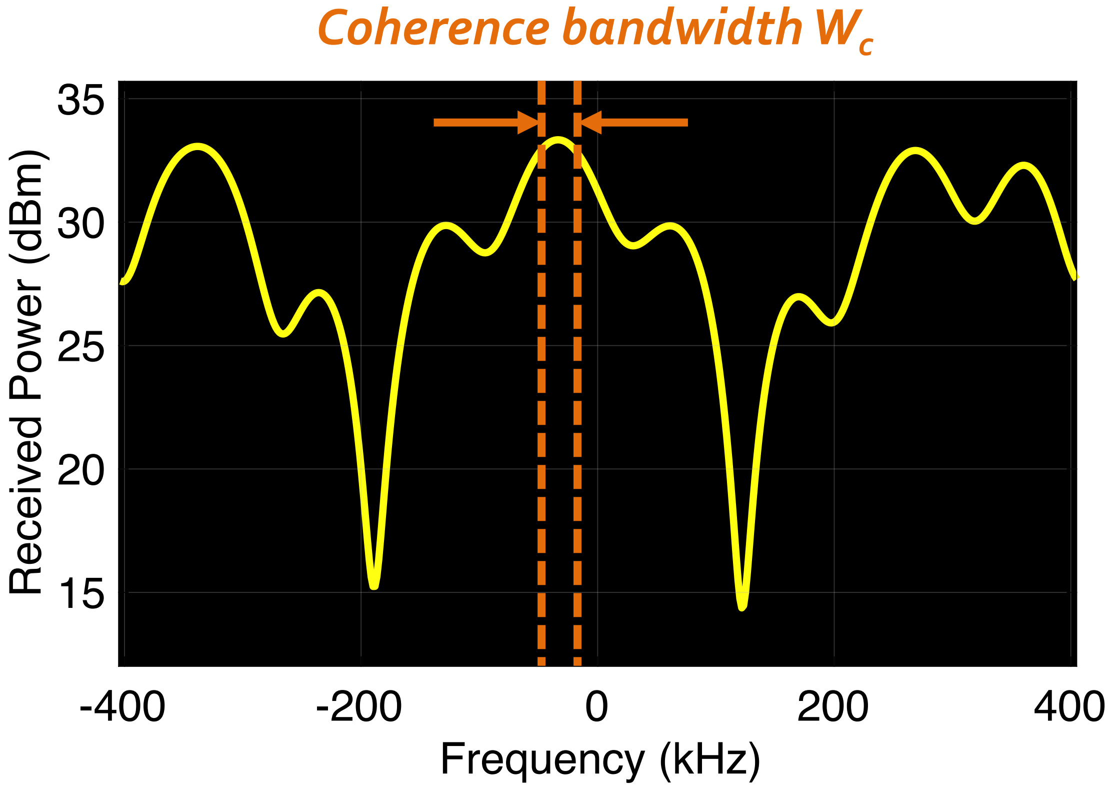
Symbol time determines frequency bandwidth



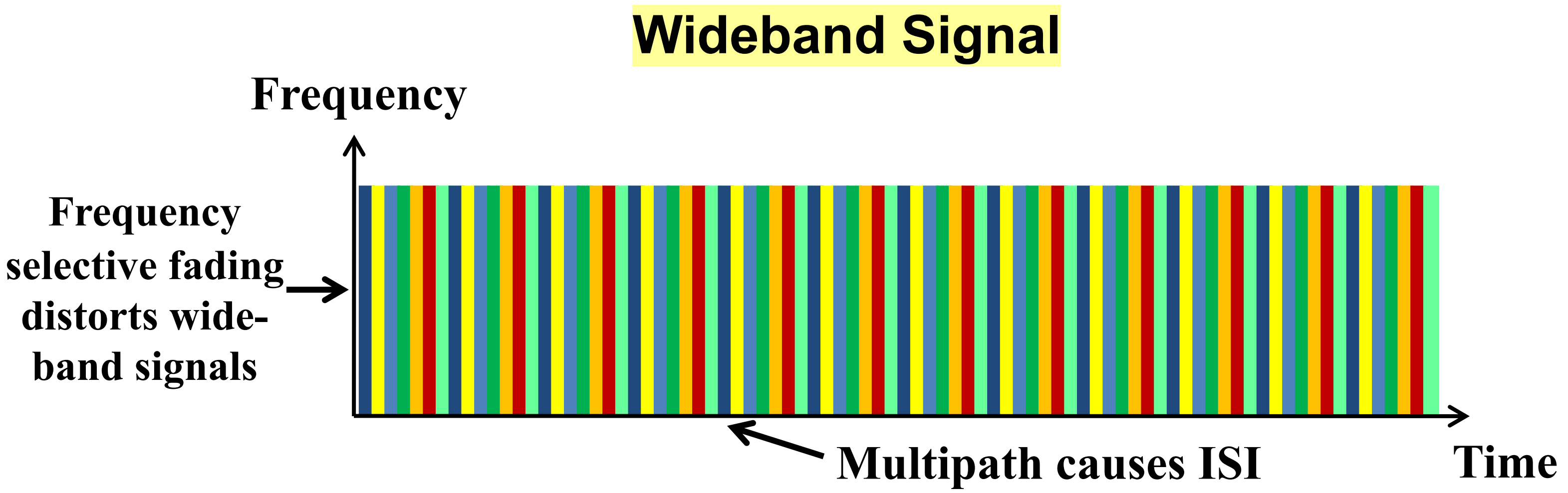
Slowing down by a factor of two **halves the frequency bandwidth** of the sender's signal

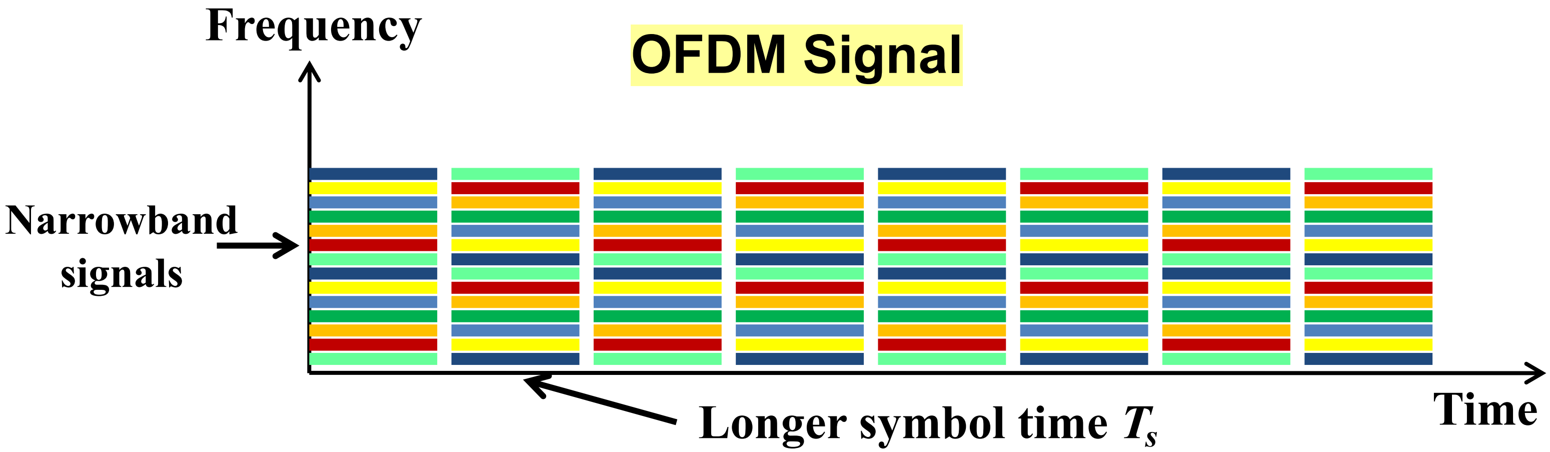
A narrowband signal "fits into" the coherence bandwidth

- Over what frequency range is the channel approximately the same? This is the ***coherence bandwidth*** $W_c \approx \frac{1}{2T_d}$



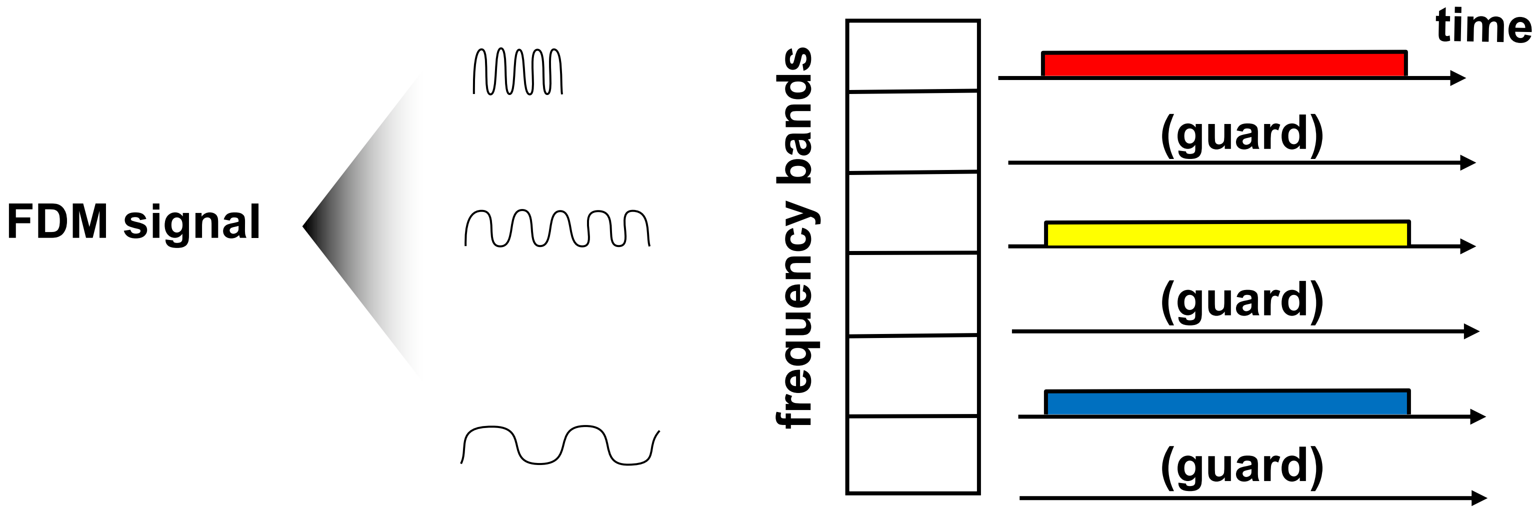
Wideband versus OFDM



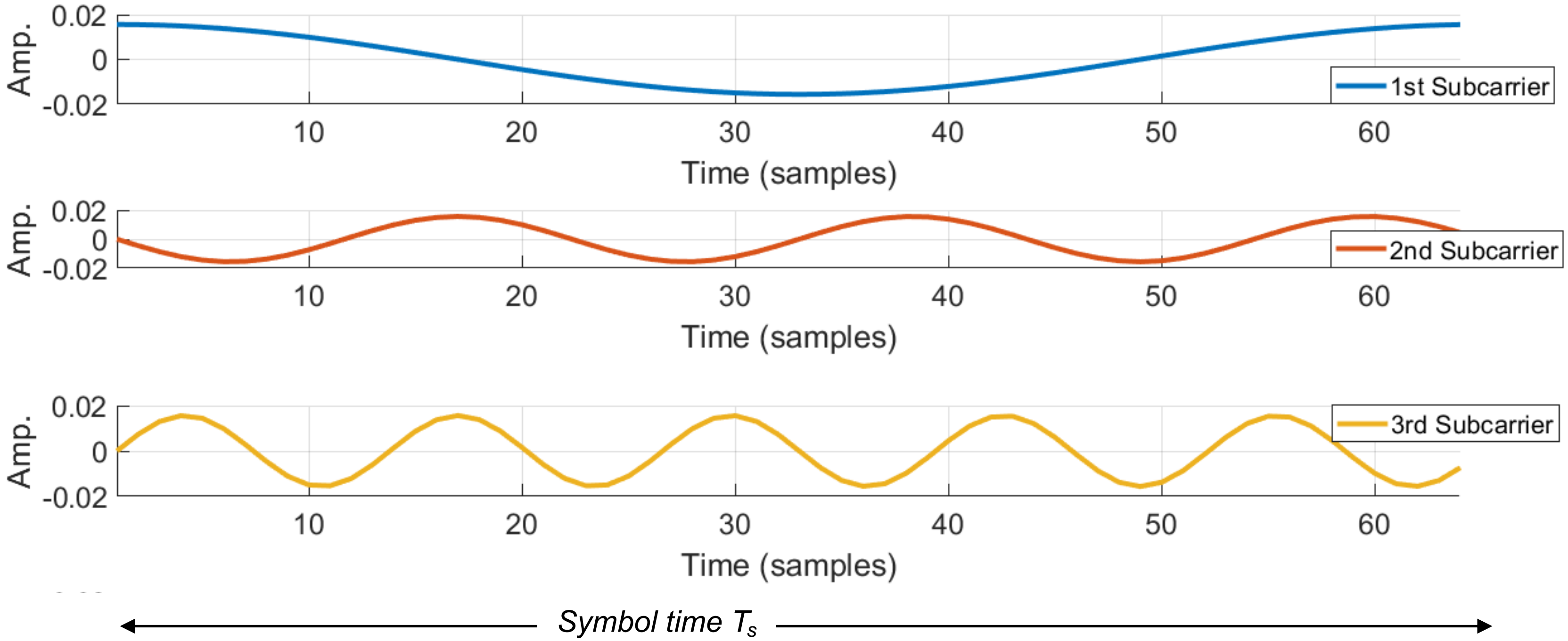


FDMA: Frequency Division Multiple Access

- Channel frequency bandwidth divided into smaller ***sub-bands***
- Each data gets **own frequency sub-band** (sinusoidal carrier)
- **But need to add quiet *guard* sub-bands** in between each sub-band
  - To **mitigate inter-sub band interference**



Long sinusoids, different frequencies



Baseband signal (In-Phase part)

- **Modulate amplitude/phase** of each above ***subcarrier*** to send information
- Different frequencies on different subcarriers **don't mutually interfere**

Integer-Multiple Frequencies

- Suppose *N* subcarriers and *N* complex valued data symbols
  - ***X*[*k*]** (0 ≤ k < N) are the **complex-valued data** we want to send

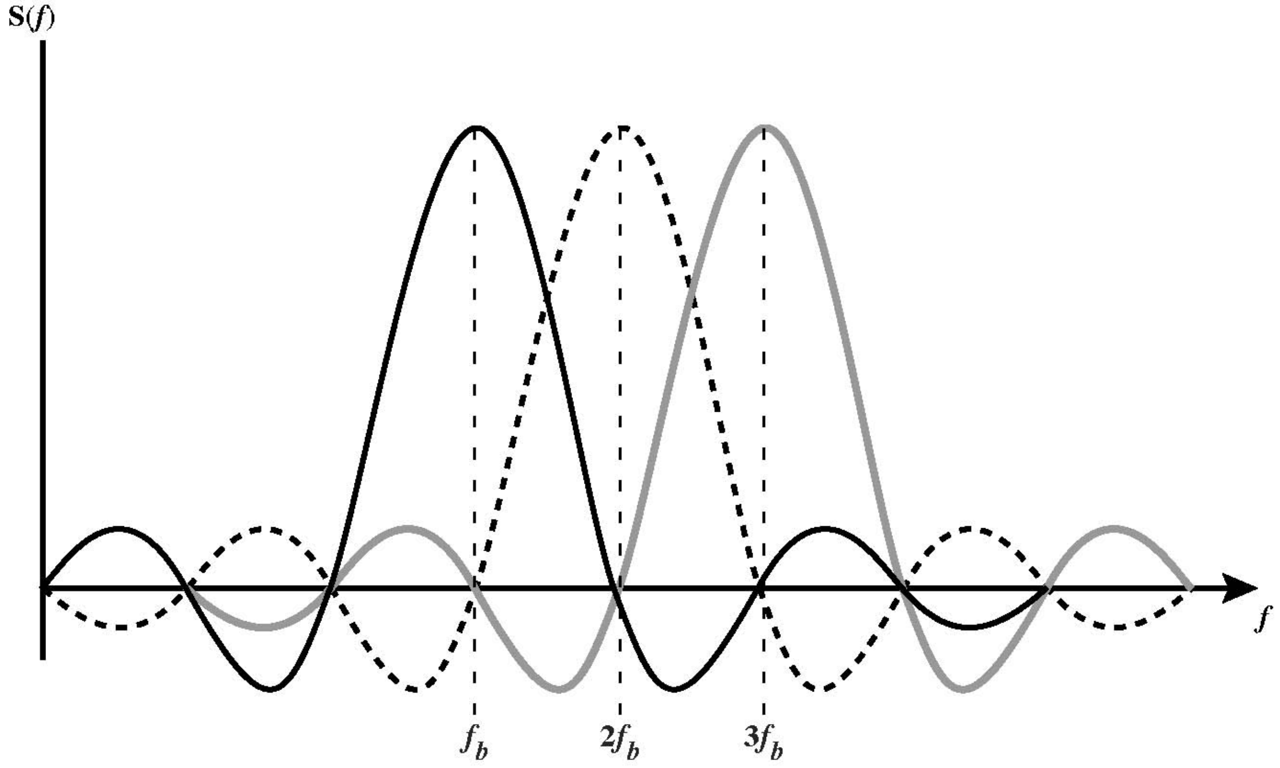
- **Goal:** for integers *k* from 0 to *N*−1:
  - **cosine** at frequency 2*πk*/*N* on the **I** channel
  - **sine** at frequency 2*πk*/*N* on the **Q** channel

- So (as before) **let each subcarrier** be the time-domain (indexed by *n*) signal:

$$X[k] \cdot \left[\cos\left(\frac{2\pi kn}{N}\right) + j\sin\left(\frac{2\pi kn}{N}\right)\right]$$

OFDM Subcarriers are "Orthogonal"

- Integer-multiple frequencies so **peaks** of each subcarrier **coincide in frequency with zeros** of other subcarriers
  - No guard bands required!



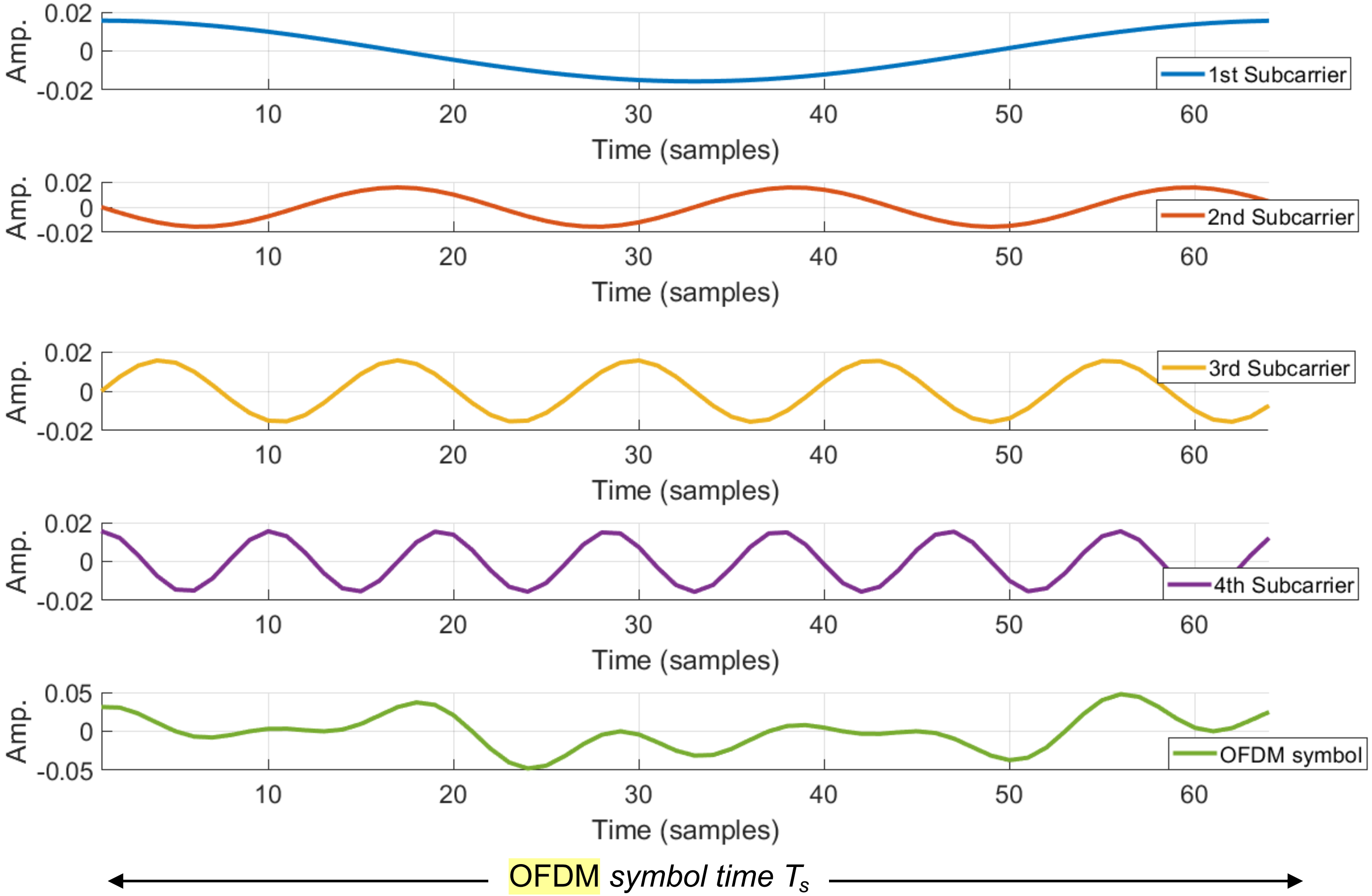
Generating an OFDM Signal

- How to generate this type of signal:

$$x[n] = \sum_{n=0}^{N-1} X[k] \cdot \left[\cos\left(\frac{2\pi kn}{N}\right) + j \sin\left(\frac{2\pi kn}{N}\right)\right]$$

- By Euler's formula, $x[n] = \sum_{n=0}^{N-1} X[k] e^{j2\pi kn/N}$
  - This is the (familiar) **Inverse DFT**
    - Transmit time domain signal $x[n]$ = $\mathbf{DFT}^{-1}\{\, X[k] \,\}$
    - So has an **efficient hardware implementation**

Transmitting one OFDM symbol in time

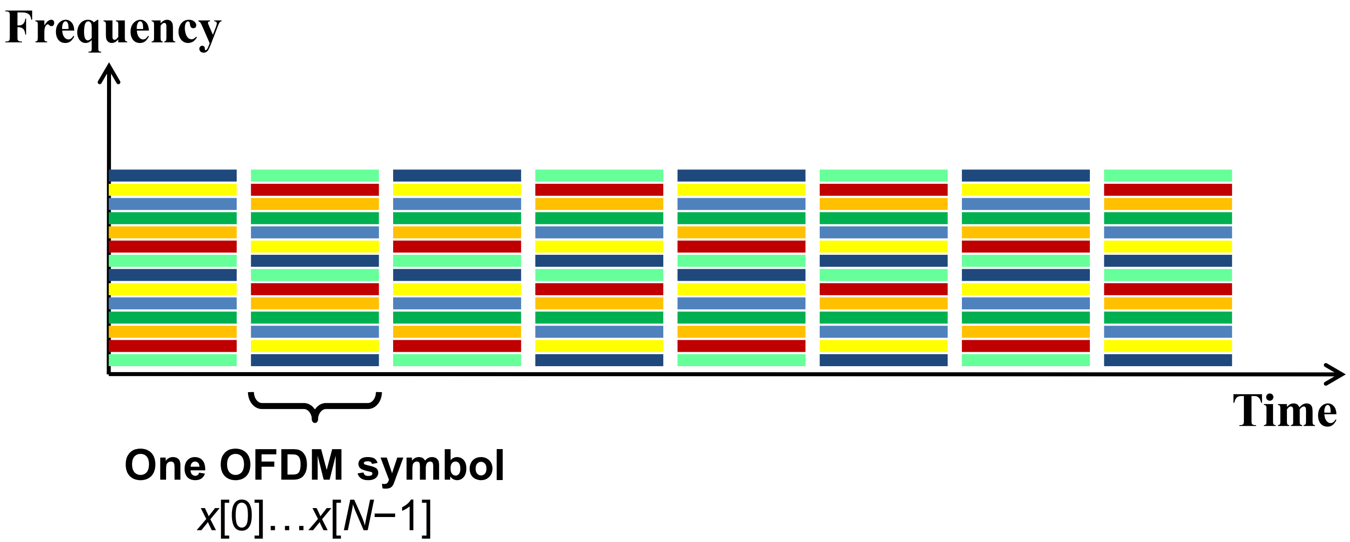


OFDM *symbol time T_s*

Receiving an OFDM signal

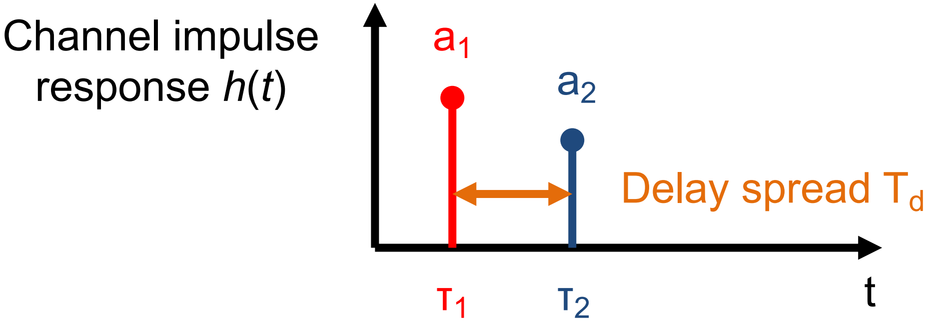
- **Apply the DFT** to each OFDM symbol individually, **to recover data:**

$$X[k] = 1/N \sum_{0}^{N-1} x[n] e^{-j2\pi kn/N}$$

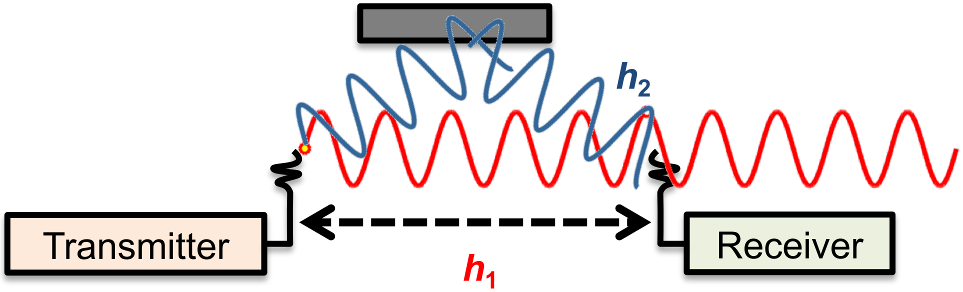


What about the wireless channel?

- **Channel impulse response** *h*(*t*) is a **function of time**
  - Impulse has an infinite bandwidth

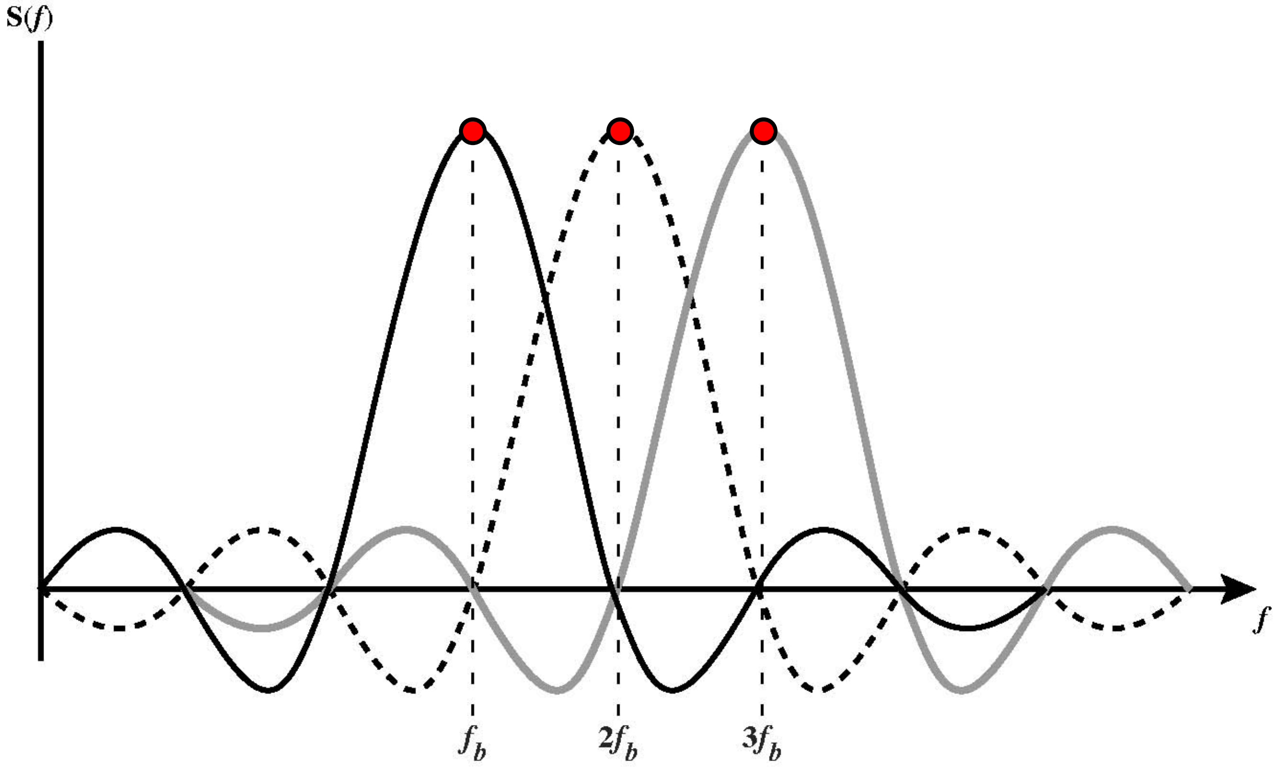


- Effect of a very narrowband channel is **one complex number *h***
  - Subcarrier eventually arrives on all paths, *e.g.* $h = h_1+h_2$:



What about the wireless channel?

- Channel used at each subcarrier's **discrete frequency location**



e.g. with three subcarriers: **k=1** **k=2** **k=3**

Estimating the Channel

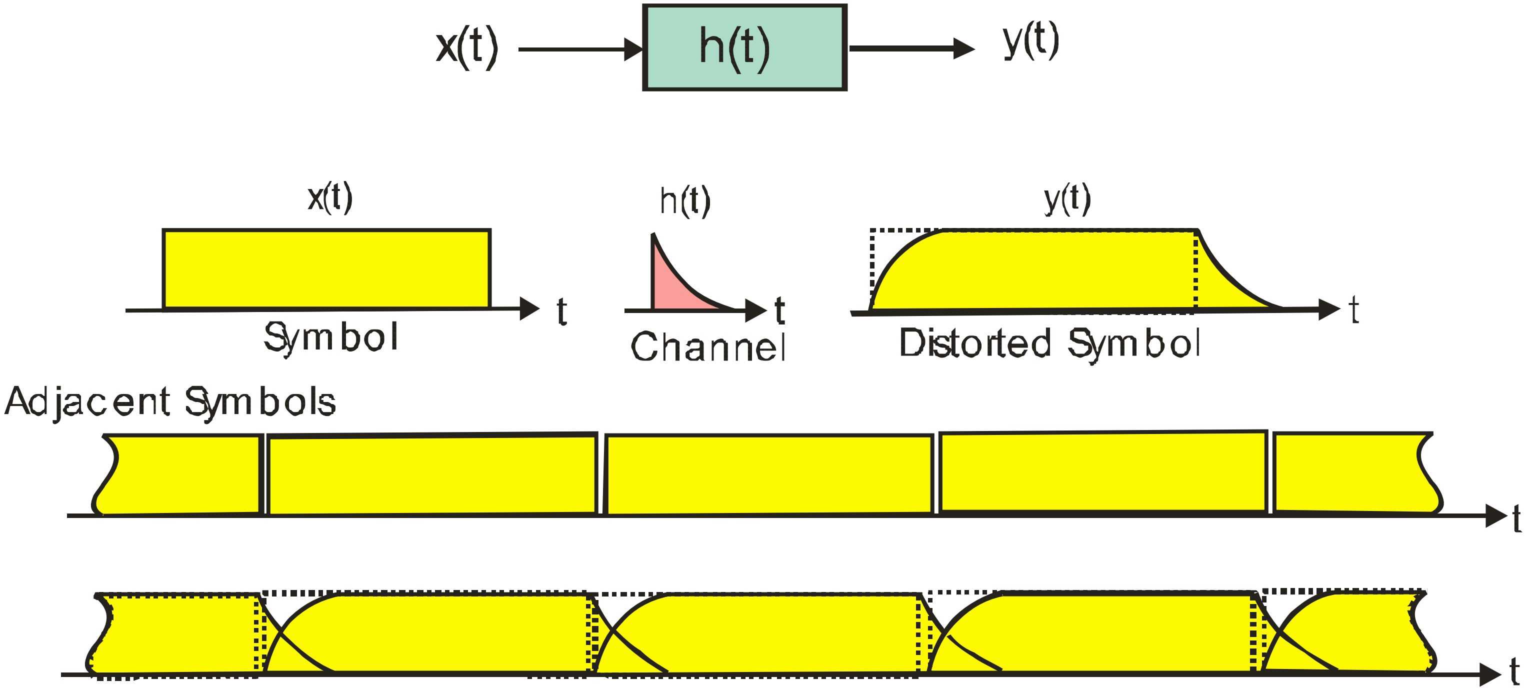
- Transmit known a OFDM preamble symbol $p[n]$
  - In frequency domain **on frequency *k*** denote preamble ***P*[*k*]**

- After DFT on the preamble, the receiver hears frequency domain value ***Y[k]***

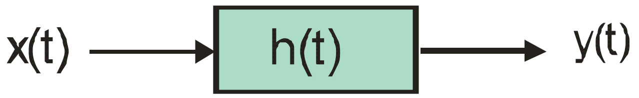
- Receiver computes channel estimate on kth subcarrier: H[k] = Y[k] / P[k]

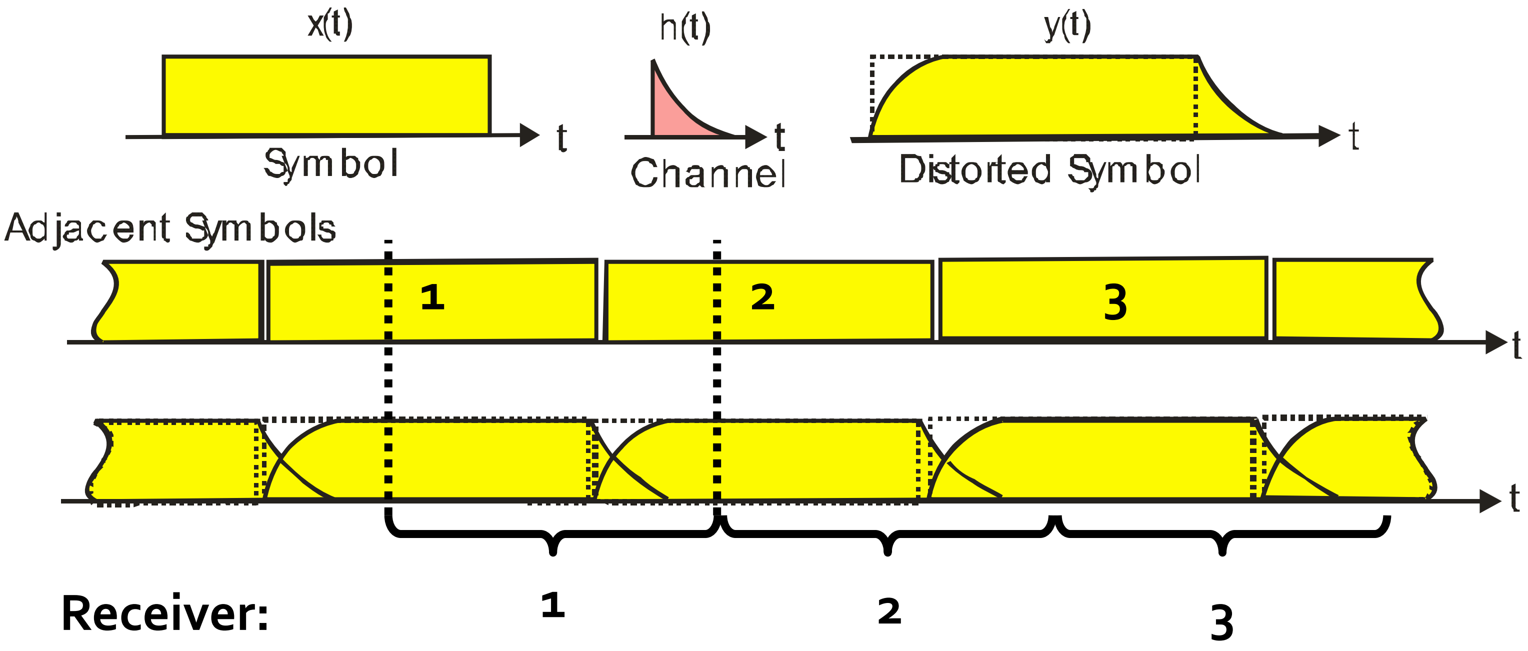
- **Aside:** To compute channel impulse response: $\mathbf{DFT}^{-1}\{ H[k] \} = h[n]$

Problem: Inter-OFDM Symbol Interference



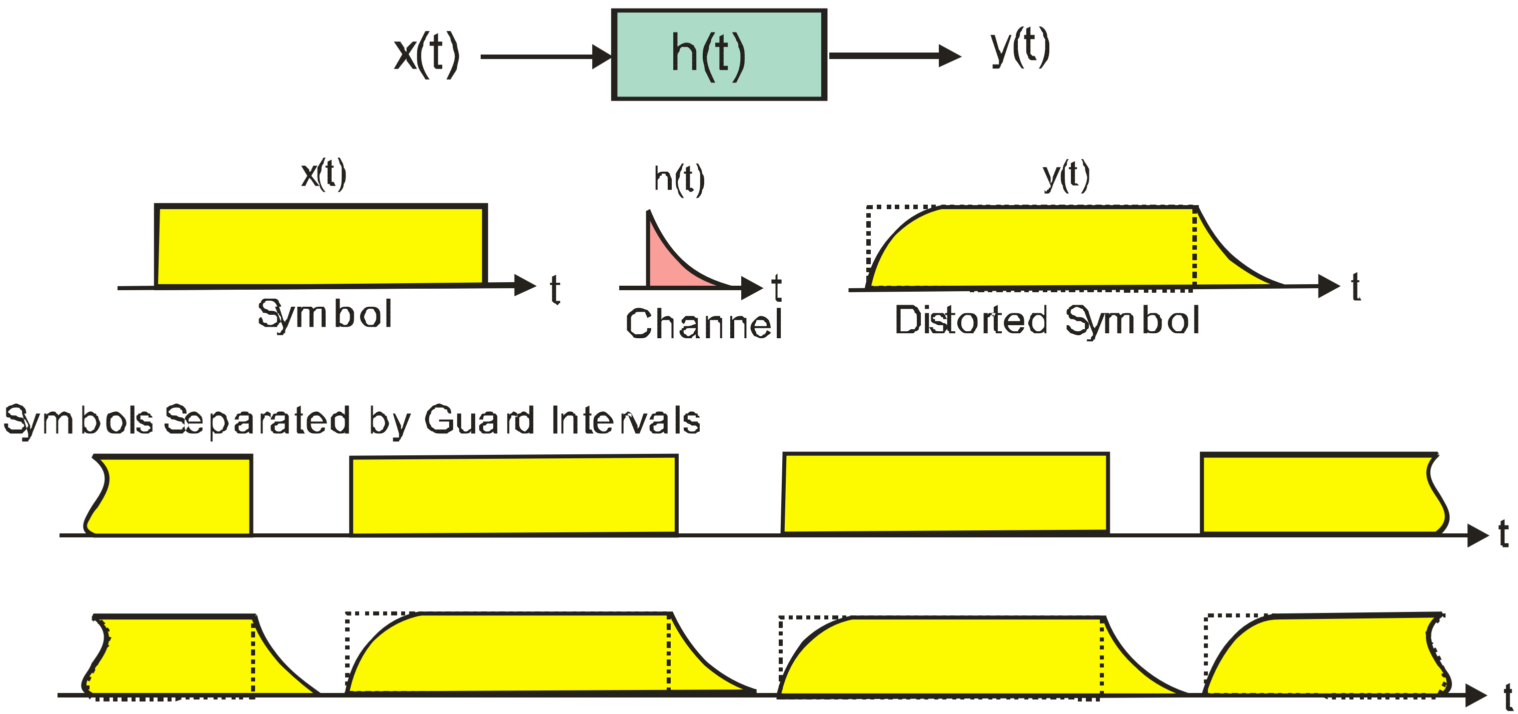
Problem: Receiver synchronization





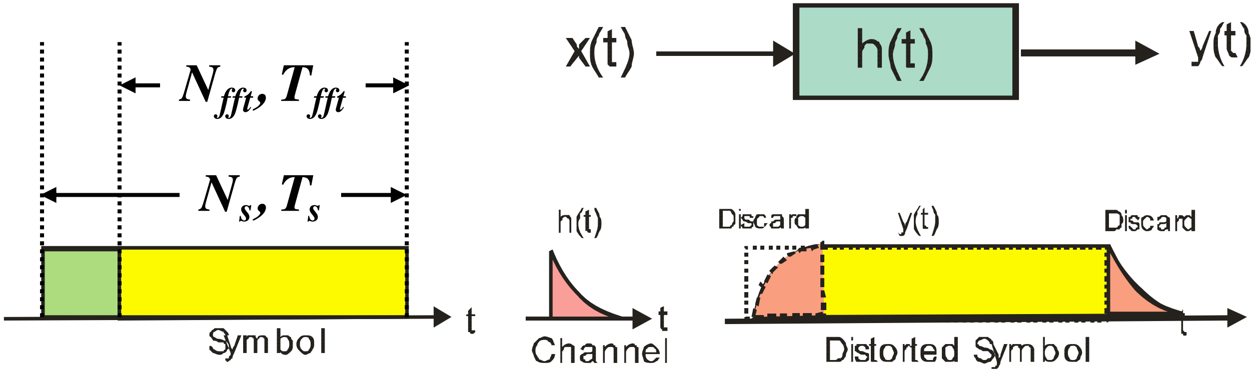
Receiver's view of Symbol 1 contains **actual Symbols 1 and 2**

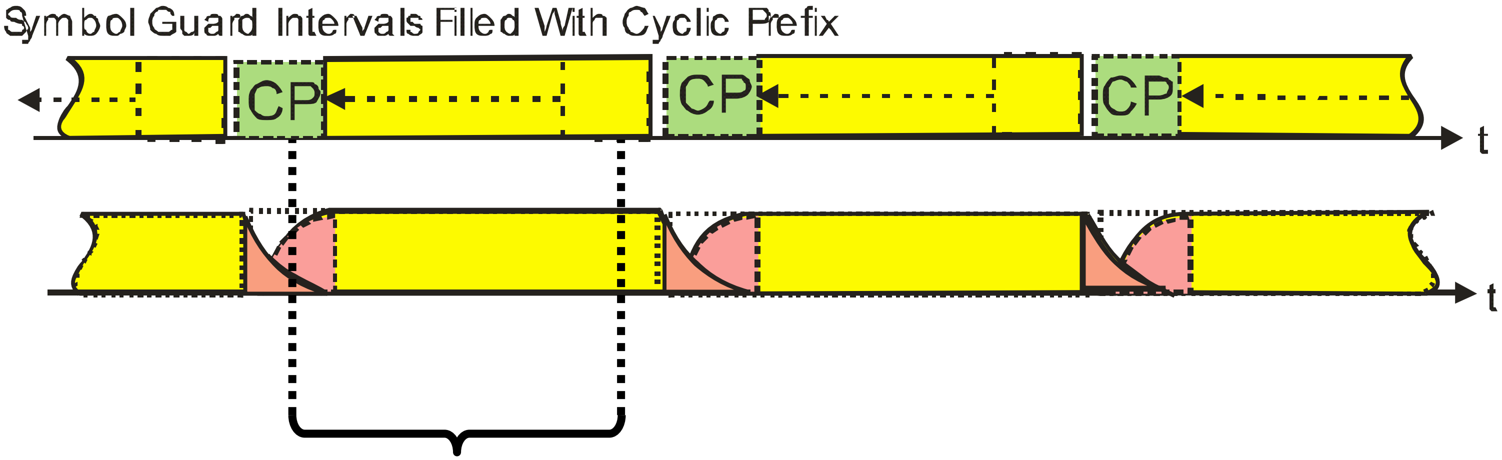
Interference solution: Inter-symbol guard interval



Guard interval between adjacent symbols mitigates adjacent symbol interference

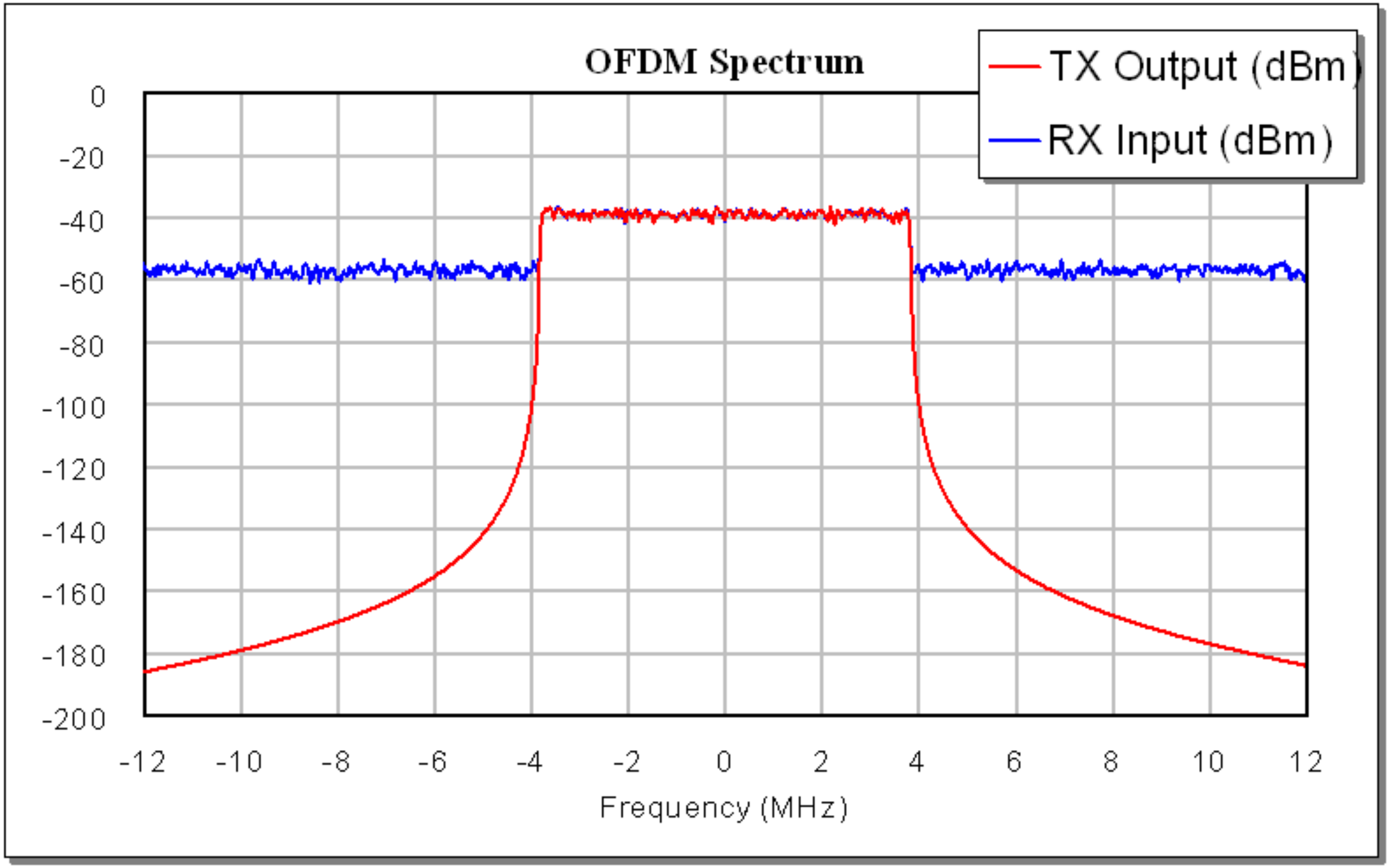
Synchronization solution: Cyclic prefix





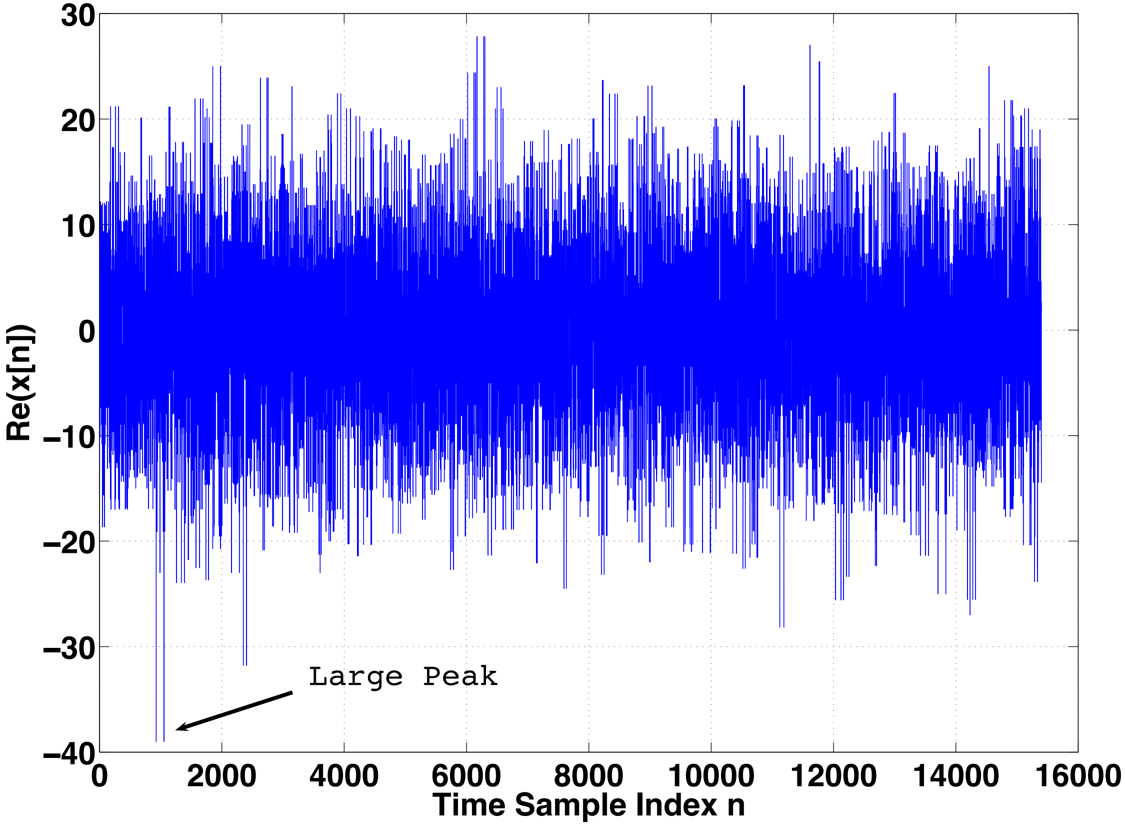
Receiver: **Symbol OK!**

OFDM signal: Frequency-Domain view



Uniform power in the frequency domain over the OFDM signal bandwidth

OFDM signal: Time Domain View

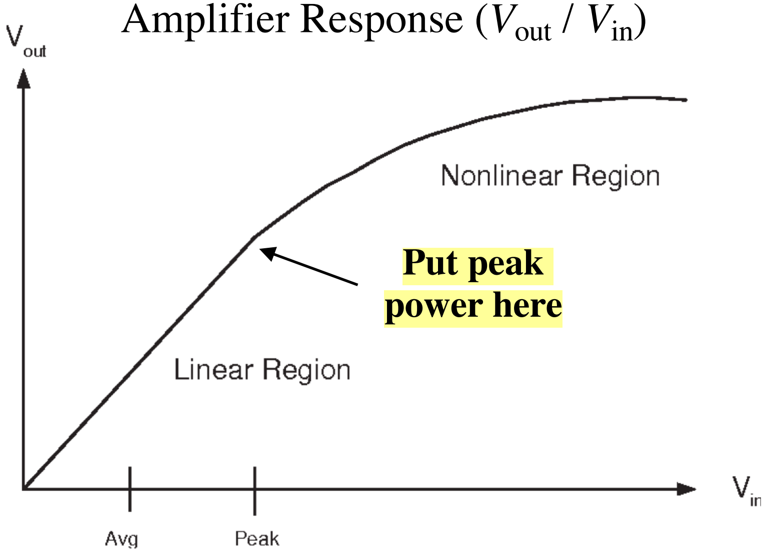


- Many low-frequency sinusoids in the time domain
- Occasionally **in time,** many will **all constructively interfere**
  - **Result:** High ratio of **peak power / average power**

Peak to Average Power & Transmit Amplifiers

- ***Transmit power amplifier*** sits just before the transmit antenna
- Peak power in non-linear region causes signal distortion
  - So **lower input signal level:**

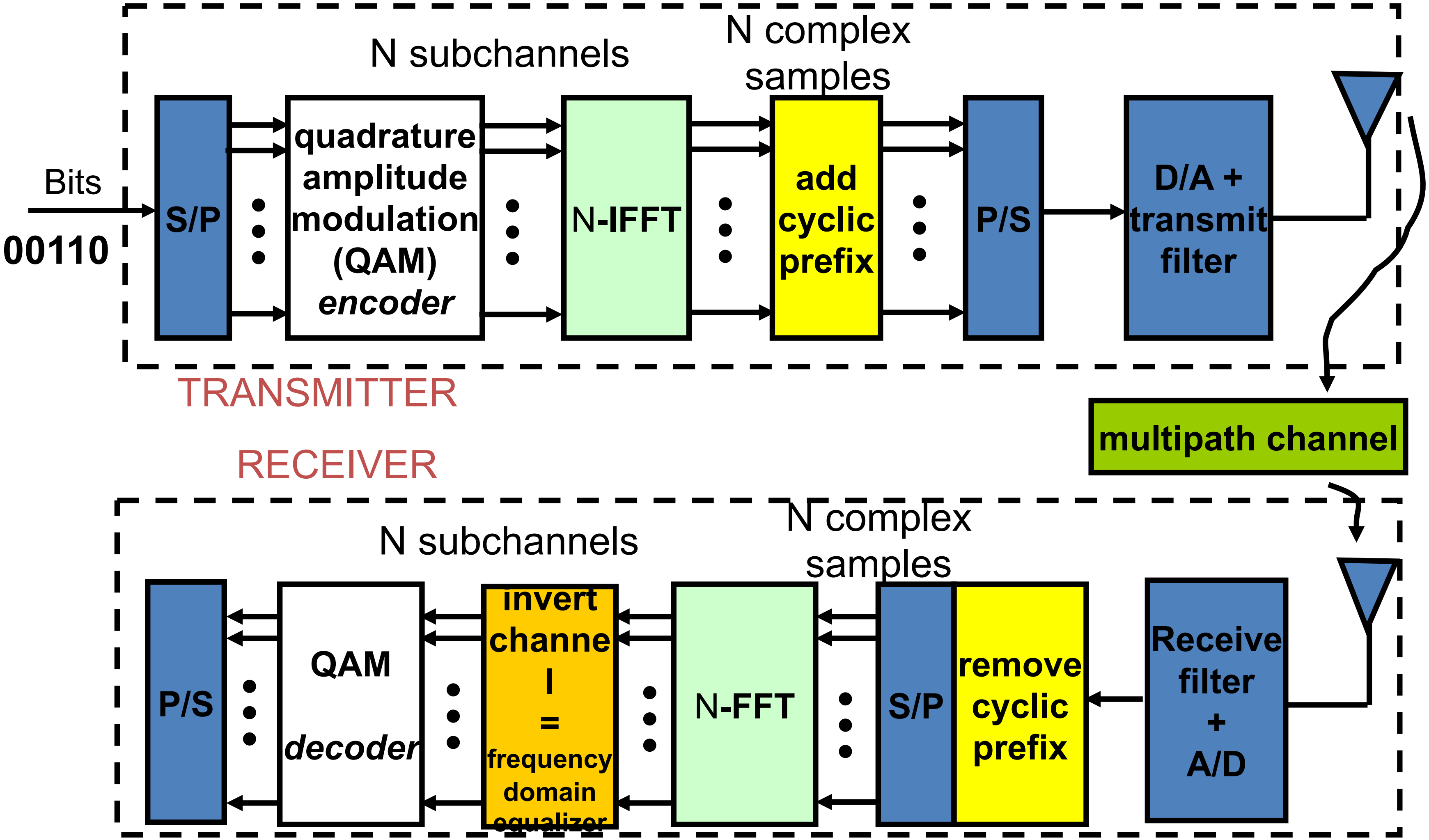
Typical Transmit Power Amplifier Response (V_{out} / V_{in})



Put peak power here

- **High** peak to average power ratio (PAPR) → **Low average** power level →
  - Signal mostly uses **fewer levels in discrete representation**, so **high quantization error** (another form of **distortion**)

An OFDM Modem



Packet detection

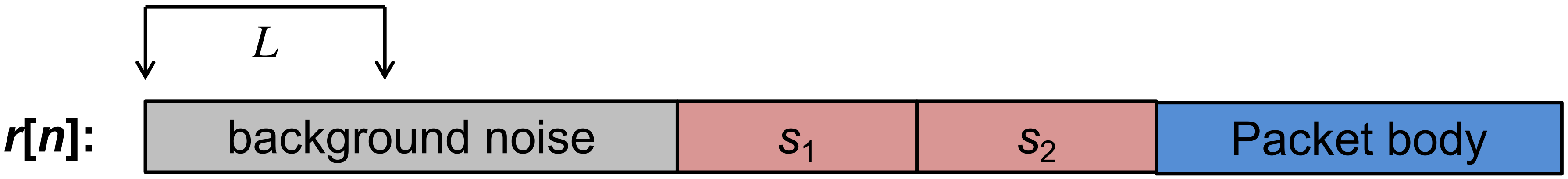
- OFDM uses **two identical, repeated symbols** s_1, s_2 in the preamble for packet detection:

- Receiver radio is **always listening, receiving samples**
  - Call this **received sample stream $r[n]$**

Searching for the preamble in noise

- Suppose each preamble symbol is of length *L*
- Receiver computes $\mathrm{c}[\mathrm{n}] = \sum_{k=0}^{L-1} r[n+k]\, r^*[n+k+L]$

Computing *c*[0]:

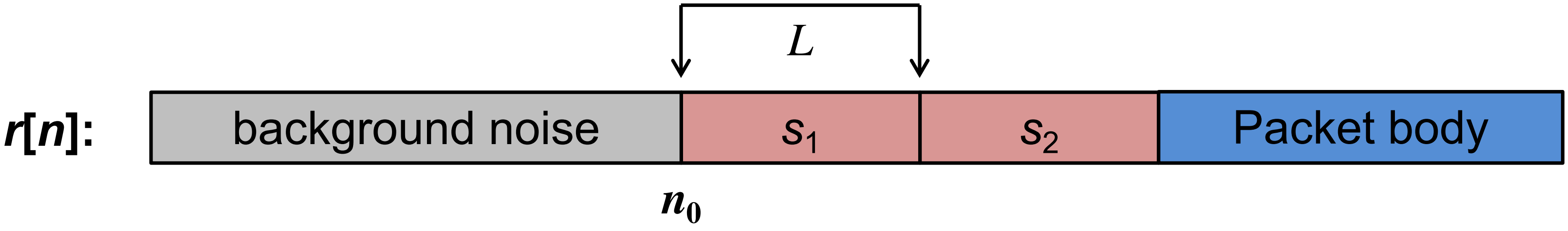


- **Angle of each term** in the sum is **random**
- **Sum** of complex numbers with **random angle ≈ 0**
  - **c[0] ≈ 0**

Search window encounters preamble

- Suppose preamble at position n_0
- Receiver computes $\mathrm{c[n]} = \sum_{k=0}^{L-1} r[n+k]r^*[n+k+L]$

Computing $c[n_0]$:



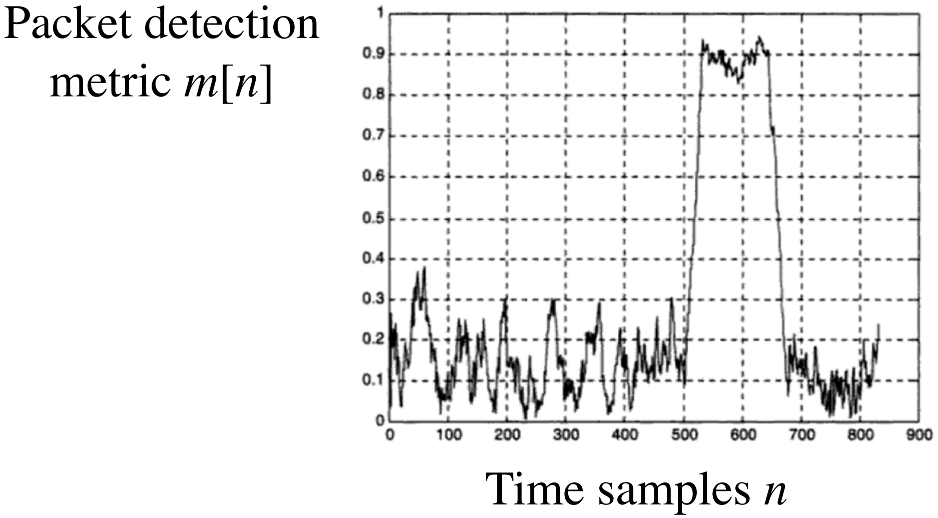
- **$\measuredangle(zz^*) = 0$, so angle of each term** in the sum is **≈ 0**
- **Sum** of complex numbers with **≈ 0 angle is large**
  - **$c[n_0]$ is large**

Schmidl-Cox Packet Detection

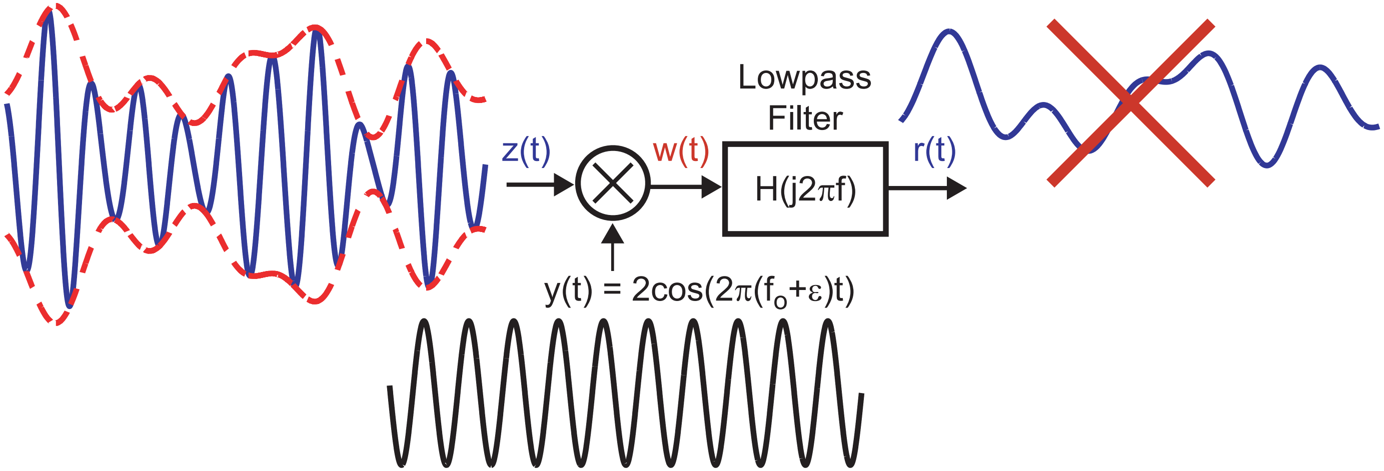
- $\mathrm{c[n]} = \sum_{k=0}^{L-1} r[n+k]r^*[n+k+L]$

- Normalize power fluctuations in r[n], by measuring power:
  - $\mathrm{p[n]} = \sum_{k=0}^{L-1} |r[n+k]|^2$

- **Schmidl-Cox Packet Detection signal:** ***m*[*n*] =** *c*[*n*] / *p*[*n*]



A Closer Look at Carrier Frequency Offset



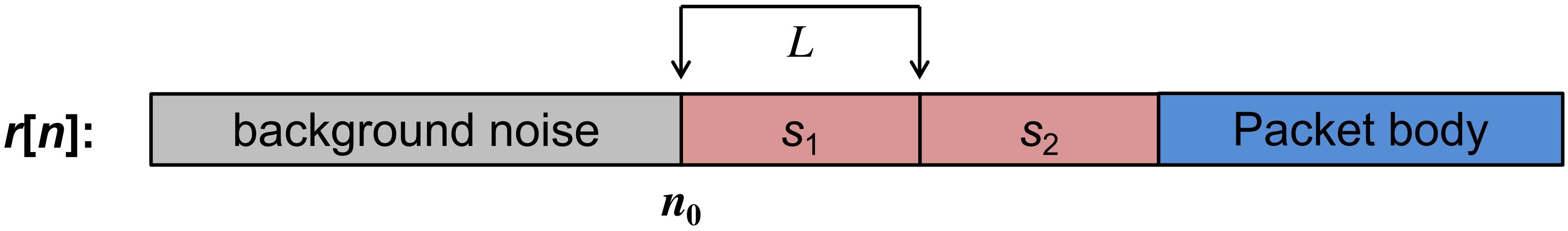
- **Limited precision** of frequency oscillators
- Up-convert baseband signal s_n to passband signal $y_n = s_n e^{j2\pi f_{tx} n T_s}$
- Down-convert passband signal y_n back to baseband:

$$r_n = s_n e^{j2\pi f_{tx} n T_s} e^{j2\pi f_{rx} n T_s} = s_n e^{j2\pi \Delta f n T_s} \ (\boldsymbol{\Delta f = f_{rx} - f_{tx}})$$

Estimating Carrier Frequency Offset

- Because of carrier frequency offset, $s_2 = s_1 e^{j2\pi\Delta f N T_s}$
  - $\mathrm{c}[n_0] = \sum_{k=0}^{L-1} r[n_0+k] r^*[n_0+k+L]$

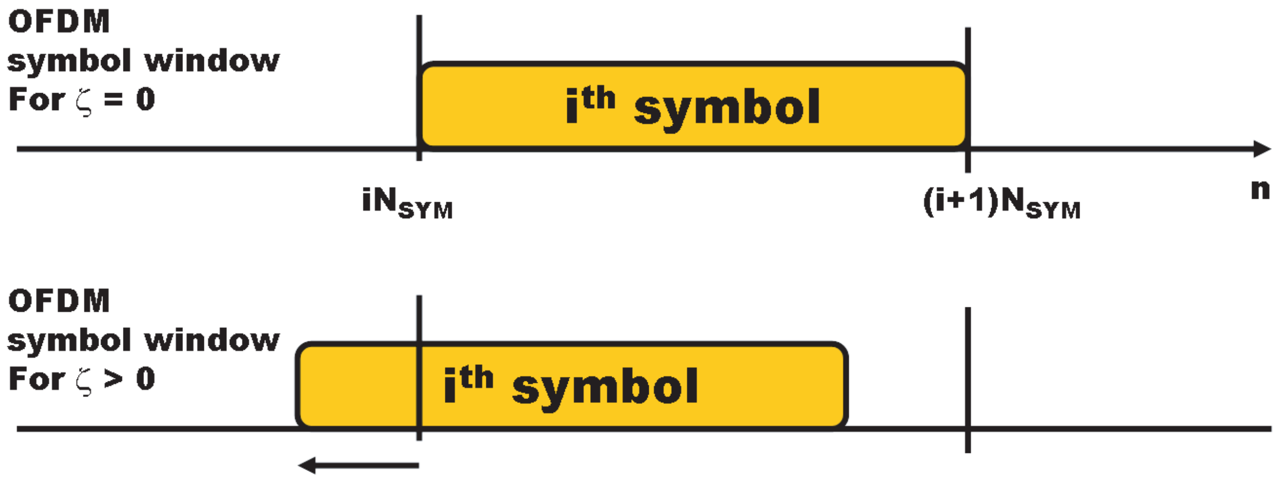
Computing $c[n_0]$:



- Consider the k^{th} term in sum: $r[n_0+k] r^*[n_0+k] e^{j2\pi\Delta f N T_s}$
  - This is equal to $e^{j2\pi\Delta f N T_s} |r[n_0+k]|^2$
    - So all terms have the **same angle** $2\pi\Delta f N T_s$

- **So,** ***carrier frequency offset estimator*** $\widehat{\Delta f} = \dfrac{\measuredangle c[n_0]}{2\pi N T_s}$

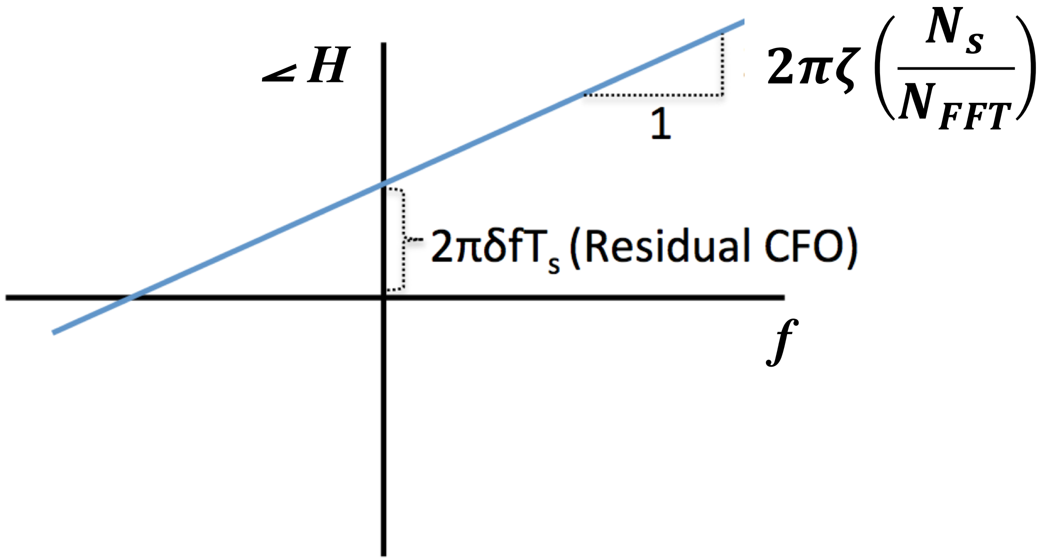
Sample Clock Offset



- The transmitter and receiver may sample the signal at slightly different rates, leading to a ***sample time offset ζ***

- All subcarriers experience the same sampling delay, but travel over different frequencies

Correcting Sample Clock Offset in the Frequency Domain



- Sample clock offset : slope
- Residual CFO: intersection with y-axis

Example: IEEE 802.11a, 802.11g

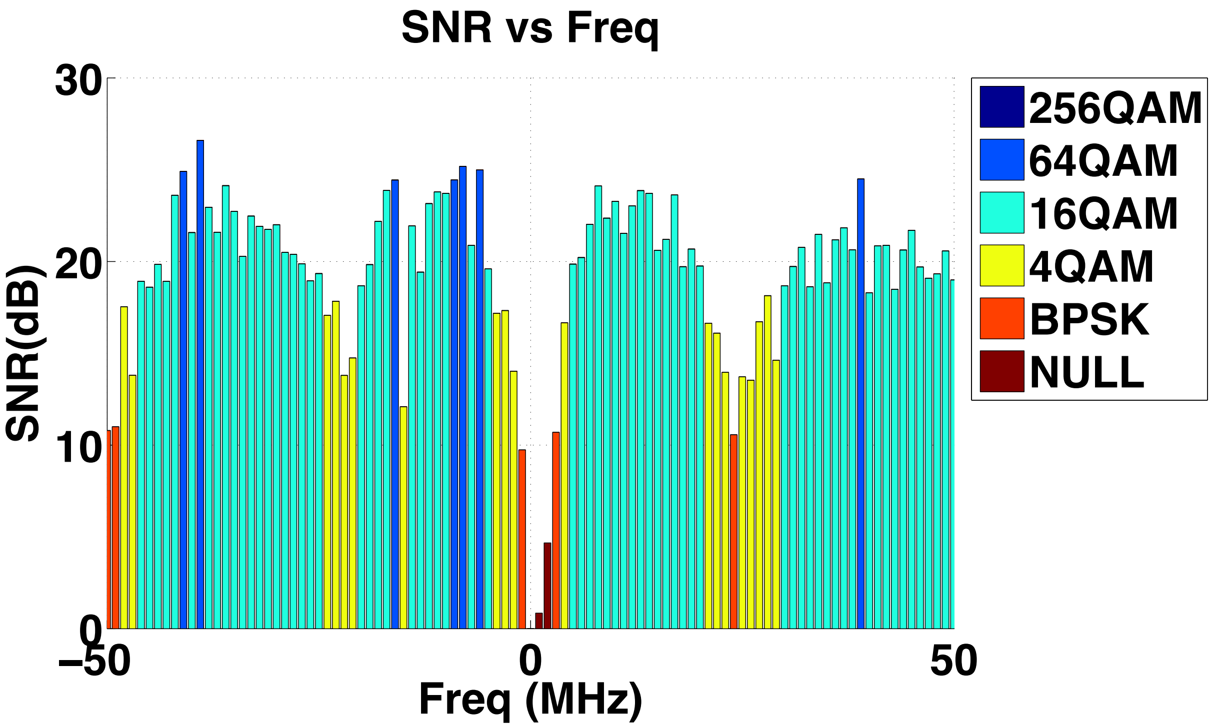
- OFDM with up to 48 subcarriers
  - Subcarrier spacing is 312.5 KHz
  - Subcarriers modulated: BPSK, QPSK, 16-QAM, or 64-QAM

- Uses a convolutional code at a rate of ½, 2/3, ¾ , or 5/6 to provide forward error correction

- Results in data rates of 6, 9, 12, 18, 24, 36, 48, and 54 MBps

- Cyclic prefix is 25% of a symbol time (16 vs 64)

Per-subcarrier Bit Rate Choice



Tuesday Topic:

MIMO I: Spatial Diversity

Friday Precept:

Lab 4: Single-carrier transceiver on the HackRF hardware