The Radio Channel



COS 463: Wireless Networks Lecture 14 Kyle Jamieson

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Radio Channel: Motivation

- The radio channel is what limits most communications systems the main challenge!
 - Understanding its properties is therefore key to understanding radio systems' design
- There is variation in many different properties
 - Carrier frequency, environment (*e.g.* indoors, outdoors, satellite, space)
- Many different models covering many different scenarios

Channel and Propagation Models

- A channel model describes what happens
 - Gives channel output power for a particular input power
 - "Black Box" no explanation of mechanism
 - Requires appropriate statistical parameters (e.g. loss, fading)
- A propagation model describes how it happens
 - How signal gets from transmitter to receiver
 - How energy is redistributed in time and frequency
 - Can inform channel model parameters

Today

- 1. Large scale channel model
 - Friis Free space model
 - How much power delivered from omnidirectional transmitter to omnidirectional receiver, in free space?

2. Small-scale channel models

Transmitting in Free Space



- Deliver P_t Watts to an omnidirectional transmitting antenna
- So then **power density** (Watts per unit area) at **range** d is $p = \frac{P_t}{4\pi d^2}$ W/m²
 - Independent of wavelength (frequency)

Idealized Receive Antenna

- **Effective aperture** A_e : fraction of incident power density *p* captured and received: $A_e = \frac{\lambda^2}{4\pi}$
 - Larger antennas at greater λ capture more power
- Therefore, **power received** P_r is the product of the power density and effective aperture:

$$P_r = p \cdot A_e = \frac{P_t \lambda^2}{(4\pi)^2 d^2}$$

Antenna Gain

- Antennas don't radiate power equally in all directions
 - Specific to the antenna design
- Model these gains in the directions of interest between transmitter, receiver:
 - Transmit antenna gain G_t
 - Receive antenna gain G_r



Friis Free Space Channel Model

• **Power received** P_r is the product of the power received by idealized antennas, times transmit and receive antenna gains:

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2}$$

Today

1. Large scale channel models

- 2. Small-scale channel models
 - Multi-path propagation
 - Motion and channel coherence time

Small-scale versus large-scale modeling



Small-scale models: Characterize the channel over at most a few wavelengths or a few seconds

Multipath Radio Propagation

- Receiver gets **multiple copies** of signal
 - Each copy follows different path, with different path length
 - Copies can either strengthen or weaken each other
 - Depends on whether they are in or out of phase
- Enables communication even when transmitter and receiver are not in "line of sight"
 - Allows radio waves effectively to propagate around obstacles, thereby increasing the radio coverage area
- Transmitter, receiver, or environment object movement on the order of λ significantly affects the outcome
 - *e.g.* 2.4 GHz → λ = 12 cm, 900 MHz → \approx 1 ft

Radio Propagation Mechanisms



- Refraction
 - Propagation wave changes direction when impinging on different medium
- Reflection
 - Propagation wave impinges on large object (compared to λ)
- Scattering
 - Objects **smaller than** λ (i.e. foliage, street signs etc.)
- Diffraction
 - Transmission path obstructed by surface with sharp irregular edges
 - Waves **bend around obstacle**, even when line of sight does not exist

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 - Frequency-domain view
 - Time-domain view
 - Motion and channel coherence time

Sinusoidal carrier, line of sight only

 Suppose transmitter is distance d (propagation time delay r = d / c) away from receiver (where c is the speed of light)



- Radio frequency transmitted signal: $\cos(2\pi f_c t) = \cos(2\pi c/\lambda \cdot t)$
 - Carrier frequency f_c corresponds to radio wavelength λ
 - **Baseband** transmitted signal in one symbol period: x = 1 + 0j

How to model the effect of the channel?

Sinusoidal carrier, line of sight only: Signal Attenuation

• Represent channel's amplitude attenuation with a real number a



 Models, e.g. attenuation due to two refractions and partial reflection as the signal passes through an indoor wall

Sinusoidal carrier, line of sight only: Signal Phase Shift

- Received signal travels distance *d*
- **One wavelength** corresponds to a 360° (2π radian) phase shift
- Represent path's **phase shift** with an **angle** (real number) $\theta = 2\pi \cdot d / \lambda$
 - "Abstract away" distance and wavelength into (one) phase shift θ



Sinusoidal carrier, line of sight only: Channel Model

- Wireless channel h attenuates by a, phase-shifts by θ
 - Therefore, $h = ae^{j\theta}$
- **Received baseband signal:** $y = h \cdot x$ (no noise)



Line-of-sight plus reflecting path: Motivation



- What if reflections (*e.g.*, indoor walls) introduce a second path?
- Wireless channel becomes the superposition of the direct path's channel h₁ and the reflection path's channel h₂

Line-of-sight plus reflecting path: Channel Model



• Channel is now $h = h_1 + h_2 = a_1 e^{j\theta_1} + a_2 e^{j\theta_2}$



Line-of-sight plus reflecting path: Channel Model



- Phase difference between paths $\Delta \theta = 2\pi / \lambda (d_1 d_2)$
 - Depends on wavelength and path length difference
- So, |h| depends on wavelength (frequency) as well as channel attenuation

Reflections cause frequency selectivity

 Interference between reflected and line-of-sight radio waves results in frequency dependent fading



Coherence bandwidth B_c: Frequency range over which the channel is roughly the same ("flat")

Practical Frequency-Selective Fading

- One 2.4 GHz Wi-Fi channel is centered at 2412 MHz and spans a 20 MHz bandwidth
- **Observe:** Frequency-selective fading





- Received phase difference between paths depends on wavelength
- Channel spans 2402–2422 MHz
 - Lowest wavelength (2402 MHz): 12.49 cm
 - Highest wavelength (2422 MHz): 12.39 cm
 - Just one millimeter wavelength difference
 - Almost the same. **Contradiction?**

Practical Frequency-Selective Fading

- Channel spans 2402–2422 MHz
 - Lowest wavelength (2402 MHz): 12.49 cm
 - Highest wavelength (2422 MHz): 12.39 cm



- 1. Recall, $\Delta \theta = 2\pi / \lambda (d_1 d_2)$ causes additive vs. destructive fading
- 2. For Wi-Fi, $d_k < 50$ m so, e.g., $d_2 d_1 \approx 20$ m, equals:
 - 160 × 12.49 cm wavelengths
 - 161 × 12.39 cm wavelengths
- So we move from e.g. constructive to destructive, to constructive fading from lowest to highest wavelength

Radio Channels are "Reciprocal"



- Forward channel (T to R) is $h_{TR} = a_1 e^{j2\pi d_1/\lambda} + a_2 e^{j2\pi d_2/\lambda}$
- Switch T and R roles, changing nothing else:
 - **Reverse channel** (R to T) is $h_{RT} = a_1 e^{j2\pi d_1/\lambda} + a_2 e^{j2\pi d_2/\lambda} = h_{TR}$
 - The reverse radio channel is "reciprocal"
- Practical radio **receiver** circuitry induces differences between h_{TR} , h_{RT}

Putting it all Together: Ray Tracing

- Approximate solutions to Maxwell's electromagnetic equations by instead representing wavefronts as particles, traveling along rays
 - Apply geometric reflection, diffraction, scattering rules
 - Compute angle of reflection, angle of diffraction
- Error is smallest when receiver is many λ from nearest scatterer, and all scatterers are large relative to λ
- Good match to empirical data in rural areas, along city streets (radios close to ground), and indoors
- Completely site-specific
 - Changes to site may invalidate model



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What does the channel look like in time?



Power delay profile (PDP)

• **Power** received via the path with excess time delay τ_i is the value (height) of the discrete PDP component $P(\tau_i)$ at τ_i

 $P(\tau)$ corresponds to $|h(\tau)|^2$



Characterizing a power delay profile

- Given a PDP $P(\tau_k)$ sampled at time steps τ_k :
- **Mean excess delay** $\overline{\tau}$: Expected value of $P(\tau_k)$:

$$\bar{\tau} = \frac{\sum_k P(\tau_k) \, \tau_k}{\sum_k P(\tau_k)}$$

- **Root mean squared (RMS) delay spread** σ_{τ} measures the spread of the power's arrival in time
 - RMS delay spread is the variance of $P(\tau_k)$:

$$\sigma_{\tau} = \sqrt{\overline{\tau^2} - (\overline{\tau})^2}$$
, where $\overline{\tau^2} = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)}$

 Maximum excess delay < X dB τ_{<X} is the greatest delay at which the PDP is greater than X dB below the strongest arrival in the PDP

Example Indoor PDP Estimation

Typical RMS delay spreads



Environment	RMS delay spread
Indoor cell	10 – 50 ns
Satellite mobile	40 – 50 ns
Open area (rural)	< 0.2 μs
Suburban macrocell	< 1 µs
Urban macrocell	1 – 3 µs
Hilly macrocell	3 – 10 μs

Indoor power delay profile



Flat Fading



- Slow down \rightarrow sending data over a **narrow bandwidth** channel
 - Channel is constant over its bandwidth
 - Multipath is still present, so channel strength fluctuates over time
 - How to model this fluctuation?

above!

Rayleigh Fading Model





- Random gain of k^{th} arriving path: $a_k = a_k^I + j a_k^Q$
- Therefore, the I and Q channel components h_I, h_Q are zero-mean Gaussian distributed

• So
$$|h| = \sqrt{h_I^2 + h_Q^2}$$
 is **Rayleigh-distributed**



Rayleigh fading example



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Stationary transmitter, moving receiver



- Suppose **reflecting wall**, fixed transmit antenna, no other objects
 - Receive antenna moving rightwards at velocity v
- Two arriving signals at receiver antenna with a path length difference of 2(d r(t))

How does fading in time arise?



Stretch Break and In-Class Question

- In the preceding example, the reflected wave and direct wave travel in opposite directions
 - What happens if we move the reflecting wall to the left side of the transmitter?



• What is the nature of the multipath fading, both over time and over frequency?

Channel Coherence Time

- A change in path length difference of λ / 2 transitions from constructive to destructive interference
 - Receiver movement of λ/4: coherence distance
 - Duration of time that transmitter, receiver, or objects in environment take to move a coherence distance: channel coherence time T_c
 - Walking speed (2 mph) @ 2.4 GHz: ≈ 15 milliseconds
 - Driving speed (20 mph) @ 1.9 GHz: ≈ 2.5 milliseconds
 - Train/freeway speed (75 mph) @ 1.9 GHz: < 1 millisecond

Another perspective: Doppler Effect

 Movement by the transmitter, receiver, or objects in the environment creates a *Doppler Shift*



Stationary transmitter, moving receiver: From a Doppler Perspective



- **Doppler Shift of a path** $\Delta f = \frac{f_c \cdot v_{radial}}{c}$
 - V_{radial} is radial component of receiver's velocity vector along the path
 - Positive ∆f with decreasing path length, negative ∆f with increasing path length
- Suppose v = 60 km/h, $f_c = 900$ MHz
 - Direct path $\Delta f = -50 Hz$, reflection path $\Delta f = +50 Hz$

Stationary transmitter, moving receiver: From a Doppler Perspective

- Channel Doppler Spread D_s: maximum path Doppler shift, minus minimum path Doppler shift
- Suppose $v = 60 \text{ km/h}, f_c = 900 \text{ MHz}$
 - Direct path $\Delta f = -50 Hz$, reflection path $\Delta f = +50 Hz$
 - Doppler Spread: 100 Hz
- Results in sinusoidal "envelope" at frequency D_s / 2:



Channel Coherence Time: From a Doppler Perspective

• Sinusoidal "envelope" at frequency $\frac{D_s}{2}$:



- Transition from 0 to peak in $\frac{1}{2D_s}$
 - So qualitatively significant change in time $T_c = \frac{1}{4D_c}$
 - Alternate definition of channel coherence time

Thursday Topic: Receiver Designs for the Wireless Channel