# Social Networks and Ranking

1

# Generalized Social Networks

- Represent relationship between entities
  - paper cites paper
  - html page links to html page
  - A supervises B

  } directed graph

  - A and B are friends
  - papers share an author
  - A and B are co-workers

  } undirected graph

2

# Hypertext

- document or part of document links to other parts or other documents
  - construct documents of interrelated pieces
  - relate documents to each other
- pre-dates Web
- Web "killer app."

3

# How use links to improve information search?

- use structure to compute score for ranking
- include more objects to rank
  - redefines "satisfying" of query?
- add to the content of a document

✧can deal with objects of mixed types
  - images, PDF, …

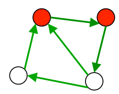
4

## Scoring using structure

- Ideas
  1. link to object suggests it valuable object

  2. distance between objects in graph represents degree of relatedness

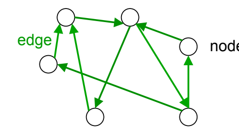
reachable by all in 2 links





## Pursuing linking and value

- Intuition: when Web page points to another Web page, it confers status/authority/ popularity to that page
- Find a measure that captures intuition
- Not just web linking
  - Citations in books, articles
  - others?






## Indegree

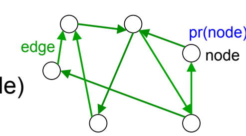
- Indegree = number of links into a node
- Most obvious idea:

  higher indegree => better node
- Doesn’t work well
- Need some feedback in system
- Leads us to Page and Brin’s PageRank



## PageRank

- Algorithm that gave Google the leap in quality
  - link structure centerpiece of scoring
- Framework
  - Given a directed graph with *n* nodes
  - Assign each node a score that represents its importance in structure: PageRank: pr(node)



# Conferring importance

Core ideas:

- A node should confer some of its importance to the nodes to which it points
  - If a node is important, the nodes it links to should be important
- A node should not transfer more importance than it has

9

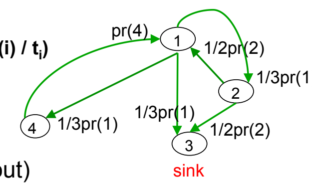
# Attempt 1

Refer to nodes by numbers 1, … , $n$ (arbitrary numbering)
Let $t_i$ denote the number of edges out of *node i* (outdegree)
Node i transfers $1/t_i$ of its importance on each edge out of it

Define

**$pr_{new}(k) = \sum_{i \text{ with edge from i to k}} (pr(i) / t_i)$**

Iterate until converges




Problems

- Sinks (nodes with no edges out)
- Cyclic behavior

10

# Attempt 2

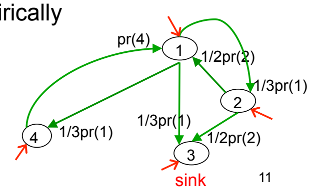
Random walk model

- Attempt 1 gives movement from node to linked neighbor with probability 1/outdegree
- Add random jump to any node

**$pr_{new}(k) = \alpha/n + (1-\alpha)\sum_{i \text{ with edge from i to k}} (pr(i) / t_i)$**

– $\alpha$ parameter chosen empirically

- Break cycles
- Escape from sinks




11

# Normalized?

- Would like **$\sum_{1\le k\le n} (pr(k)) = 1$**
- Consider **$\sum_{1\le k\le n} (pr_{new}(k))$**

$= \sum_{1\le k\le n}( \alpha/n + (1-\alpha)\sum_{i \text{ with edge from i to k}}(pr(i) / t_i) )$ (1)

$= \sum_{1\le k\le n}( \alpha/n)+\sum_{1\le k\le n}((1-\alpha)\sum_{i \text{ with edge from i to k}}(pr(i) / t_i))$ * (2)

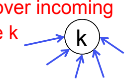
$= \alpha + (1-\alpha)\sum_{1\le k\le n} \sum_{i \text{ with edge from i to k}}(pr(i) / t_i)$ (3)

$= \alpha + (1-\alpha)\sum_{1\le i\le n}\sum_{k \text{ with edge from i to k}}(pr(i) / t_i)$ * (4)

$\alpha + (1-\alpha)\sum_{1\le i\le n} pr(i) \sum_{k \text{ with edge from i to k}}(1/ t_i)$ * (5)

$= \alpha + (1-\alpha)\sum_{i \text{ with edge from i}} pr(i)$ (6)

*inner sum $\sum_i$ over incoming edges for one k

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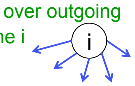
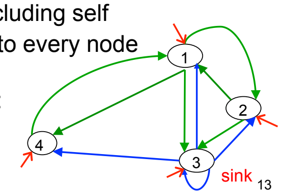



12

## Problem for desired normalization

- Have
  $\sum_{1\le k\le n}(\mathbf{pr_{new}(k)}) = \alpha + (1\text{-}\alpha)\sum_{i \text{ with edge from } i} \mathbf{pr(i)})$
- Missing **pr(i)** for nodes with no edges from them
  - sinks!
- Solution: add n edges out of every sink
  - Edge to every node including self
  - Gives 1/n contribution to every node

Gives desired normalization:
If $\sum_{1\le k\le n}(pr_{initial}(k)) = 1$
then $\sum_{1\le k\le n}(pr(k)) = 1$






## Matrix formulation

- Let E be the n by n adjacency matrix
  E(i,k) = 1 if there is an edge from node i to node k
  = 0 otherwise
- Define new matrix L:
  For each row i of E ($1\le i\le n$)
  If row i contains $t_i$ >0 ones, $L(i,k)=(1/t_i)\,E(i,k)$, $1\le k\le n$
  If row i contains 0 ones, $L(i,k) = 1/n$, $1\le k\le n$
- Vector ***pr*** of PageRank values defined by
  $\boldsymbol{pr} = (\alpha/n, \alpha/n, \dots \alpha/n)^T + (1-\alpha)\, L^T \boldsymbol{pr}$
- has a solution representing the steady-state values pr(k)



## Eigenvector Formulation

- $\boldsymbol{pr} = (\alpha/n, \alpha/n, \dots \alpha/n)^T + (1-\alpha)\, L^T \boldsymbol{pr}$
  $= (\alpha/n)\, J\boldsymbol{pr} + (1-\alpha)\, L^T \boldsymbol{pr}$
  $= (\,(\alpha/n)\, J + (1-\alpha)\, L^T\,)\, \boldsymbol{pr}$
  $= (\qquad M \qquad)\, \boldsymbol{pr}$
- J is the matrix of all 1's
- $J\boldsymbol{pr} = (1, 1, \dots 1)^T$ because $\sum_{1\le k\le n}(pr(k)) = 1$
- ***pr*** is the principal eigenvector of M
  $A\boldsymbol{v} = \lambda \boldsymbol{v}$, $\lambda=1$



## Calculation Choices

1. ***pr*** = M ***pr*** : Find principle eigenvector of M
   solves n simultaneous equations:
   $pr(k) = \alpha/n + (1\text{-}\alpha)\sum_{1\le i\le n} L(i,k)pr(i)$
2. Use iterative calculation - power method
   (where we started)
   – Initialize $pr_{initial}(k) = 1/n$ for each node k
   – Until converges {
   For each node k
   $pr_{new}(k) = \alpha/n + (1\text{-}\alpha)\sum_{1\le i\le n} L(i,k)pr(i)$
   For each node k
   $pr(k) = pr_{new}(k)$
   }

## Power method

- Convergence
  - In practice choose convergence criterion
    - e.g. stop iteration when
      $\text{Max}^{n}_{k=1}( |pr\_new(k) - pr(k)| ) < \varepsilon$
      $\varepsilon = 10^{-3}? \; 10^{-4}? \; 10^{-5}?$
- Choice $\alpha$
  - No single best value
  - 1-$\alpha$ determines rate of convergence
    - Second eigenvalue
  - $\alpha = 0.15$ common
    - gives $10^{-4}$ accuracy in about 60 iterations regardless of size of graph [Chu; Wu]



## PageRank Observations

- Can be calculated for *any* directed graph
- Google calculates on entire Web graph
  - query independent scoring
- Huge calculation for Web graph
  - precomputed
  - 1998 Google published:
    - 52 iterations for 322 million links
    - 45 iterations for 161 million links
- PageRank must be combined with query-based scoring for final ranking
  - Many variations
  - What Google exactly does secret
  - Can make some guesses by results



## HITS

**H**yperlink **I**nduced **T**opic **S**earch

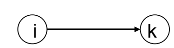
- Second well-known algorithm
- By Jon Kleinberg while at IBM Almaden Research Center
- Same general goal as PageRank
- Distinguishes 2 kinds of nodes
  - Hubs: resource pages
    - **Point to many authorities**
  - Authorities: good information pages
    - **Pointed to by many hubs**



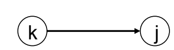
## Mutual reinforcement

- Authority weight node j: a(j)
  - Vector of weights ***a***
- Hub weight node j: h(j)
  - Vector of weights ***h***
- Update:

$a_{new}(k) = \sum_{i \text{ with edge from } i \text{ to } k} (h(i))$




$h_{new}(k) = \sum_{j \text{ with edge from } k \text{ to } j} (a(j))$



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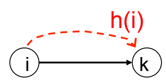


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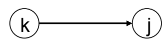


21

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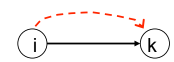


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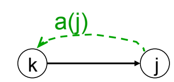



22

## Matrix formulation

Steady state:

| | |
|---|---|
| $\boldsymbol{a} = E^T \boldsymbol{h}$ | $\boldsymbol{a} = E^T E \boldsymbol{a}$ |
| $\boldsymbol{h} = E \boldsymbol{a}$ | $\boldsymbol{h} = E E^T \boldsymbol{h}$ |

Interpretation?

23

## Look inside

- $E^T(i,k)$ 1 where k→i
- $E(k,j)$ 1 where k→j
- Row i of $E^T$:
  1's where k's→i
- Column j of E:
  1's where k's→j
- $E^TE(i,j)$ is number of nodes pointing to both i and j

- $E(i,k)$ 1 where i→k
- $E^T(k,j)$ 1 where j→k
- Row i of E:
  1's where i→k's
- Column j of $E^T$
  1's where j→k's
- $EE^T(i,j)$ is number of nodes pointed to by both i and j

24

## Matrix formulation

Steady state:

$\boldsymbol{a} = E^T\boldsymbol{h}$ $\boldsymbol{a} = E^TE\boldsymbol{a}$

$\boldsymbol{h} = E\boldsymbol{a}$ $\boldsymbol{h} = EE^T\boldsymbol{h}$

Interpretation:

- $E^TE(i,j)$: number nodes point to both node i and node j
  - "Co-citation"
- $EE^T(i,j)$: number nodes pointed to by both node i and node j
  - "Bibliographic coupling"



## Iterative Calculation

$\boldsymbol{a} = \boldsymbol{h} = (1, \ldots, 1)^T$

While (not converged) {

$\boldsymbol{a}_{new} = E^t\boldsymbol{h}$

$\boldsymbol{h}_{new} = E\boldsymbol{a}$

$\boldsymbol{a} = \boldsymbol{a}_{new} / ||\boldsymbol{a}_{new}||$ normalize to unit vector

$\boldsymbol{h} = \boldsymbol{h}_{new} / ||\boldsymbol{h}_{new}||$ normalize to unit vector

}

Provable convergence by linear algebra



## Use of HITS

original use **after** find Web pages satisfying query:

1. Retrieve documents satisfy query and rank by term-based techniques
2. Keep top *c* documents: root set of nodes
   – *c* a chosen constant - tunable
3. Make base set:
   a) Root set
   *b)* *Plus* nodes pointed to by nodes of root set
   *c)* *Plus* nodes pointing to nodes of root set

   using links to expand matches!
4. Make base graph: base set plus edges from Web graph between these nodes
5. Apply HITS to base graph



## Results using HITS

- Documents ranked by authority score a(doc) and hub score h(doc)
  – Authority score primary score for search results
- Heuristics:
  – delete all links between pages in same domain
  – Keep only pre-determined number of pages linking into root set ( ~200)
- Findings (original paper)
  – Number iterations in original tests ~50
  – most authoritative pages **do not** contain initial query terms

## Observations

- HITS can be applied to any directed graph
- Base graph **much smaller** than Web graph
- Kleinberg identified bad phenomena
  - Topic diffusion: generalizes topic when expand root graph to base graph
    - example: want *compilers* - generalized to *programming*

29

## PageRank and HITS

- designed independently around 1997
- indicates time was ripe for this kind of analysis
- lots of embellishments by others

30

## Revisit: How use links in ranking documents?

- use structure to compute score for ranking
  - PageRank, HITS
- include more objects to rank
  - saw in use of HITS

➢ use anchor text (HTML)
- anchor text labels link
- include anchor text as text of ***document pointed to***

31

## Anchor text

- HTML text:

All assignments will be made available on
<a href="https://piazza.com/princeton/spring2017/cos435/home">the Piazza course account</a>.

- Renders as:

All assignments will be made available on
the Piazza course account.

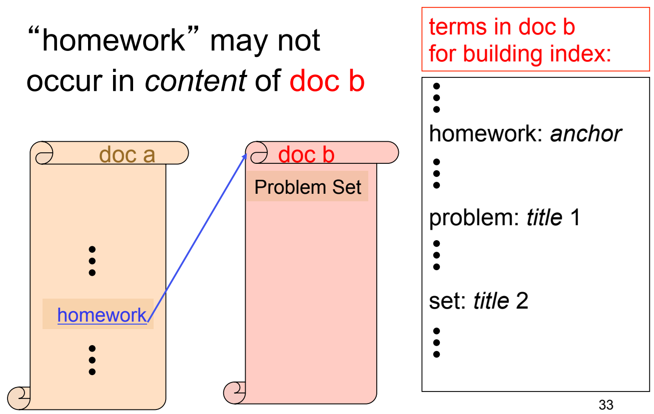
- Anchor text:

"the Piazza course account" is anchor text

32

## Using anchor text

“homework” may not occur in *content* of doc b



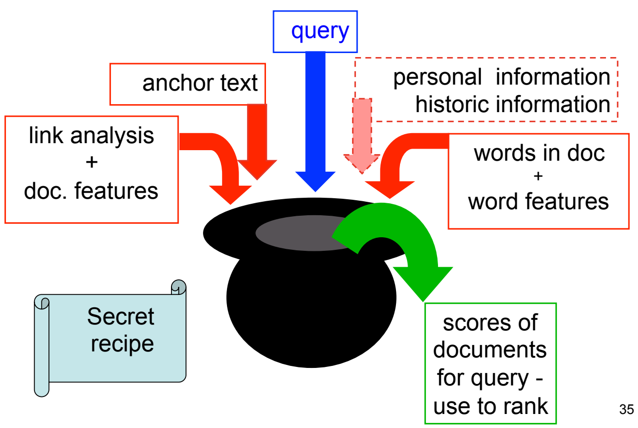

33

## Summary

- Link analysis
  - a principal component of ranking by modern Web search engines
  - must be combined with content analysis
- Extend document content with link info
  - anchor text
  - text of URLs
    - e.g. princeton.edu, aardvarksportsshop.com
- Expand set of satisfying docs using links
  - less often used

34

## Ranking documents w.r.t. query




35

## General Framework

- Have set of n features (aka signals) to use in determining ranking score
  - Features depend on query:
    vector $\Psi(d_i,q)$ of feature values $f_k$ for doc $d_i$, query q
    - eg tf.idf score is feature
  - Features are conditioned to be comparable
- Have parameterized function to combine signals
  - simple: linear $\alpha_0 + \sum_{i=1}^{n} \alpha_i{*}(f_i)$
  - $\alpha_i$ are adjustable weights - how choose?
    - intuition
    - experimentation
    - machine learning

36

# Machine Learning

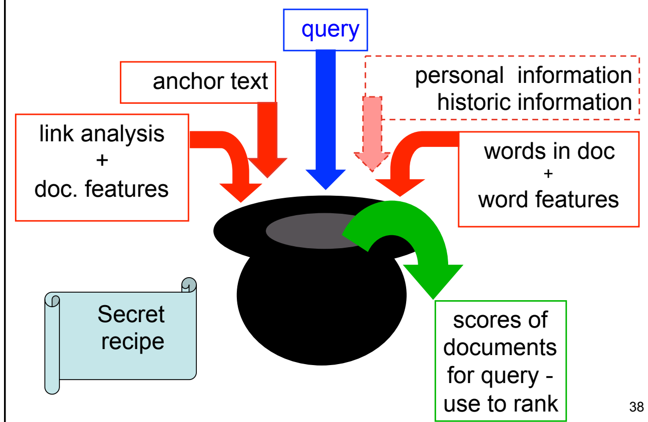
Many possibilities – overview of one

Ordinal Regression Model

- Goal: get comparison of doc.s correct
- capture goal
  - Let $\boldsymbol{\omega}$ represent vector $(\alpha_1 , \ldots , \alpha_n )$
  - want $\boldsymbol{\omega}^T \bullet \Psi(d_i,q) - \boldsymbol{\omega}^T \bullet \Psi(d_j,q) > 0$ if and only if
    $d_i$ more relevant than $d_j$ for query q
  - find $\boldsymbol{\omega}$ that works
- techniques train on known correct data:
  - humans rank a set of documents for various queries

37

# Ranking documents w.r.t. query




38