PHY IV: Rateless Codes, MIMO

COS 598a: Wireless Networking and Sensing Systems

Kyle Jamieson

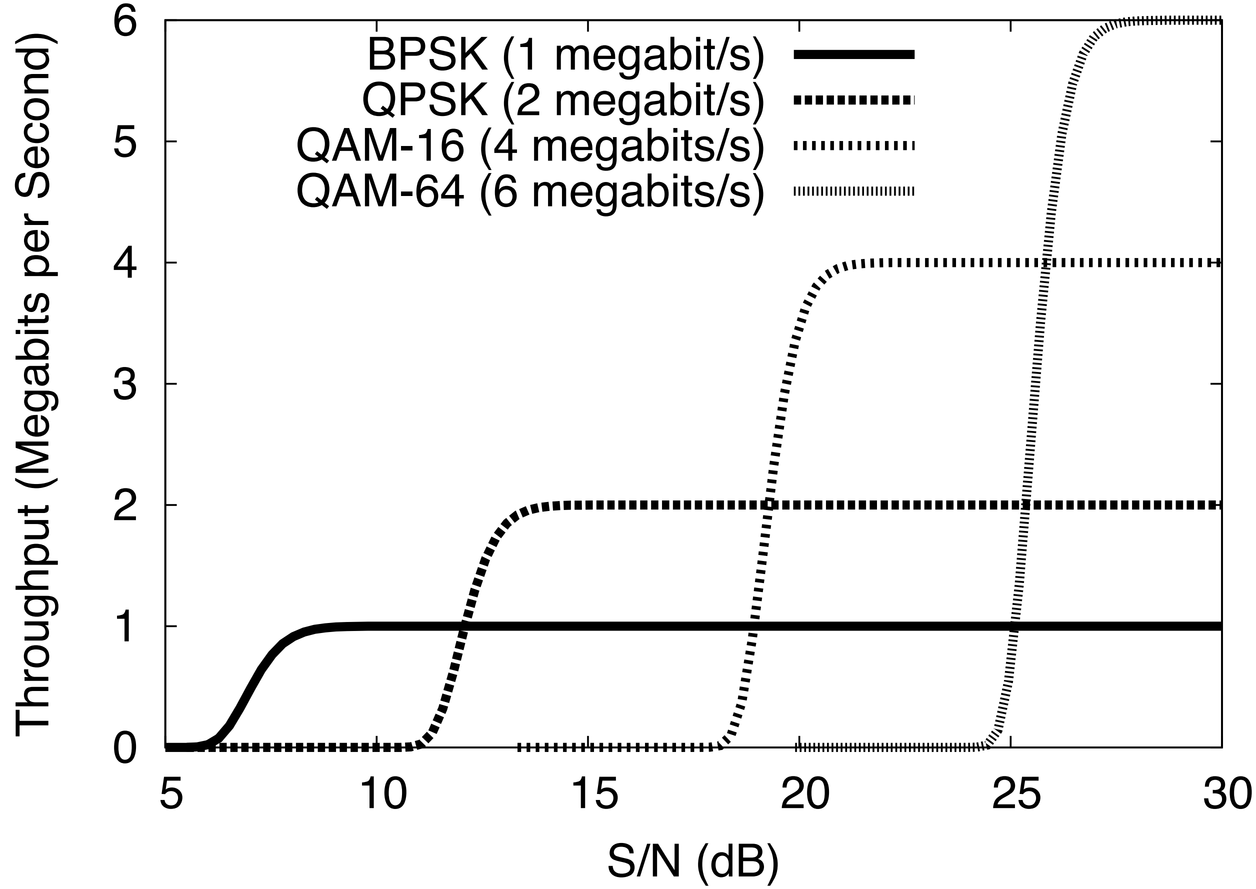
Today

1. **Spinal Codes**

2. Introduction to MIMO

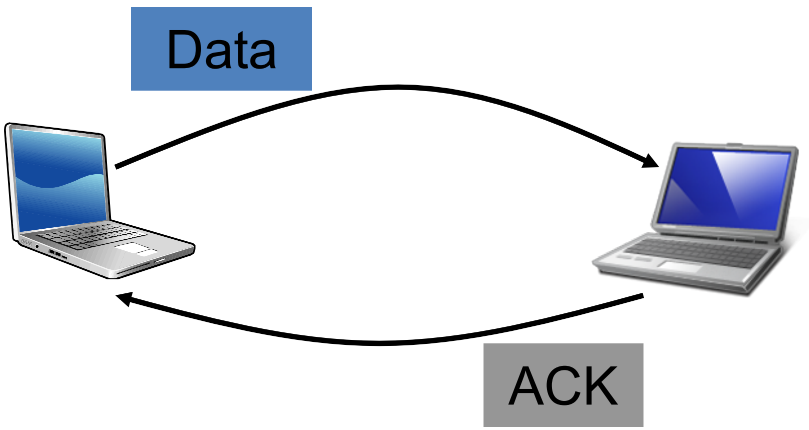
3. SoftRate

Fixed-rate codes *require* channel adaptation



Existing rate adaptation algorithms

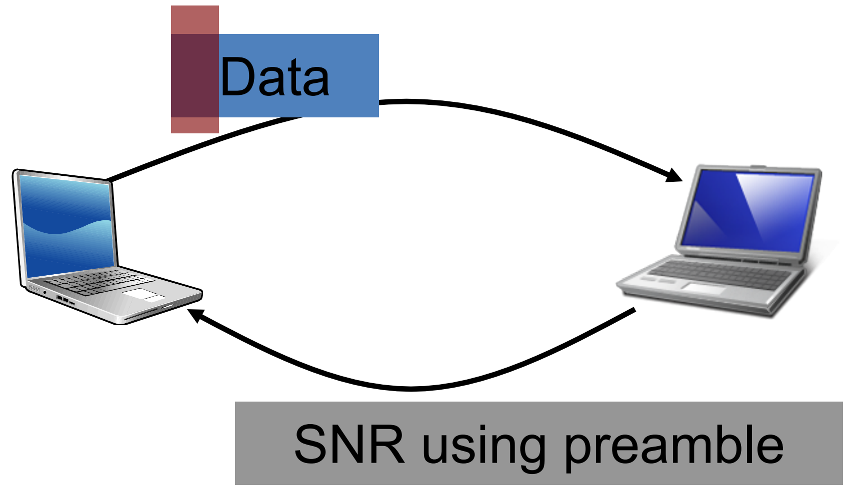
Frame-based



Estimate frame loss rate at each bit rate

- **RRAA**, Wong *et al.*, 2006.
- **SampleRate**, Bicket, 2005.
- **ARF, ONOE**

SNR/BER-based

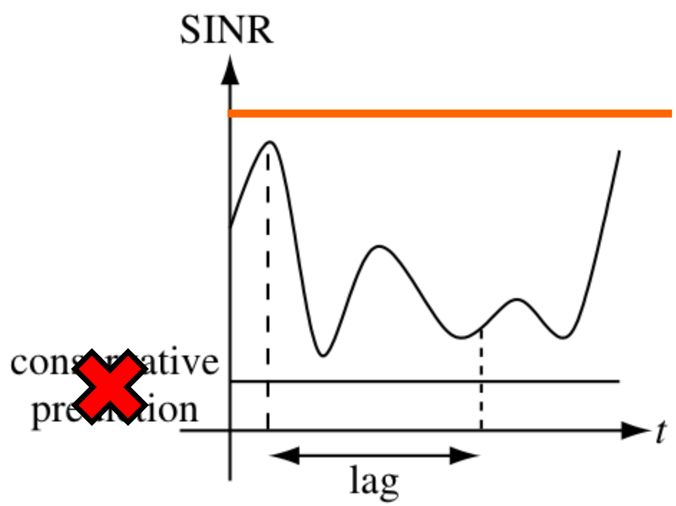


Lookup table:
SNR/BER → best rate

- **RBAR**, Holland *et al.*, 2001.
- **CHARM**, Judd *et al.*, 2008.
- **SoftRate**, Vutukuru *et al.*, 2009

Rateless codes

- **Idea:** Sender transmits information at a rate **higher** than the channel can sustain
  - At first glance, this sounds disastrous!



- Receiver extracts information at the rate the channel can sustain **at that instant**
  - **No adaptation loop is needed!**

Spinal Codes: Outline

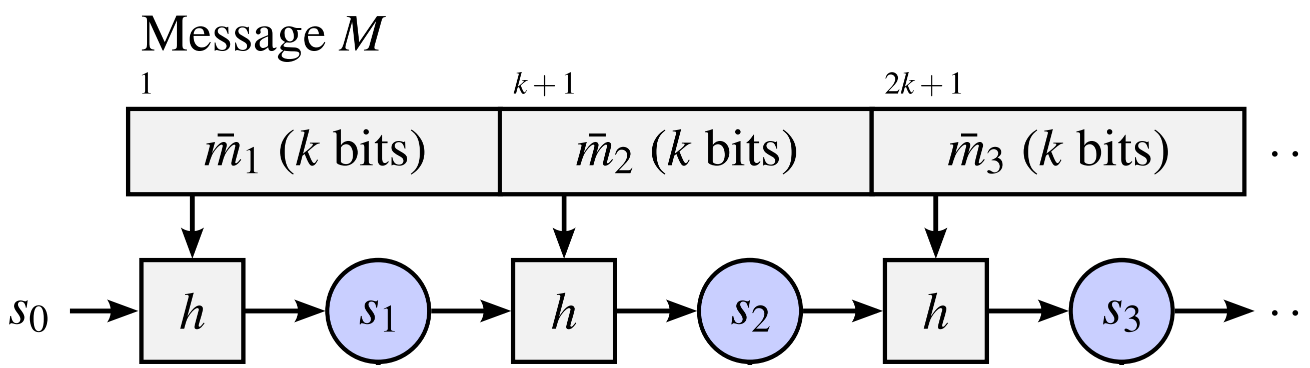
Perry, Ianucci, Fleming, Balakrishnan, Shah. Spinal Codes, SIGCOMM 2012.

1. **Encoding Spinal Codes**

2. Decoding Spinal codes

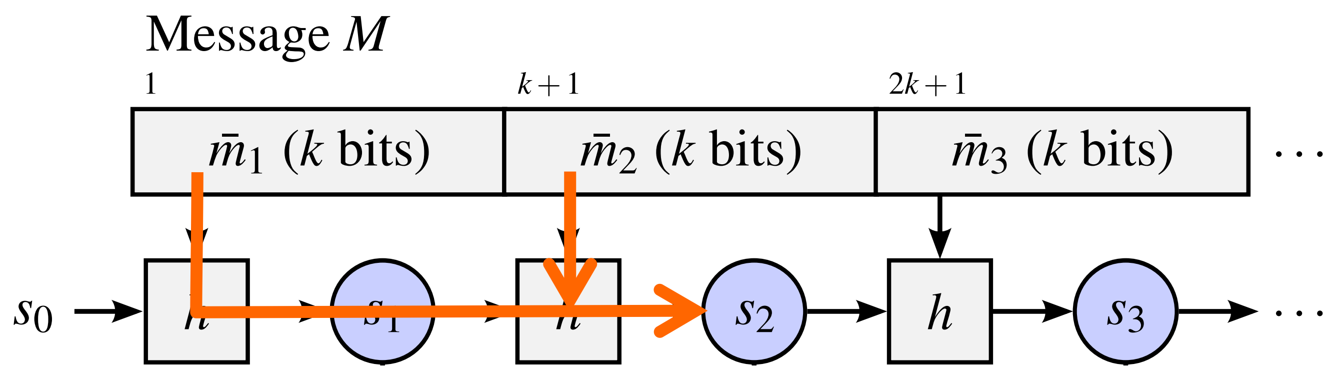
3. Implementation and evaluation

Spinal encoder: Computing the spines



- Start with a hash function *h* and an initial random *v*-bit ***state*** s_0
  - Sender and receiver agree on *h* and s_0 *a priori*
- Sender divides its *n*-bit ***message*** *M* into *k*-bit ***chunks*** m_i
- *h* maps the state and a message chunk into a new state
  - The *v*-bit states $s_1, \ldots, s_{\lceil n/k \rceil}$ are the ***spines***

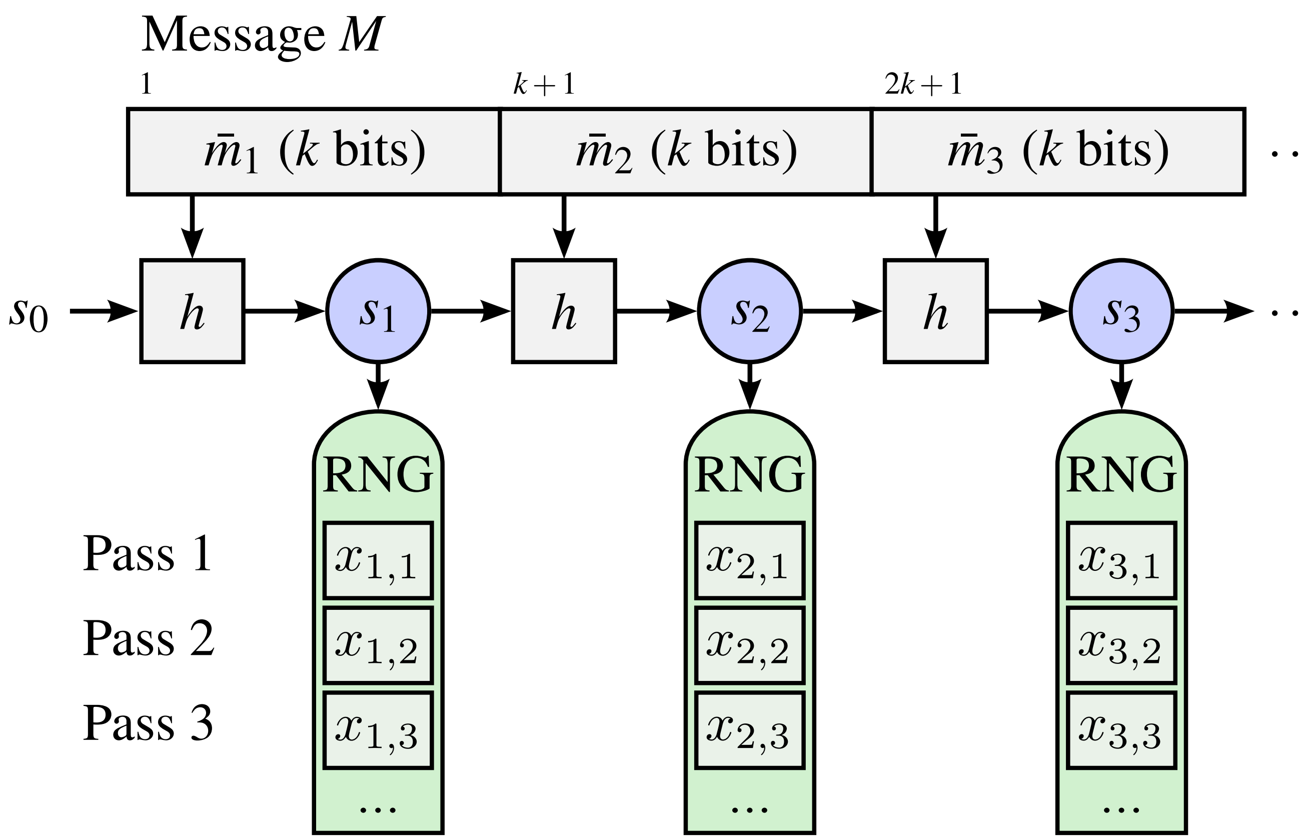
Spinal encoder: Information flow



- Observe: State s_i contains information about chunks $m_1, \ldots, m_i$
  - A stage's state depends on the message bits **up to** that stage

- So only state $s_{\lceil n/k \rceil}$ has information about entire message

Spinal encoder: Computing the spines

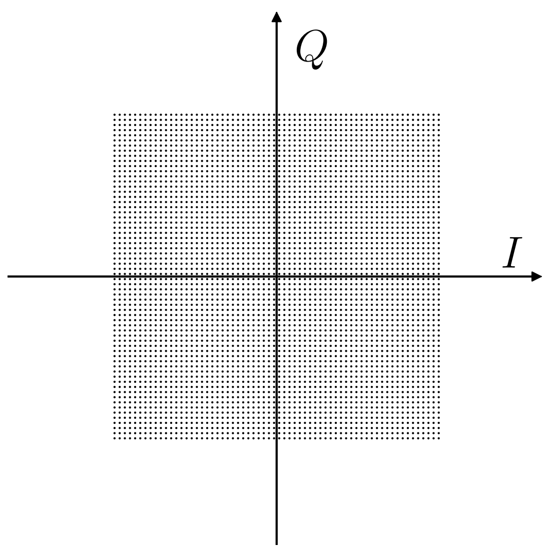


- Each spine seeds a random number generator ***RNG***
- RNG generates a sequence of *c*-bit numbers
- Encoder output is a series of ***passes*** of $\lceil n/k \rceil$ ***symbols*** $x_{i,l}$ each

Spinal encoder: RNG to symbols

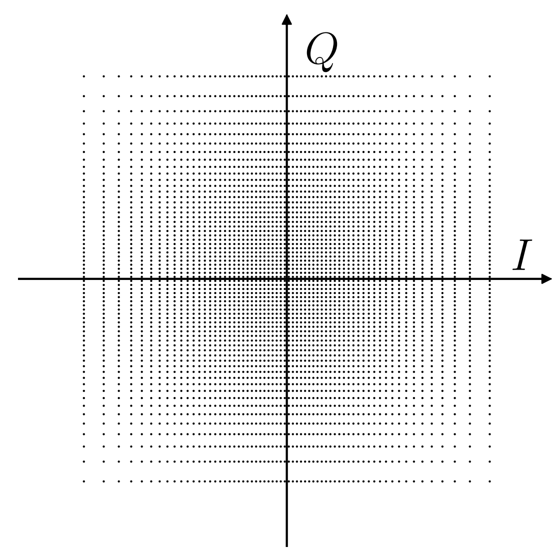
- A constellation mapping function translates c-bit numbers $x_{i,l}$ from the RNG to in-phase (I) and quadrature (Q)
  - Generates in-phase (I) and quadrature (Q) components **independently from two separate** $x_{i,l}$

Uniform

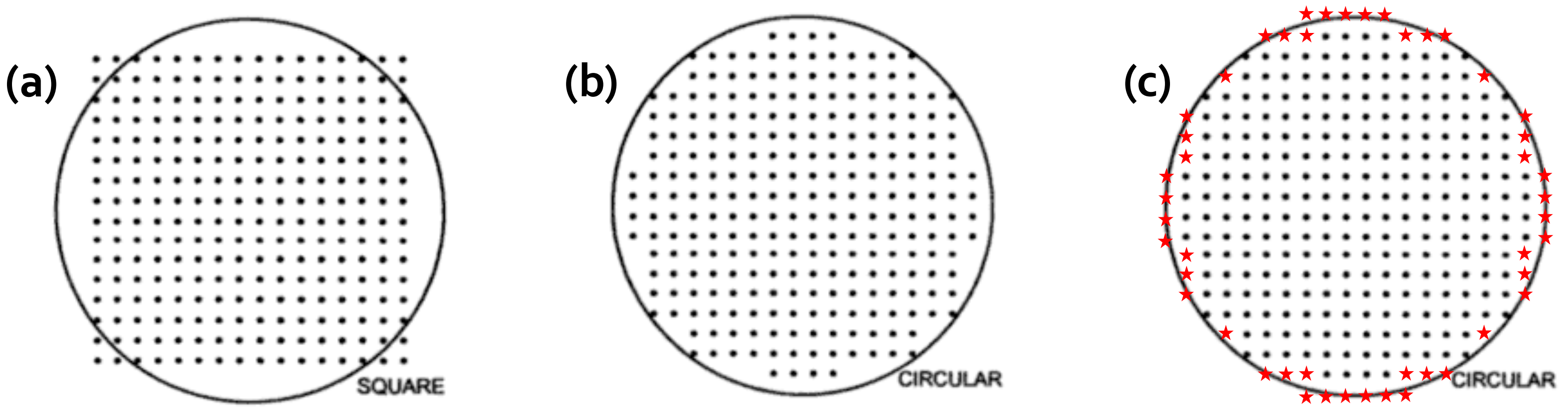


$$x_{i,l} \mapsto \frac{x_{i,l} + \frac{1}{2} - 2^{c-1}}{2^c \sqrt{6P}}$$

Truncated Gaussian



Digression: What's the best constellation shape?



a) Start with a square constellation
   – Recall, distance of each symbol from origin determines power
   – So, a circle traces constant power points

b) Maintaining inter-point spacing, move points inside circle
   – This is ***shaping gain:*** *we* maintain error probability, hence throughput, but reduce the average signal power
   – Now can add more points, **increasing throughput (c)** to restore average power to as it was before.

Spinal Codes: Outline

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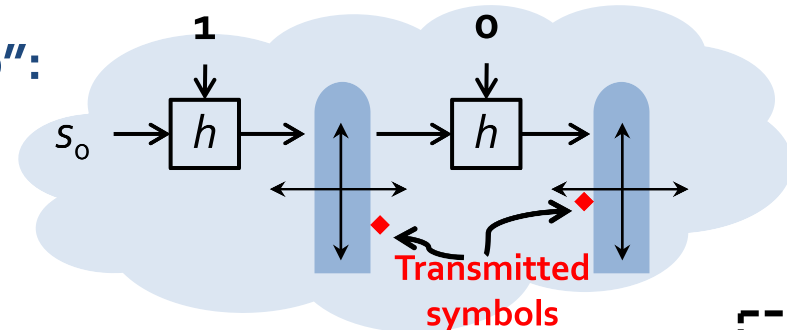
1. Encoding Spinal Codes

2. **Decoding Spinal codes**
   - **"Maximum-likelihood" decoding**
   - The Bubble Decoder
   - Puncturing for higher rate

3. Implementation and evaluation

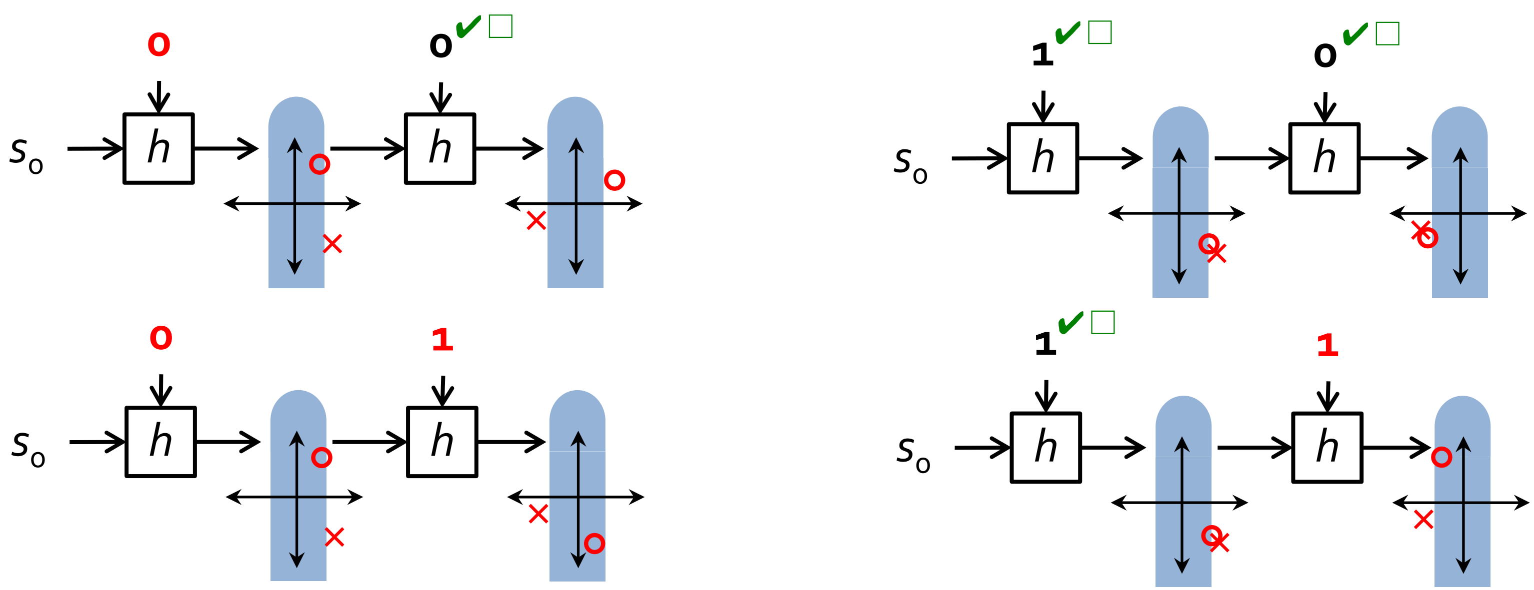
Decode by replaying the encoder

Sender transmits “1”, “0”:



Instead of inverting the hash function, the decoder *replays* all four possibilities:

- ○ Replayed symbol
- × Received symbol

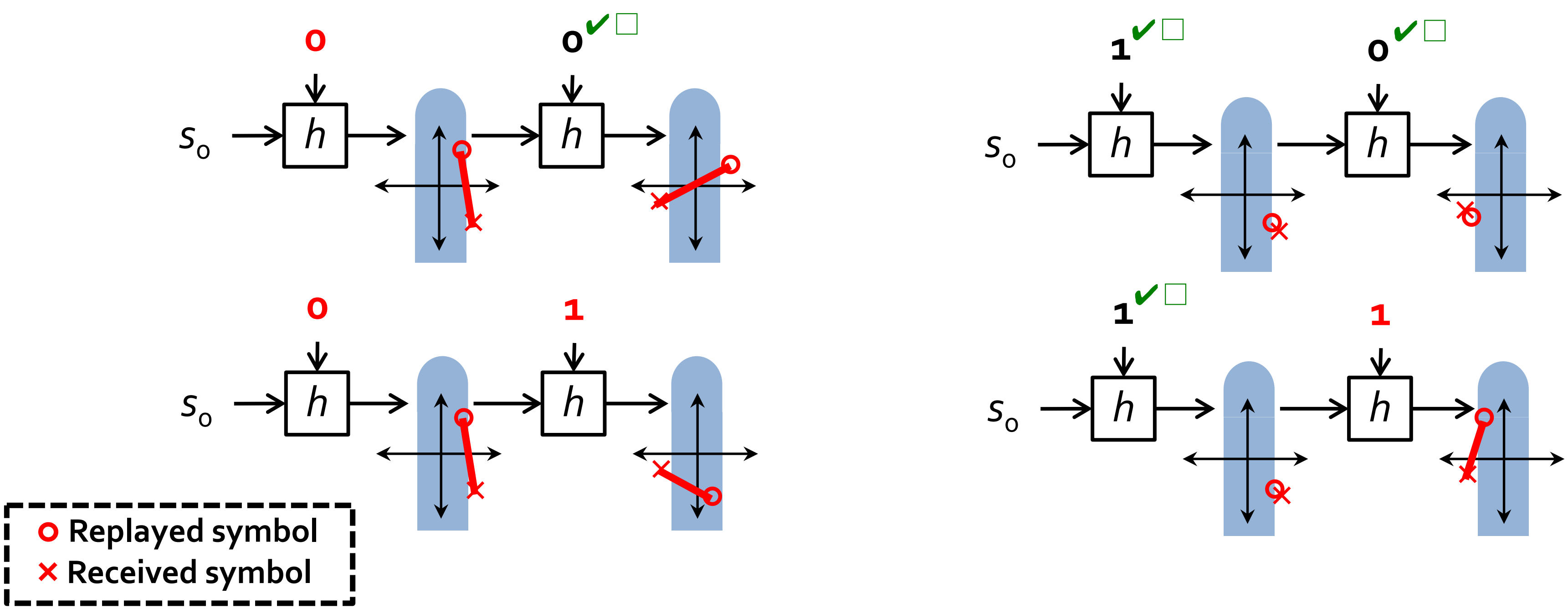


Decode by measuring distance

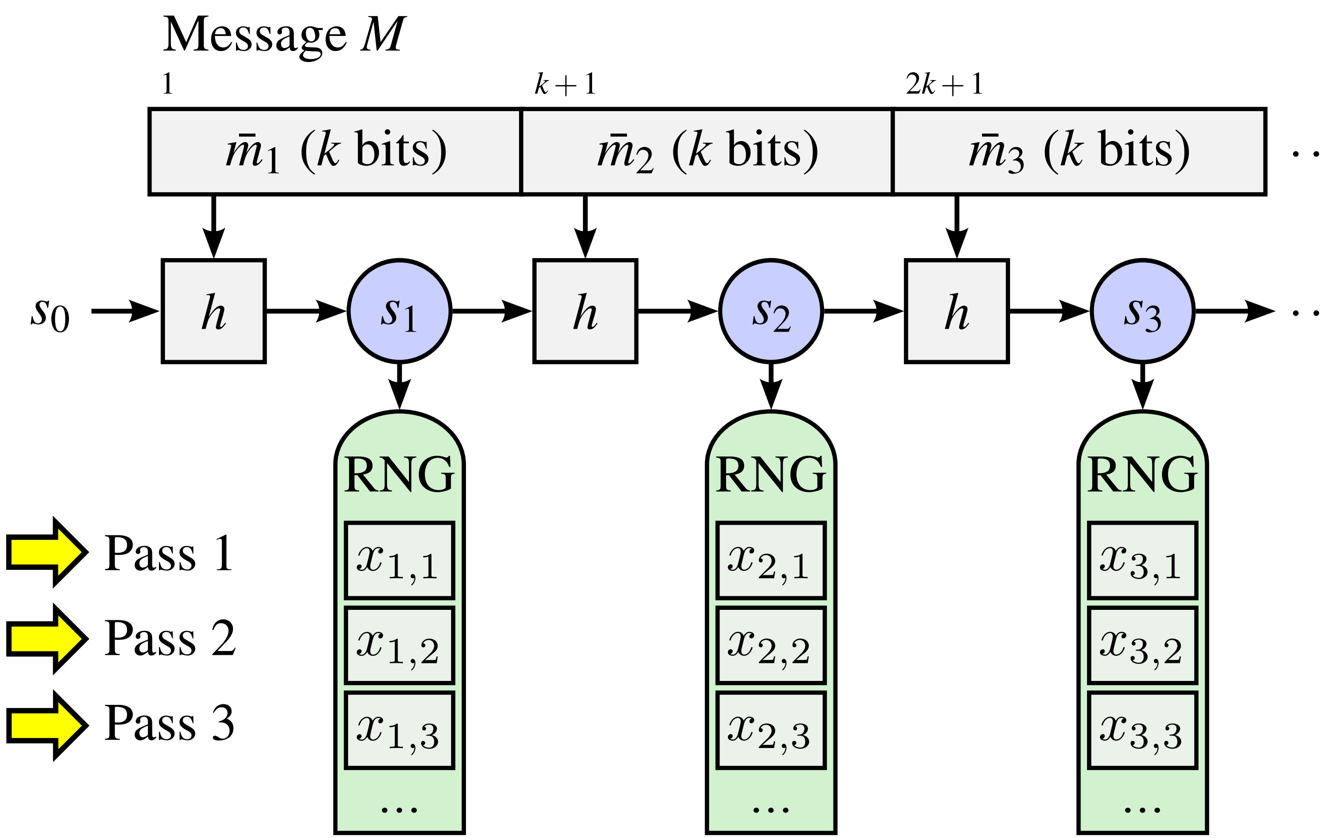
- *How to decide between the four possible messages?*

- **Measure total distance** between:
  – Received symbols, corrupted by noise (×), and
  – Replayed symbols (○)

- Sum across stages: the distance increases at **first incorrect symbol**



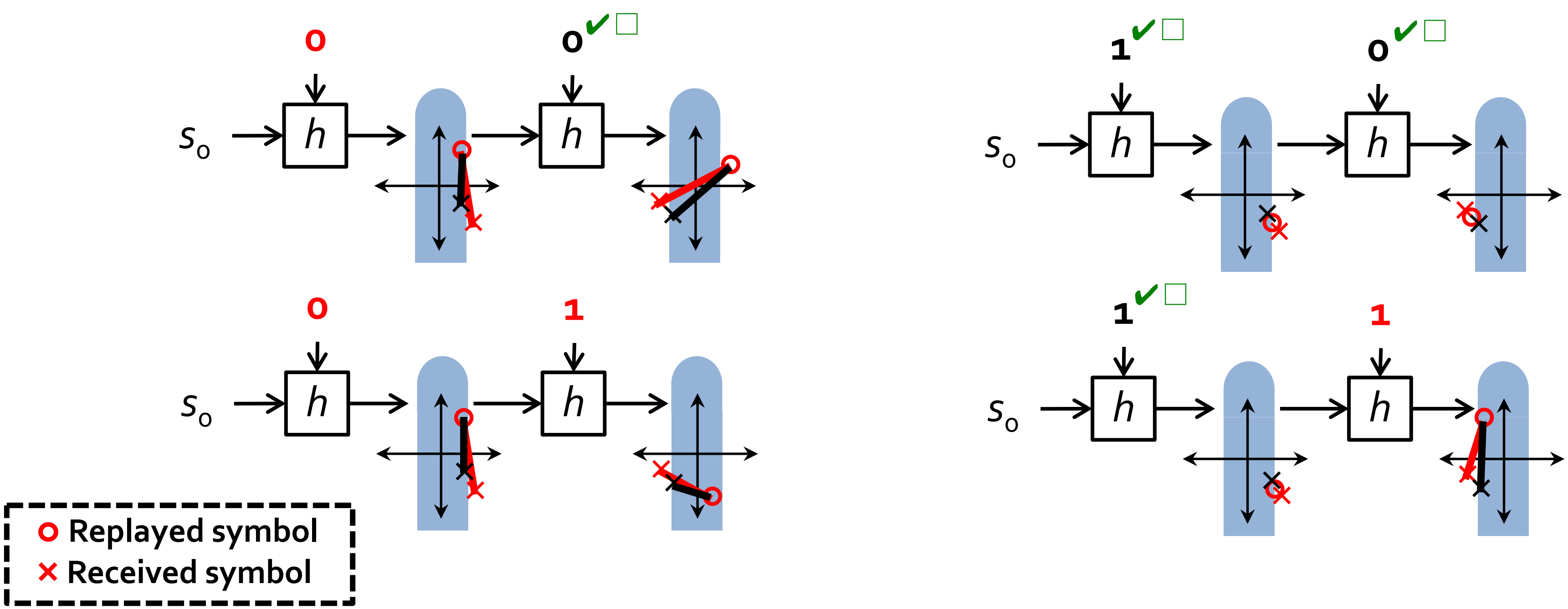
Adding additional passes



- **Recall:** The encoder sends **multiple passes** over the same message blocks

Adding additional passes

- What's a reasonable strategy for decoding now?
- Take the **average distance** from the replayed symbol (○), **across all received symbols** (×, ×)
  - Intuition: As number of passes increases, noise and bursts of interference average out and impact the metric less



The Maximum Likelihood (ML) decoder

- Consider all 2^n possible messages that could have been sent
  - The ML decoder **minimizes probability of error**

- Pick the message M' that minimizes the vector distance between:
  - The vector of all received constellation points $\mathbf{y}$
  - The vector of constellation points sent **if M' were the message,** $\mathbf{x}(M')$

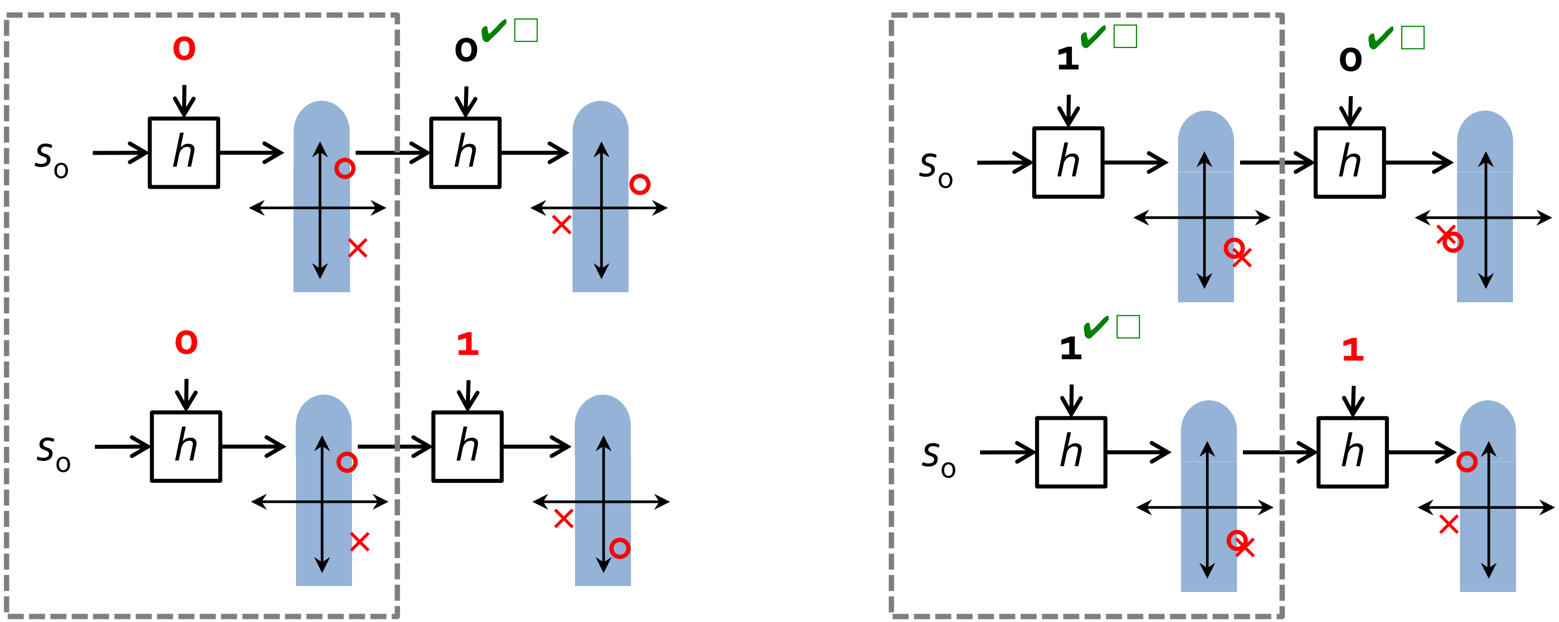
$$\hat{M} = \arg \min_{M' \in \{0,1\}^n} \left\| \mathbf{y} - \mathbf{x}\left(M'\right) \right\|^2$$

- In further detail:
  1. $x_{t,l}(M')$: t^{th} constellation point **sent** in the l^{th} pass for M'
  2. $y_{t,l}$: t^{th} constellation point **received** in the l^{th} pass

$$\hat{M} = \arg \min_{M' \in \{0,1\}^n} \sum_{\text{all } t,l} \left| y_{t,l} - x_{t,l}\left(M'\right) \right|^2$$

ML decoding over a tree

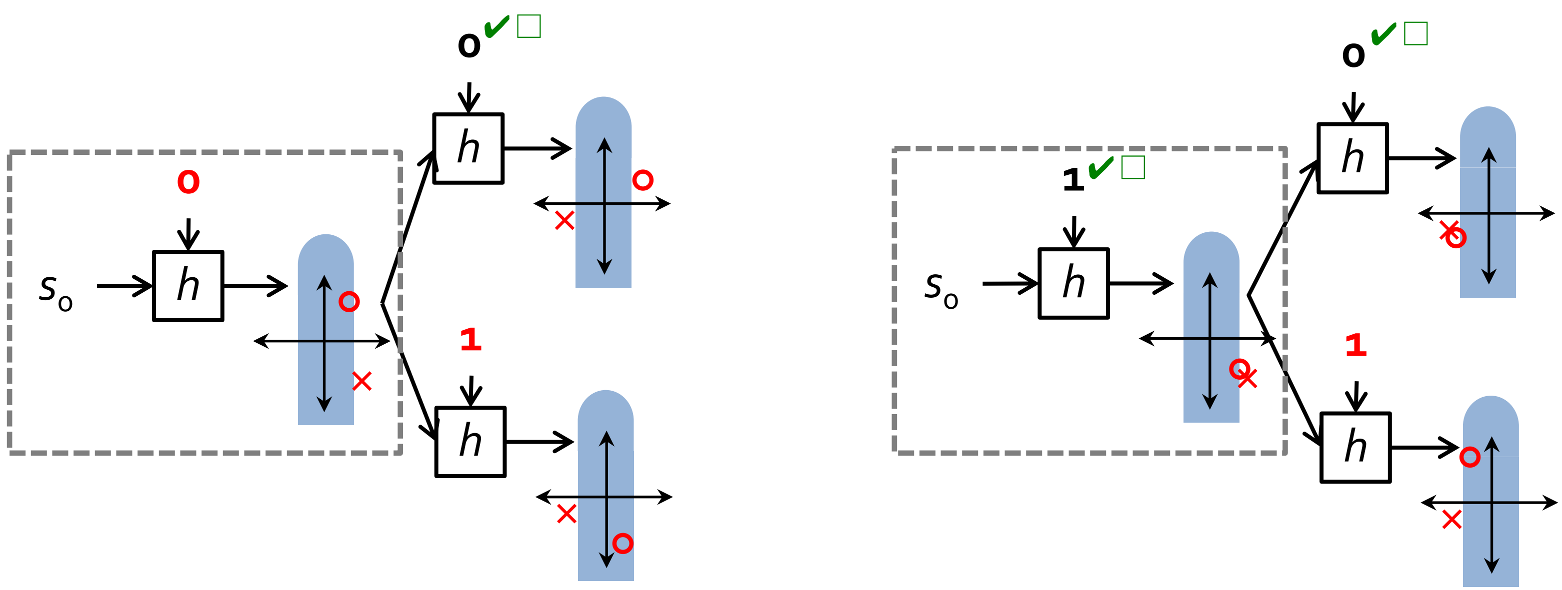
- Observe: Hypotheses whose initial stages share the same symbol guesses are **identical** in those stages



ML decoding over a tree

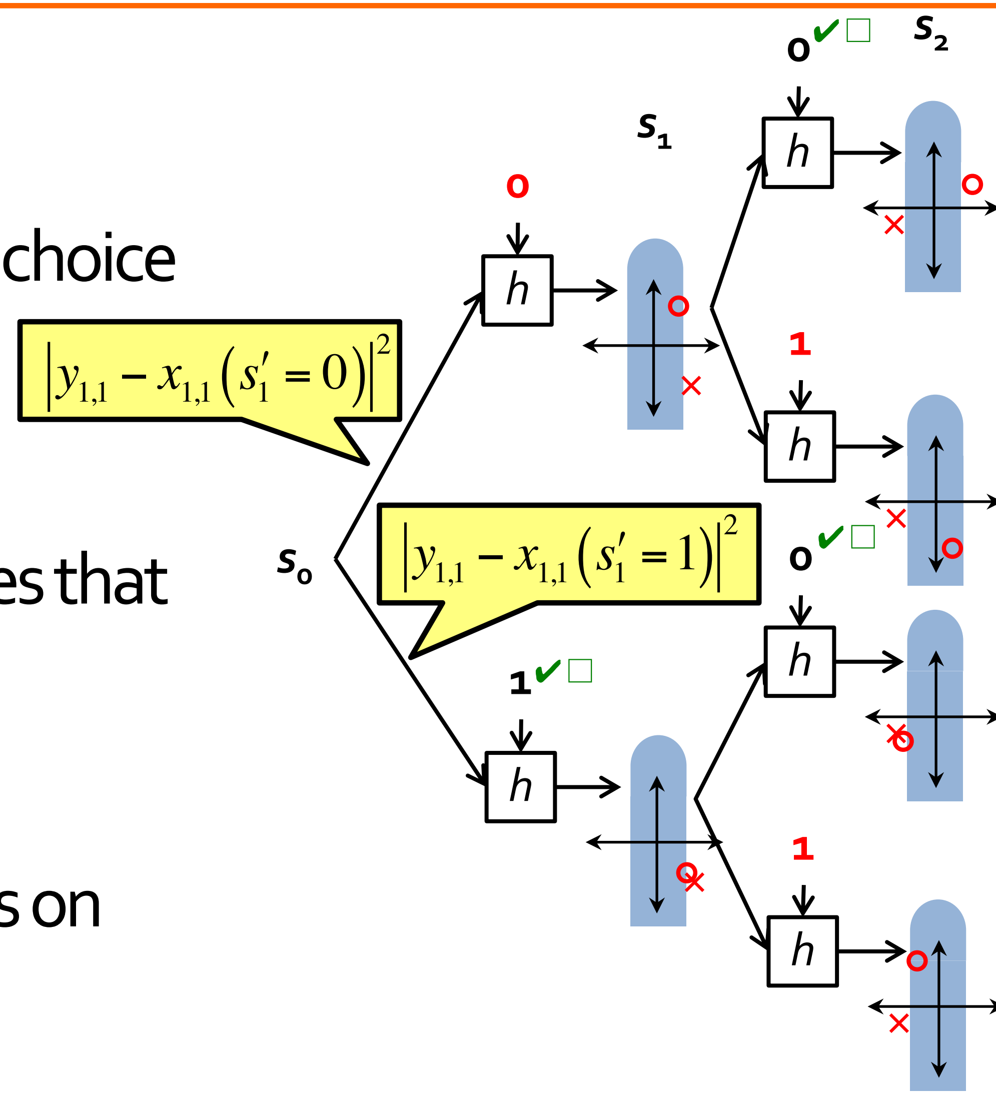
- Observe: Hypotheses whose initial stages share the same symbol guesses are **identical** in those stages

- Therefore we can **merge** these initial identical stages:



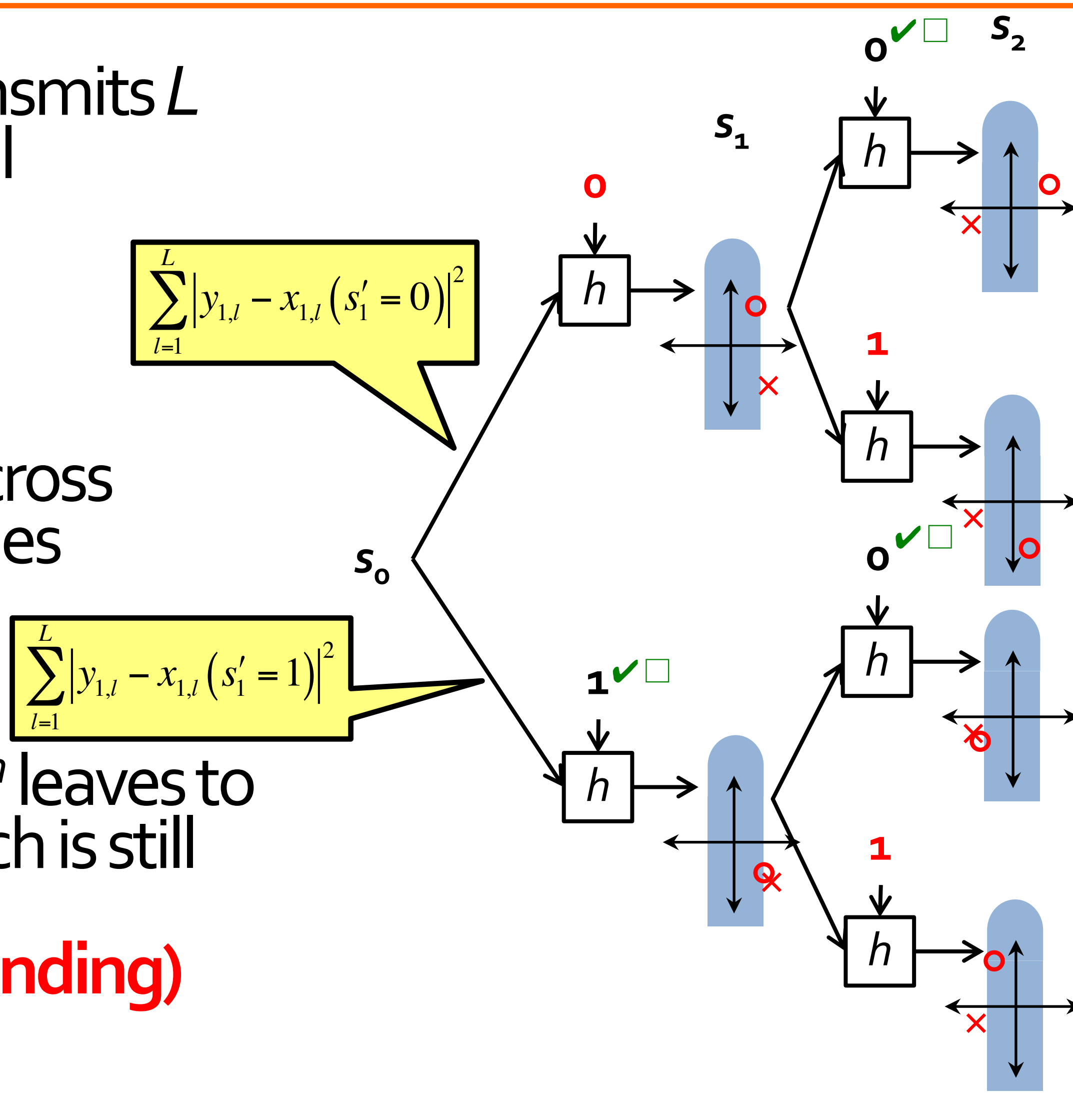
ML decoding over a tree

- General tree properties:
  - n/k levels, one per spine
  - Branching factor 2^k (per choice of k-bit message chunk)
- Let s'_t be the t^{th} spine value associated with all messages that share s'_t
- We find cost of a particular message by summing costs on path from root to leaf



ML decoding over a tree: Multiple passes

- Suppose the sender transmits L passes, in a poor channel
- Average (sum) metric across passes, and label branches
- However, the tree has 2^n leaves to compare so this approach is still **impracticable (too computationally demanding)**



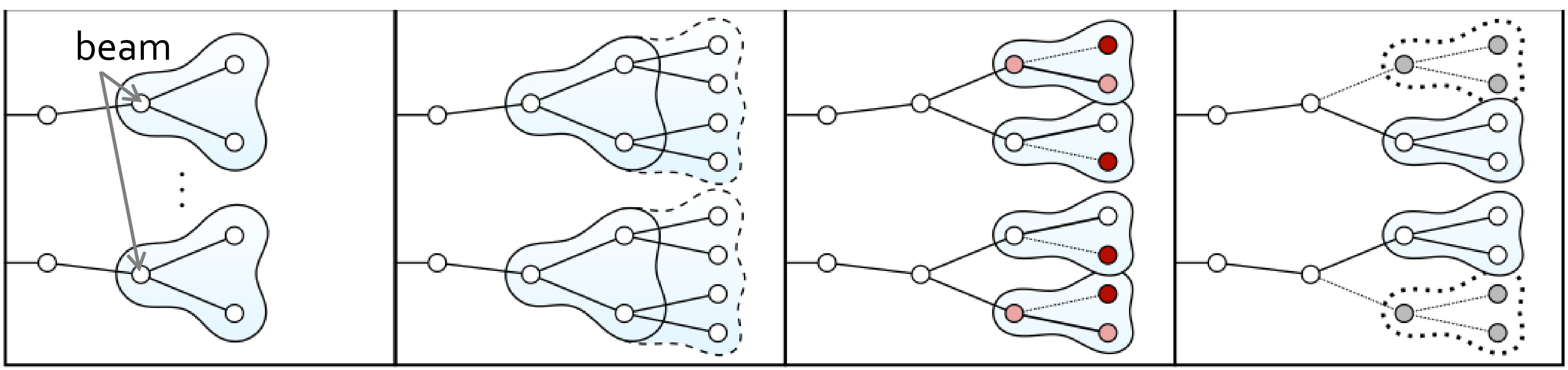
Efficiently exploring the tree

- Observation: Suppose the **ML message *M**** and some other message *M′* differ only in the i^{th} bit
  - Only symbols including and after index $\lceil i/k \rceil$ will disagree
  - So the **earlier** the error in *M′*, the **larger** the cost
  - Can show that the "runners-up" to *M** differ only in the last O(log *n*) bits

- Consider the **best 100 leaves** in the ML tree:
  - Tracing back through the tree, they will have a **common ancestor** with *M** in O(log *n*) steps
  - This suggests a strategy in which we only **keep a limited number of ancestors**

Bubble decoder

- Maintain a ***beam*** of ***B*** tree node ancestors to explore, each to a certain depth ***d***
- Expand each ancestor, score every child, propagate best child score for each ancestor, pick *B* best survivors
- Example: $B = d = 2$, $k = 1$ (lighter color = better score)



(a) After step $i-1$ (b) Expand (c) Score (d) Pop

Decoding complexity

- The bubble decoder operates in $n/k - d$ **steps**
  - Each step explores $B{\cdot}2^{kd}$ nodes, evaluating the RNG L times
  - Selecting the best B candidates takes $B{\cdot}2^{k}$ comparisons

- **Overall cost:** $O((n/k)BL{\cdot}2^{kd})$ hashes, $O((n/k)B{\cdot}2^{k})$ comparisons

- Comparison with LDPC **belief propagation** algorithms
  - These operate in iterations, too, involve all message bits
  - But, these are also quite parallelizable
  - Hard to give exact head-to-head comparison

Adjusting the rate

- Spinal codes as described so far uses different numbers of passes to adjust the rate

- Two problems in Spinal codes **as described so far:**

  1. Must transmit one full pass, so max out at k bits/symbol
     - Increase k? No: Decoding cost is **exponential** in k

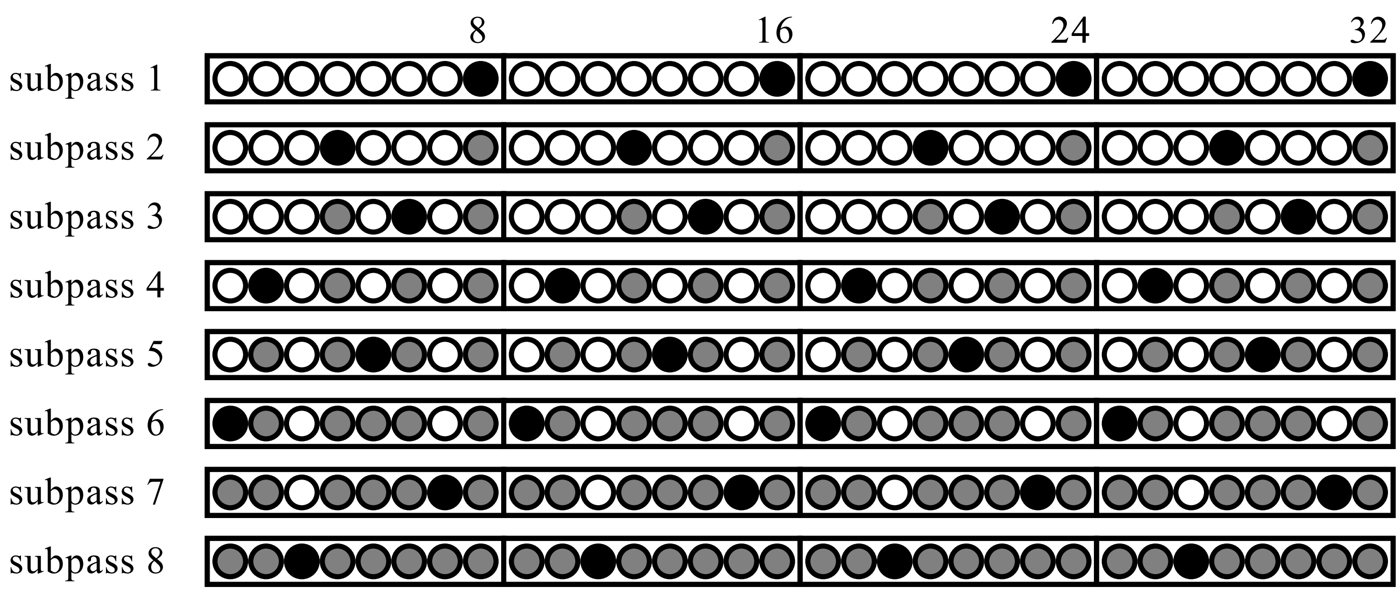
  1. Sending L passes reduces rate to k/L—**abrupt drop**
     - Introduces plateaus in the rate versus SNR curve

Puncturing for higher and finer-controlled rates

- Idea: Systematically skip some spines
  - Sender and receiver agree on the pattern beforehand
  - Receiver can now attempt a decode before a pass concludes

- Decoder algorithm unchanged, missing symbols get zero score

- Max rate of this puncturing: 8·*k* bits/symbol



Framing at the link layer

- Sender and receiver need to maintain synchronization
  - Sender uses a short sequence number protected by a highly redundant code
- Unusual property of Spinal codes: **Shorter** message length *n* is **more** efficient
  - This is in opposition to the trend most codes follow
  - Divide the link-layer frame into shorter checksum-protected ***code blocks***
- If half-duplex radio, when should sender wait for feedback?
  - For more information, see *RateMore* (MobiCom '12)

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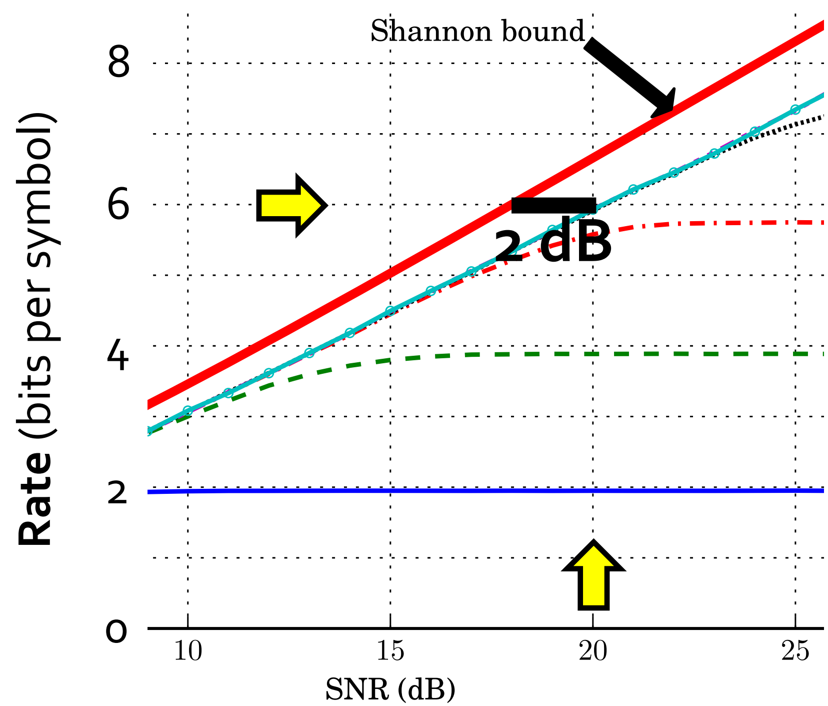
3. **Performance evaluation**

Methodology

- Software simulation: Simulated wireless channel (additive white Gaussian noise and Rayleigh fading)

- Hardware platform: **Airblue** (Xilinx Virtex-5 FPGA, USRP2 radio)
  - Real 10, 20 MHz bandwidth channels in 2.4 GHz ISM band

- ***Gap to capacity:*** How much more noise could a capacity-achieving code tolerate at same rate?
  - Smaller gap is better
  - *e.g.*: This code achieves six bits/symbol at 20 dB SNR, for a **2 dB gap to capacity**

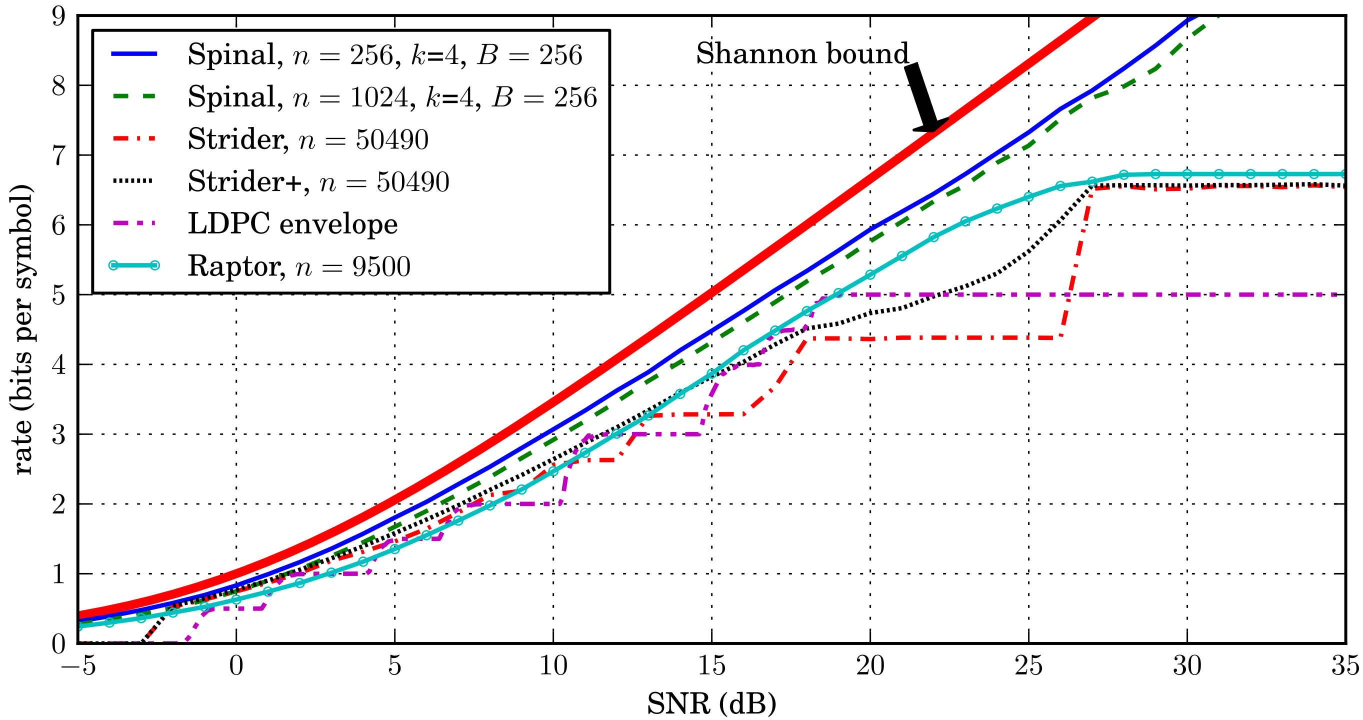


Performance evaluation: Questions

1. How well do Spinal codes perform versus other codes:
   - Rateless codes such as Raptor and Strider?
   - Rated codes such as LDPC?

2. How should one choose various parameters:
   - Bits per chunk k, beam width B, output bits c?

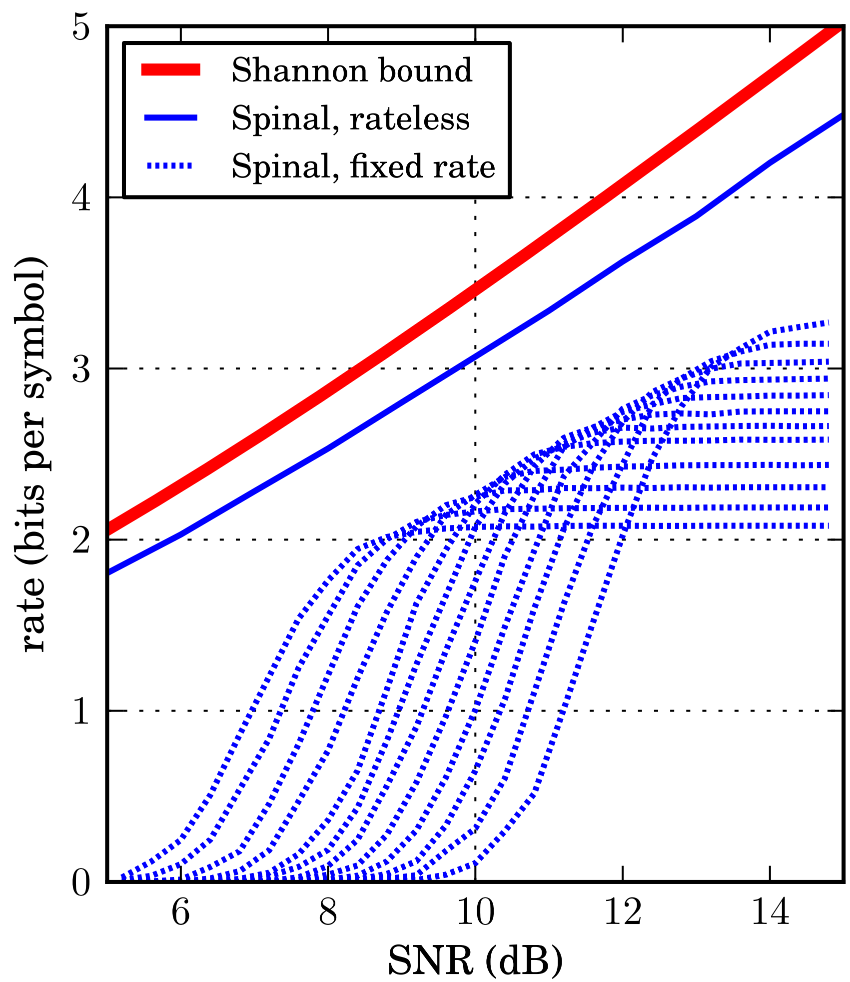
Spinal codes: Higher rate on AWGN channel



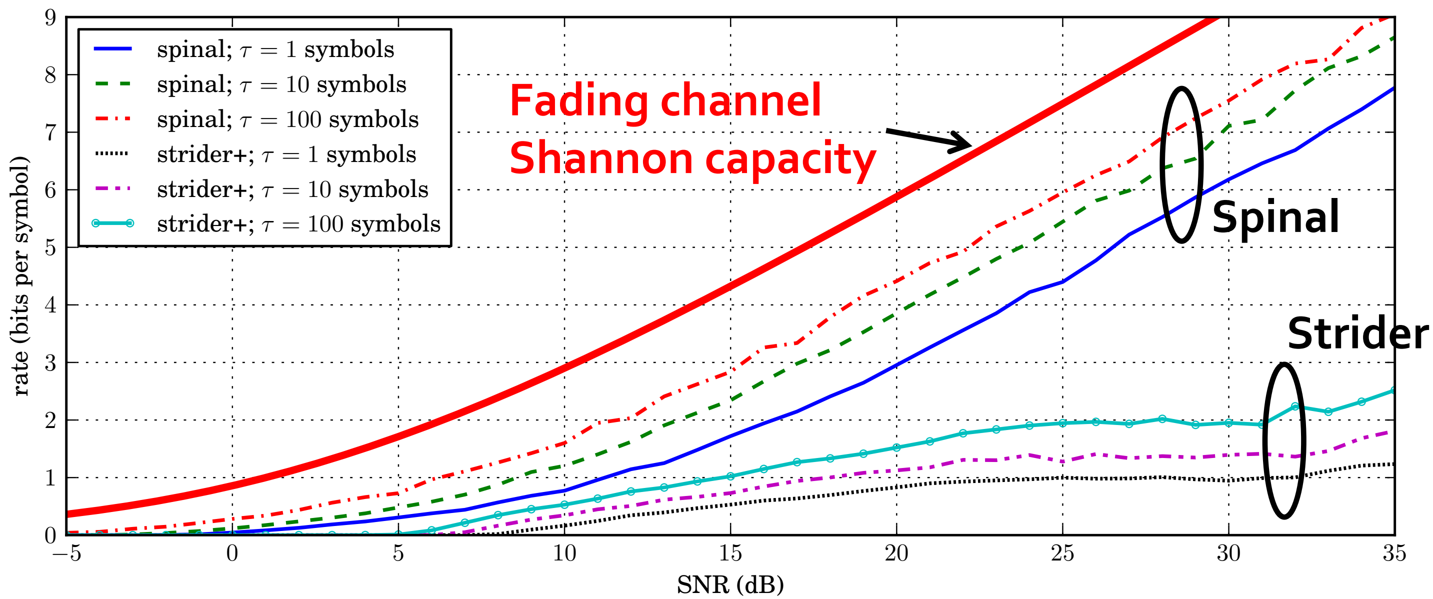
- Simulated AWGN channel: no link-layer performance effects here
- **LDPC envelope:** Choose best-performing rated LDPC code at each SNR to mimic the best a rate adaptation strategy could do
- **Strider+:** Strider + puncturing: finer rate control, but significant gap to capacity

Rateless codes can "hedge their bets"

- Constant SNR means constant **average** noise power
  - But, noise impacting **any particular symbol(s)** may be higher or lower
- Rated codes must be risk averse (send at lower rate)
- Rateless codes can decode with **fewer symbols** when noise is **momentarily lower**
- But this result **requires perfect and instantaneous feedback** so the rateless code knows when to stop



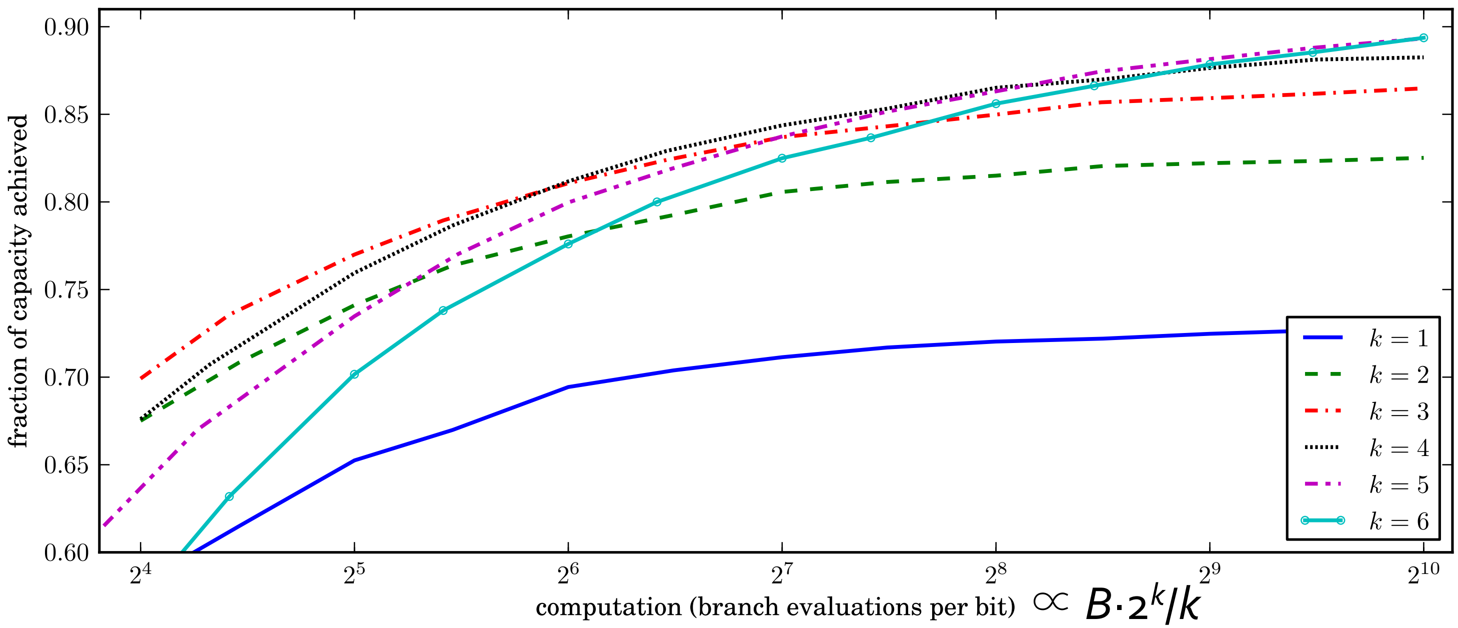
Spinal codes tolerate unknown channels well



- Rayleigh fading channel changing every **τ** symbols (multipath fading) with average SNR as shown on x-axis

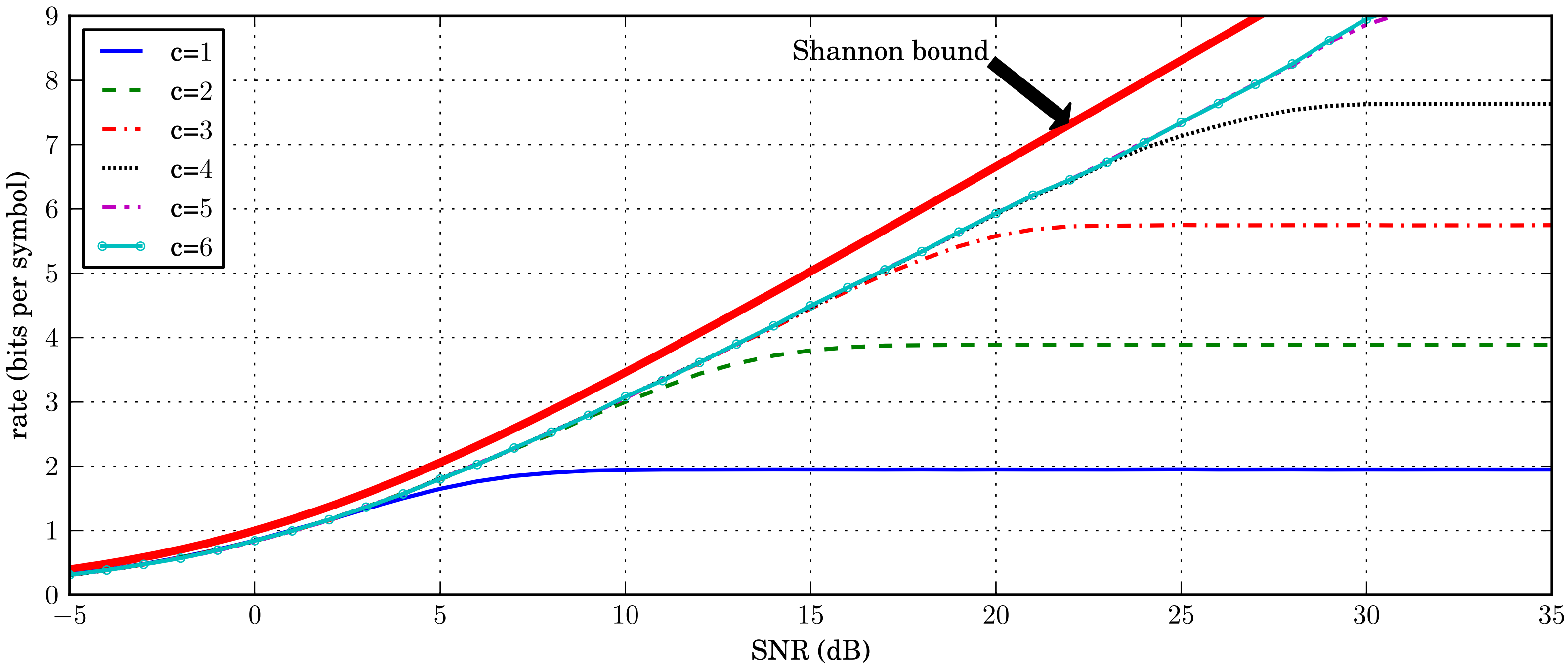
- Measure codes' performance without knowing τ
  - Shorter coherence time is harder on the code
  - Conclude that Spinal can adapt to unknown channel conditions better than Strider+

Choosing chunk length



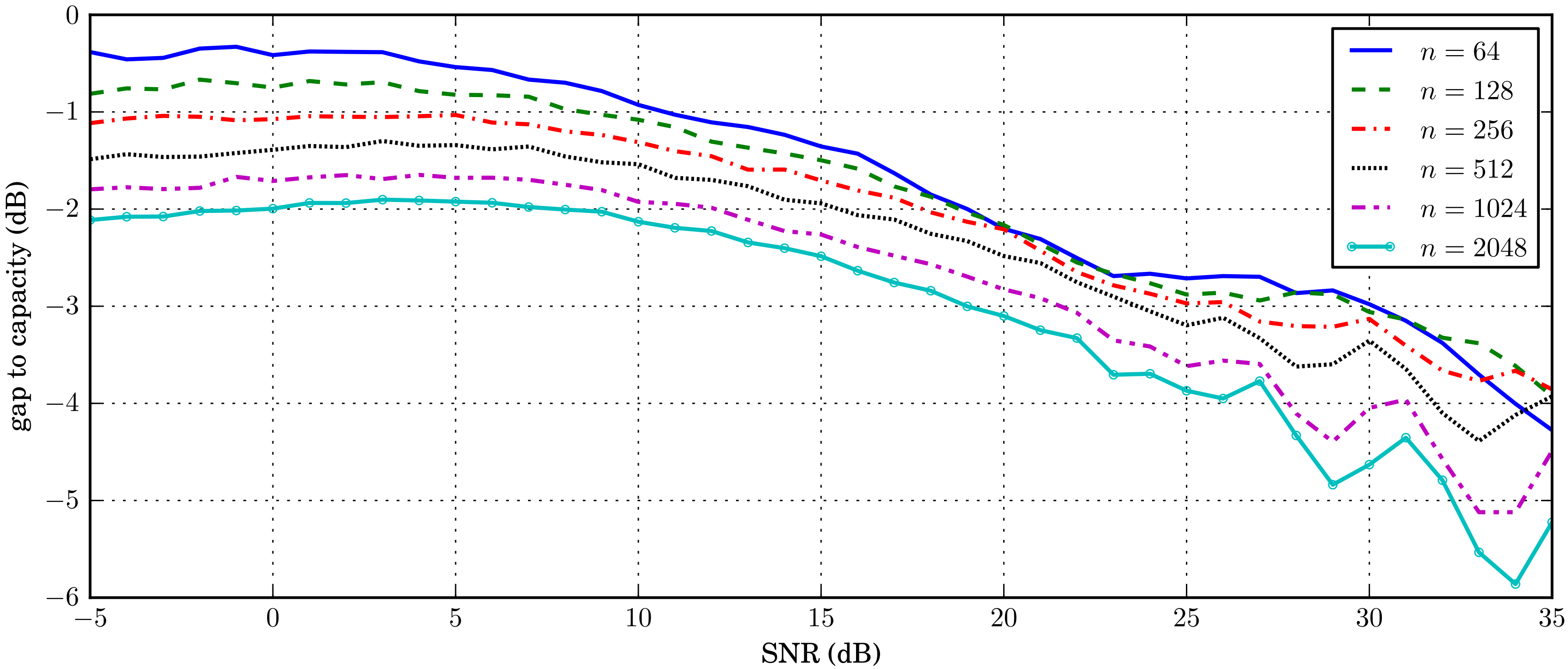
- Each decoder can choose *B* without restriction; how to choose *k*?
  - Consider ***decoder compute budget:*** $B \cdot 2^k$ operations per *k* bits
  - Conclude that $k = 4$ is a good choice (maximizes rate)
  - Also claim that $B = 256$ is a reasonable choice

Choosing number of output bits *c*



- Can send at most 2·*c* output bits per symbol, so caps maximum rate
- Choose *c* so that the rate cap isn't a problem at operational SNRs
- *c* = 6 is a reasonable choice

Spinal codes: Better at sending short messages



- **Longer** code block means more opportunities to **prune correct path**
  - So Spinal codes achieves **better** performance (smaller gap to capacity) with **smaller** code block length *n*

- We can see artifacts due to puncturing at higher SNRs

Spinal Codes: Conclusion

- Spinal Codes give performance close to Shannon capacity
- Eliminate the need to run a bit rate adaptation algorithm
- Simpler design and better performance

Today

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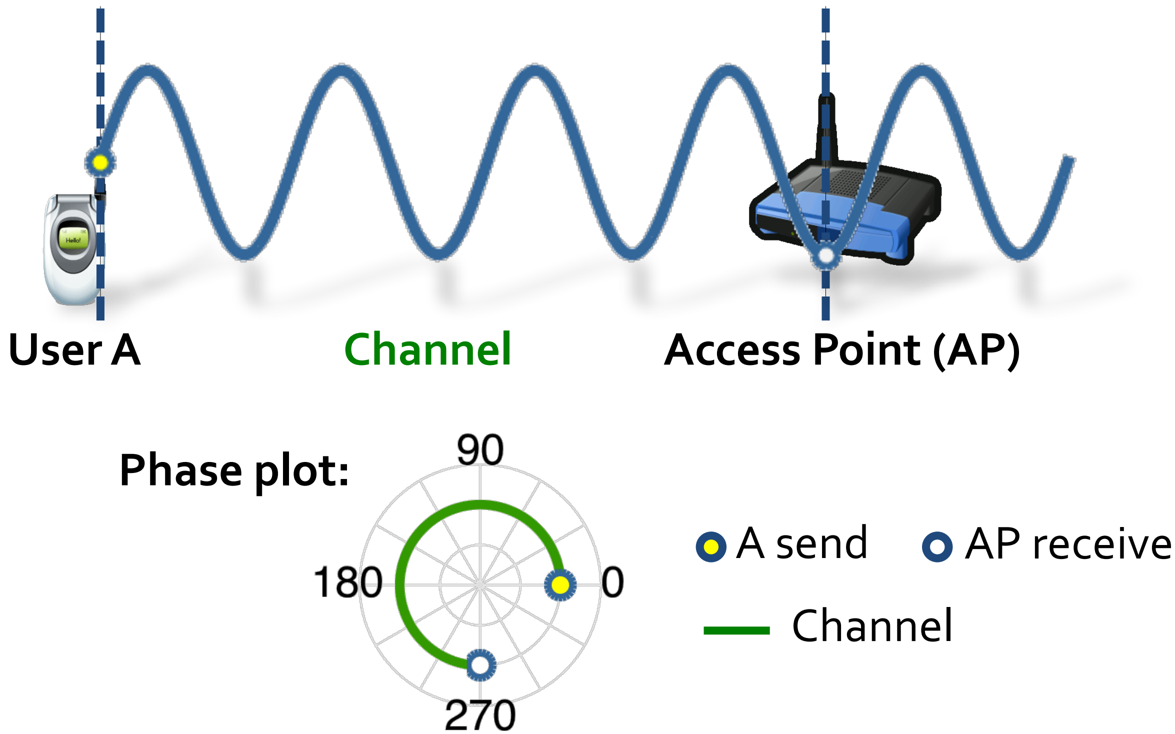
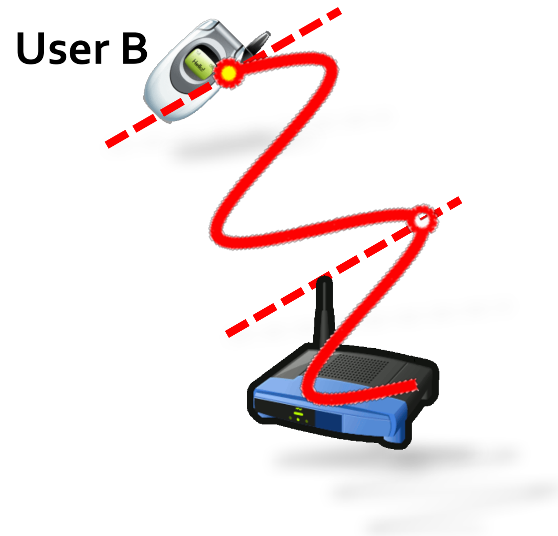
2. **Introduction to MIMO**
3. SoftRate

Single-input, single-output (SISO)

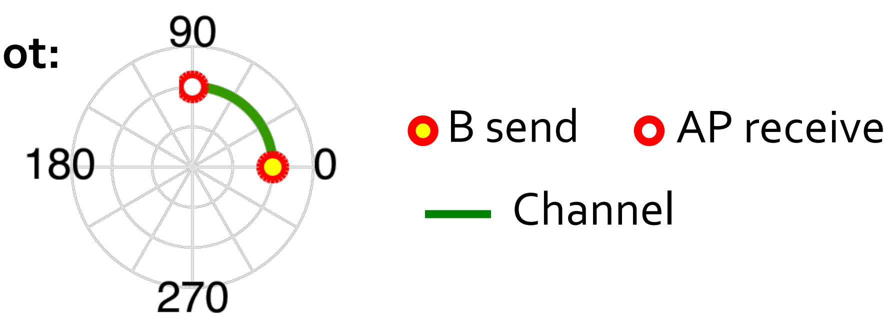


AP can estimate the channel, so can decode User A's signal ( )

Single-input, single-output (SISO)

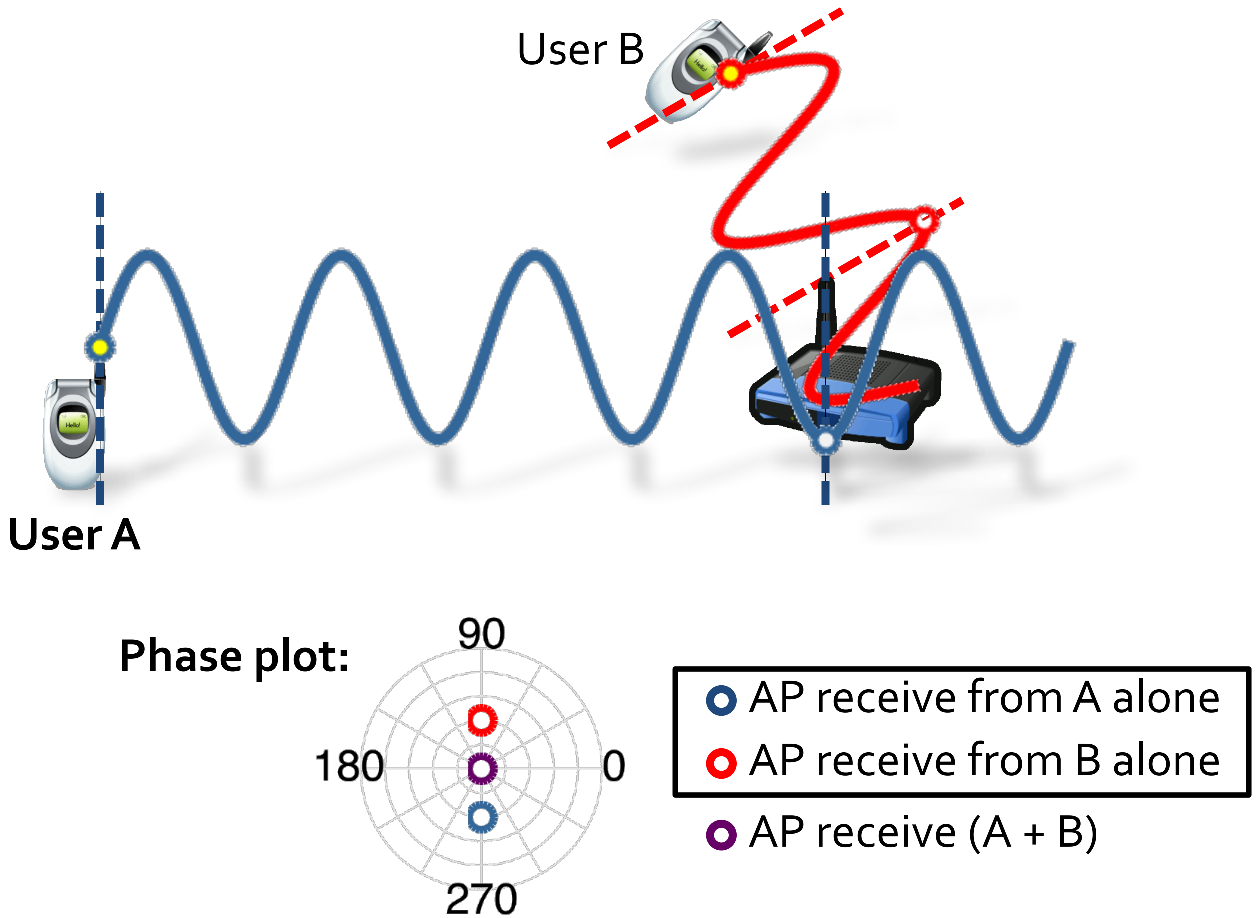


Phase plot:



AP can estimate the channel, so can decode User B's signal ( )

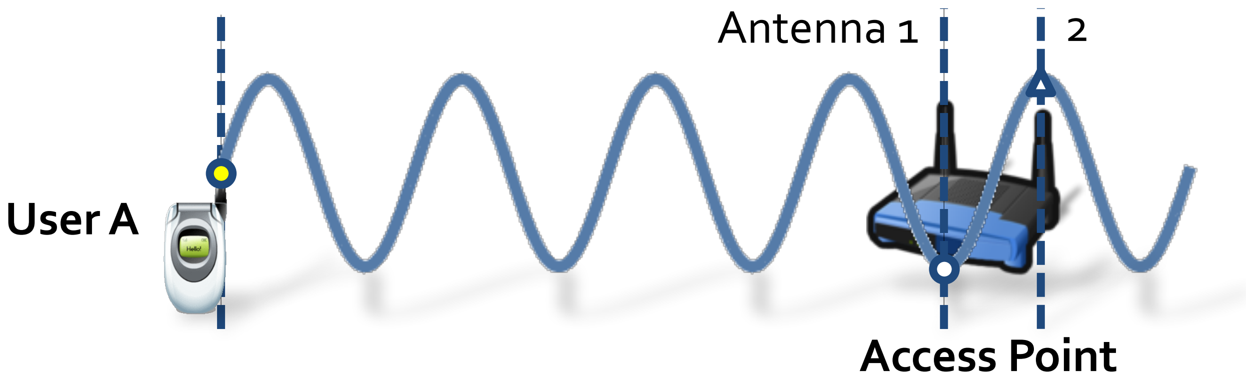
Interferring transmissions in SISO

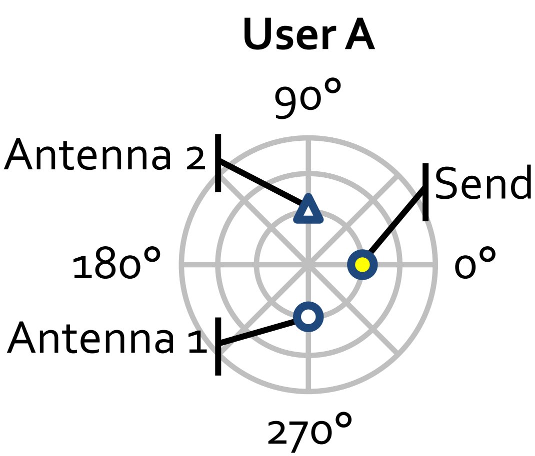


One received signal (), two sent (,), so **AP can't decode**

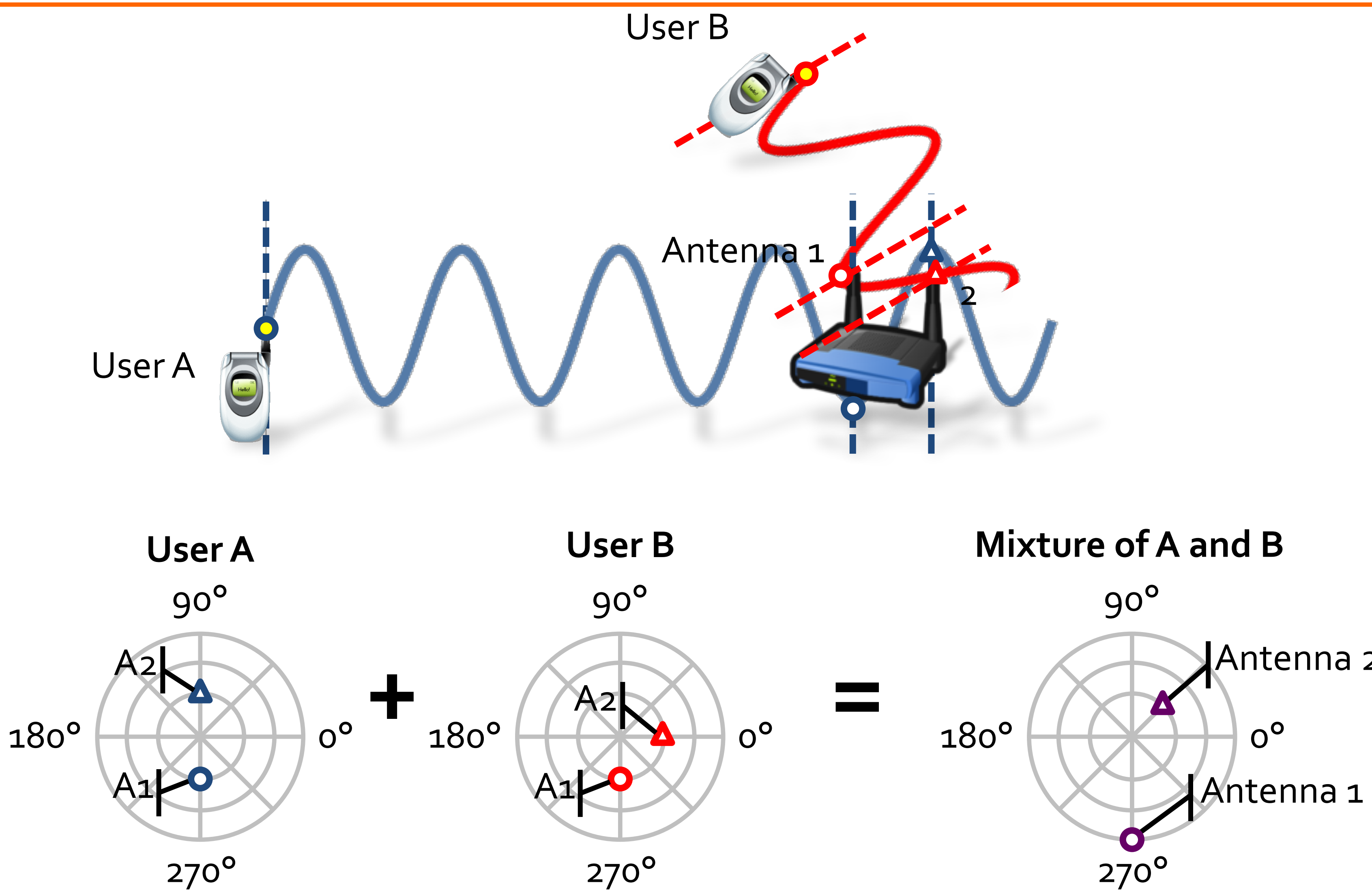
Multiple-input, multiple-output: MIMO

- Now, the AP hears **two** received signals, one on each antenna:





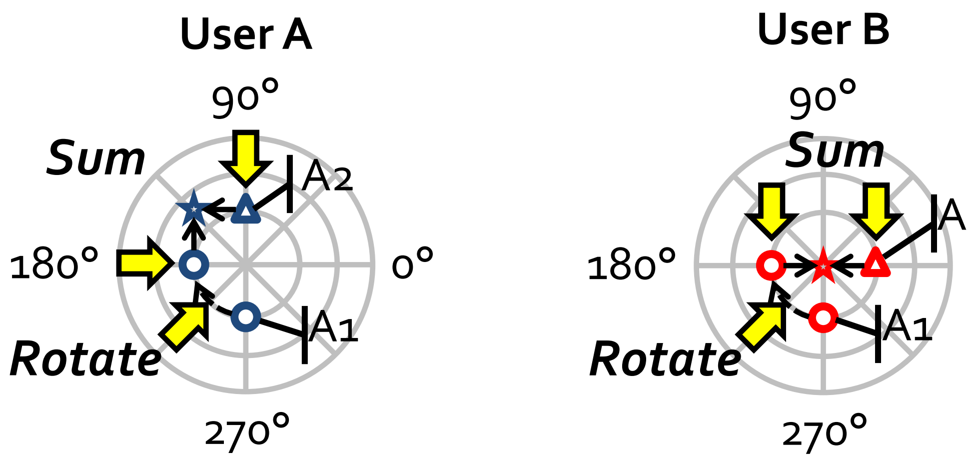
Leveraging MIMO to detect two users



Zero-forcing overcomes interference

- MIMO ***zero-forcing*** (Paulraj *et al.*, Foschini *et al.*):

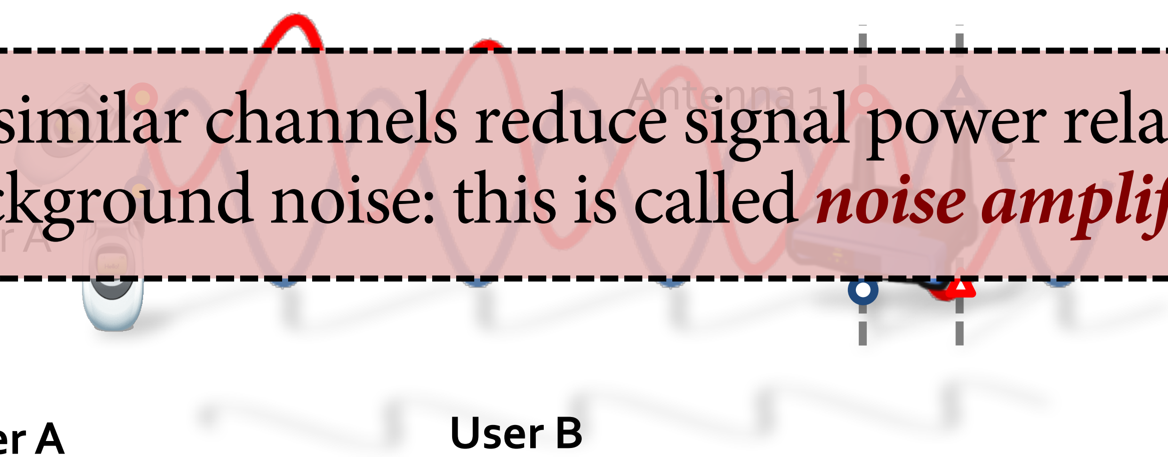
1. **Rotate** one antenna's signal (○)
2. **Sum** the two antennas' signals together (○+△)



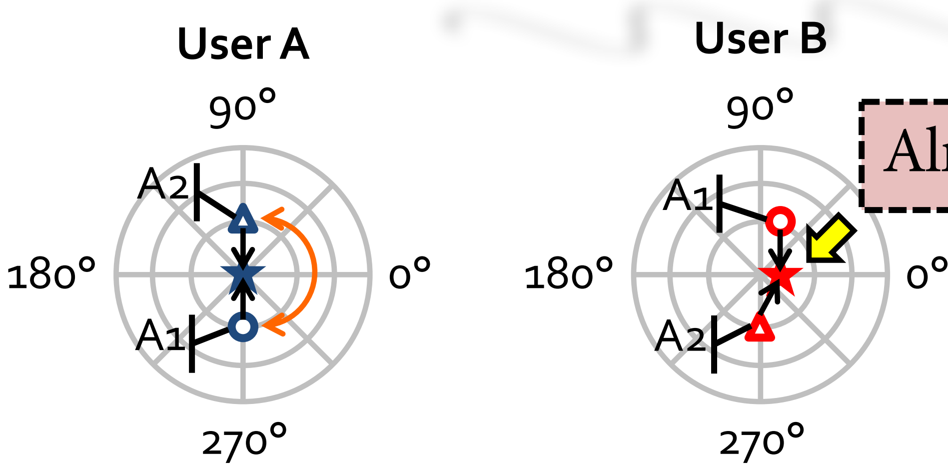
Zero-forcing **cancels** B, **revealing** A

Can re-run to cancel A, revealing B

Does zero-forcing work all the time?



Very similar channels reduce signal power relative to the background noise: this is called ***noise amplification***



Almost zero