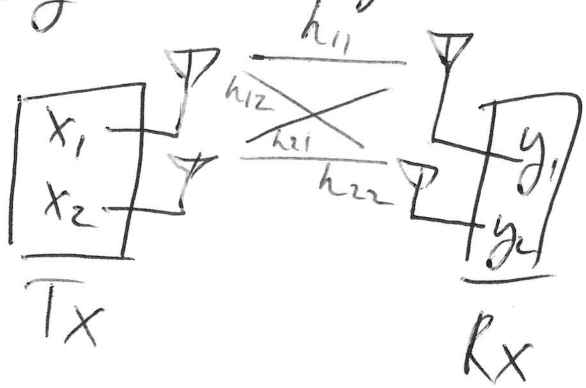
COS 598a : MIMO

- **MIMO**: Multiple-Input, Multiple-output (antennas)
- **M × N MIMO** has M transmitting antennas, N receiving

Decoding in MIMO systems:



- assumption: channel described by a single complex number h_{ij} (tx ant $i \rightarrow$ rx ant j)
  - narrowband or one OFDM subcarrier: OK!

$y_1 = h_{11} x_1 + h_{21} x_2$ (+ noise) because of superposition of the E-M field.

Likewise, $y_2 = h_{12} x_1 + h_{22} x_2 + \text{noise}$

- note, the signals interfere with each other.

rearranging...

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\vec{y} = \underbrace{H}_{\text{channel matrix}} \cdot \vec{x} + \underbrace{\vec{n}}_{\text{noise}}$$

- If H is known, we can solve for $\vec{x} = H^{-1}\vec{y}$.

$$\left(\begin{aligned} &= H^{-1}(H\vec{x} + \vec{n}) \\ &= \vec{x} + \underbrace{H^{-1}\vec{n}}_{\text{noise term.}} \end{aligned} \right)$$

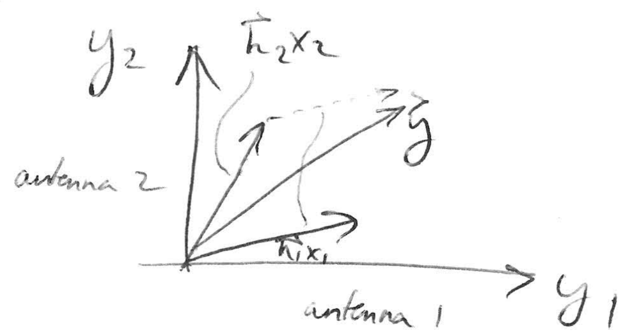
- The upshot: with a 1×1 system, can send one packet at a time
  2×2 ——— " — two packets
  - this is called spatial multiplexing gain.

- Assumptions: ① H is invertible
  ② $H^{-1}\vec{n}$ is not large
  - Statistically OK when antennas @ transmitter and receiver are spaced $> \approx 1.5\lambda$
    ↳ Multipath from tx 1 → rx 1 differs (statistically) from tx 2 → rx 1.

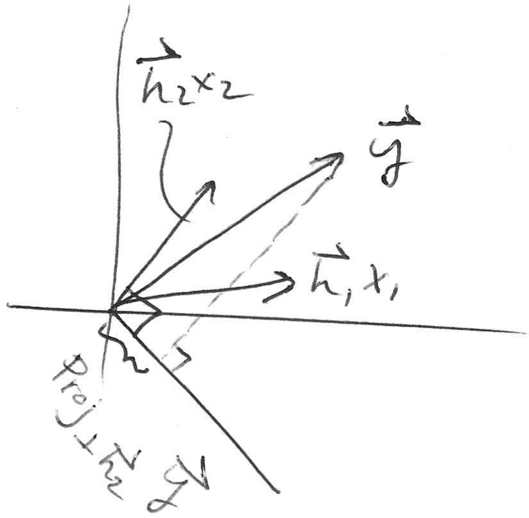
- Geometrical interpretation:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{12} \end{bmatrix} x_1 + \begin{bmatrix} h_{21} \\ h_{22} \end{bmatrix} x_2$$

$$\vec{y} = \vec{h}_1 x_1 + \vec{h}_2 x_2$$



Solve for x_1 by projecting $\vec{y}$ onto a direction orthogonal to $\vec{h}_2$; thus removing all power from $\vec{h}_2 x_2$:

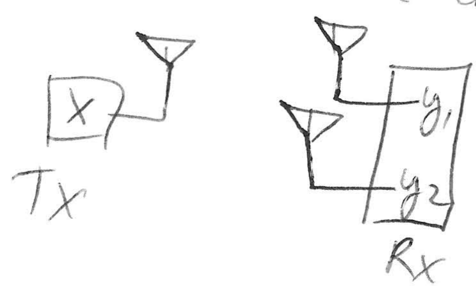


— lose some of the signal strength from $\vec{h}_1 x_1$.

MIMO Gain:

- Multiplexing gain ($M \times M$ MIMO) $\Rightarrow$ M simul. pkts.
- Diversity gain: ($1 \times M$ MIMO)

Diversity Gain: assume channels are 1 (both).



$$y_1 = x + n_1$$
$$y_2 = x + n_2$$

variance σ^2

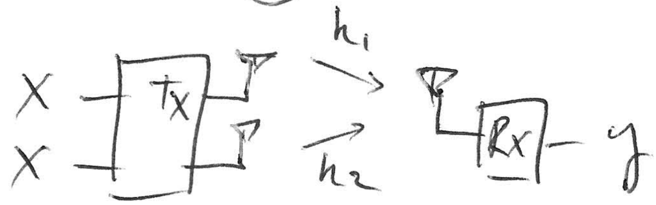
$$y_1 + y_2 = 2x + n_1 + n_2$$

$$SNR_{2Rx} = \frac{\text{Power (signal)}}{\text{Power (noise)}} = \frac{E[(2x)^2]}{E[(n_1+n_2)^2]} = \frac{4E[x^2]}{E[n_1^2] + E[n_2^2]} = \frac{2E[x^2]}{\sigma^2}$$

$$SNR_{1Rx} = \frac{E[x^2]}{\sigma^2}$$

So two Rx antennas doubles SNR, compared to one Rx ant.

Transmit diversity



$$y = \underbrace{(h_1 + h_2)}_{\text{problem when } h_1 + h_2 \approx 0!} x + n$$

To accont for this, lets send instead:

Time	t	$t+1$
Tx ant 1:	x_1	x_2
Tx ant 2:	x_1	x_2
Receive:	~~$y[1]$~~	~~$y[2]$~~

$\Longrightarrow$

Time	t	$t+1$
	x_1	$-x_2^*$
	x_2	x_1^*
	$y[1]$	$y[2]$

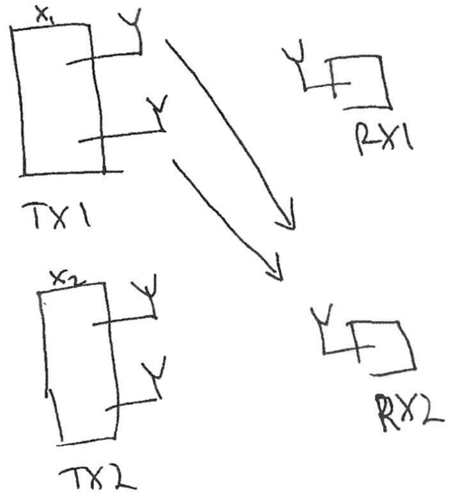
$$\left[y[1]\ y[2]\right] = [h_1\ h_2]\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \left[n[1]\ n[2]\right]$$

$$\begin{bmatrix} y[1] \\ y[2]^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n[1] \\ n[2]^* \end{bmatrix}$$

orthogonal columns, so when $y[1]$ is faded, $y[2]^*$ is high-power, and vice-versa.

[This page adapted from D. Katabi]

Interference Nulling: Each transmitter uses 2nd antenna to null his transmission at the other receiver



$$y_2 = h_{11}x_1 + h_{21}x_1$$

$$\Downarrow$$

instead ~~of~~ transmit $-h_{21}x_1$ and $h_{11}x_1$

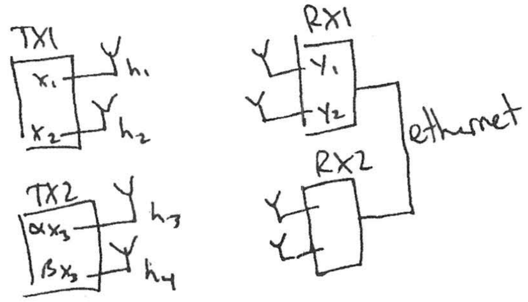
$$\Downarrow$$

$$y_2 = h_{11}(-h_{21})x_1 + h_{21}h_{11}x_1 = 0$$

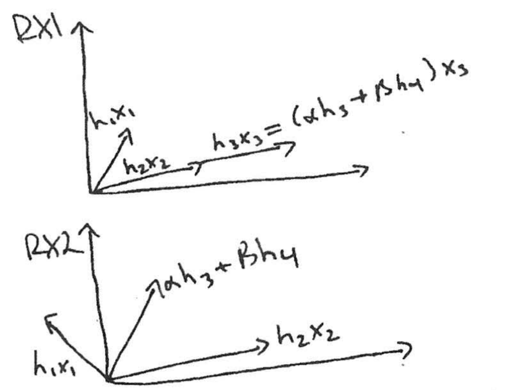
Each additional antenna on the receiver needs two additional on the transmitter to perform interference nulling.

Interference Alignment:

Uplink:



Aligning 2 of the vectors



$$(\alpha\vec{h}_3 + \beta\vec{h}_4)x_3$$

$\uparrow$

set $\alpha\vec{h}_3 + \beta\vec{h}_4$ proportional to $\vec{h}_2$

(h's are vectors, 2 equations and 2 unknowns)

RX2 receives $\vec{h}_1x_1$ over ethernet, subtracts it, and solves for remaining two, sends back to RX1

With this, can transmit 3 packets instead of 2 at once