N-BODY SIMULATION

- overview
- problem decomposition
- the physics
- bugs
- universes
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N-body simulation

Simulate the motion of $n$ bodies, subject to Newton’s laws.
Physics and math

Newton’s law of gravity.  \[ F = \frac{G m_1 m_2}{r^2} \]

Newton’s second law of motion.  \[ F = m a \]

“Leapfrog” method.  For numerical integration of differential equations.

don’t worry (this is not a math or physics course)
Applications. Cosmology, semiconductors, fluid dynamics, ....

http://www.youtube.com/watch?v=ua7YIN4eL_w
Programming goals

- Use standard input, standard output, and standard drawing for I/O.
- Use parallel arrays.
- Decompose a large program into small, manageable steps.

key to becoming a good programmer
Before you begin

Carefully read assignment specification; skim checklist.

Check that standard libraries are available to Java.
  • Already configured if you used auto-installer.
  • Remember to use javac-introcs and java-introcs at command line.

Useful programs from lecture/precept.
  • Students.java
  • BouncingBallDeluxe.java
  • Distinct.java

Download sample data files and create working directory.
  • Download nbody.zip from assignment specification.
    (see Assignment FAQ for extracting zip file)
  • Have all data files available in a folder, say nbody.
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Decompose problem into individual steps

Develop program incrementally, decomposing into 6 individual steps.

1. Parse command-line arguments.
2. Read universe from standard input.
3. Initialize standard drawing.
4. Play music on standard audio.
5. Simulate the universe.
   - A. Calculate net forces.
   - B. Update velocities and positions.
   - C. Draw universe to standard drawing.
6. Print universe to standard output.

Advice. Although final code will appear in order 1–6, we recommend implementing these steps in the order 1, 2, 6, 3, 4, 5B, 5C, 5A.

Q. Why?
A. Easier to test and debug.
public class NBody {
    public static void main(String[] args) {

        // Step 1. Parse command-line arguments.
        // Step 2. Read universe from standard input.
        // Step 3. Initialize standard drawing.
        // Step 5. Simulate the universe.
        // Step 5A. Calculate net forces.
        // Step 5B. Update velocities and positions.
        // Step 5C. Draw universe to standard drawing.

        // Step 6. Print universe to standard output.
    }
}
Command-line arguments

Step 1. Parse command-line arguments.
- Read stopping time $T$ and increment $\Delta t$ from command line.
- Print values of each variable (as debugging aid).

Note. Easy, but you should still test it!

```bash
% java NBody 10 1
  tau = 10.0
  dt = 1.0

% java NBody 157788000.0 25000.0
  tau = 1.57788E8
  dt = 25000.0
```
### Step 2. Read universe from standard input.

This file contains the sun and the inner 4 planets of our Solar System.

<table>
<thead>
<tr>
<th>number of bodies</th>
<th>radius of universe</th>
<th>planet</th>
<th>initial x-position</th>
<th>initial y-position</th>
<th>initial x-velocity</th>
<th>initial y-velocity</th>
<th>mass</th>
<th>image filename</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.50e+11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4960e+11</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>2.9800e+04</td>
<td>5.9740e+24</td>
<td>earth.gif</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2790e+11</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>2.4100e+04</td>
<td>6.4190e+23</td>
<td>mars.gif</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.7900e+10</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>4.7900e+04</td>
<td>3.3020e+23</td>
<td>mercury.gif</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>1.9890e+30</td>
<td>sun.gif</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0820e+11</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>3.5000e+04</td>
<td>4.8690e+24</td>
<td>venus.gif</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This file contains the sun and the inner 4 planets of our Solar System.

Optional description
Step 2. Read universe from standard input.
- Read number of bodies $n$ from standard input.
- Read $radius$ of universe standard input.
- Create 6 parallel arrays, each of length $n$, to store the 6 pieces of information associated with each body.
- Read the data associated with each body and store in parallel arrays.

Hint. Recall Students.java.

```bash
% java-introcs NBody 157788000.0 25000.0 < planets.txt
[no output]
```

Q. How to test?
A. Do Step 6 (print universe).
Step 6. Print universe to standard output.

- Write a loop to iterate over the 6 parallel arrays.
- Use StdOut.printf() for formatted output (see checklist for hint).

% java-introcs NBody 157788000.0 25000.0 < planets.txt
5
2.50e+11
  1.4960e+11 0.0000e+00 0.0000e+00 2.9800e+04 5.9740e+24    earth.gif
  2.2790e+11 0.0000e+00 0.0000e+00 2.4100e+04 6.4190e+23    mars.gif
  5.7900e+10 0.0000e+00 0.0000e+00 4.7900e+04 3.3020e+23    mercury.gif
  0.0000e+00 0.0000e+00 0.0000e+00 0.0000e+00 1.9890e+30       sun.gif
  1.0820e+11 0.0000e+00 0.0000e+00 3.5000e+04 4.8690e+24    venus.gif
Standard drawing

Step 3. Initialize standard drawing.

- Enable double buffering by calling `StdDraw.enableDoubleBuffering()`.
- Default $x$- and $y$-scale supports coordinates between 0 and 1;
- Change scale to be between $-radius$ and $+radius$.

Hint: `StdDraw.setXscale()` and `StdDraw.setYscale()`.

Q. How to test?
Step 4. Play music.
- Call StdAudio.play("2001.wav").
- Easy (but optional).
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The simulation loop (the "big time loop")

Step 5. Simulate the universe. At each time step $t$:
   A. Calculate the net force on each body.
   B. Update the velocities and positions.
   C. Draw the universe.

Q. In which order should I implement these 3 sub-steps?
A. 5B, 5C, 5A because calculating forces is hardest.

Q. Can I combine 3 sub-steps into a single big one?
A. No. Not only is it bad design, but it ruins the physics.
   (need position of all bodies at time $t$, not some at time $t + \Delta t$)

Hint. See BouncingBallDeluxe.java.
Measuring time

Time loop. From $t = 0$ up to (but not including) $T$, incrementing by $\Delta t$.

Hint. Easy, but also easy to get wrong. $\Rightarrow$ Test!

<table>
<thead>
<tr>
<th>$T = 23.0$, $\Delta t = 2.5$</th>
<th>$T = 25.0$, $\Delta t = 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0.0$</td>
<td>$t = 0.0$</td>
</tr>
<tr>
<td>$t = 2.5$</td>
<td>$t = 2.5$</td>
</tr>
<tr>
<td>$t = 5.0$</td>
<td>$t = 5.0$</td>
</tr>
<tr>
<td>$t = 7.5$</td>
<td>$t = 7.5$</td>
</tr>
<tr>
<td>$t = 10.0$</td>
<td>$t = 10.0$</td>
</tr>
<tr>
<td>$t = 12.5$</td>
<td>$t = 12.5$</td>
</tr>
<tr>
<td>$t = 15.0$</td>
<td>$t = 15.0$</td>
</tr>
<tr>
<td>$t = 17.5$</td>
<td>$t = 17.5$</td>
</tr>
<tr>
<td>$t = 20.0$</td>
<td>$t = 20.0$</td>
</tr>
<tr>
<td>$t = 22.5$</td>
<td>$t = 22.5$</td>
</tr>
</tbody>
</table>

Don’t include 25.0
Updating the velocities and positions

Step 5B. [for now, forces and accelerations are 0]

- Update the velocity of each body: \( v_x = v_x + a_x \Delta t, \ v_y = v_y + a_y \Delta t. \)
- Update the position of each body: \( p_x = p_x + v_x \Delta t, \ p_y = p_y + v_y \Delta t. \)

Warning. Cut-and-paste errors are common.

Q. How to test?
A. Artificial universe that is easy to check by hand.

% java-introcs NBody 191 1 < 3body-zero-gravity.txt
3
5.12e+02
1.9200e+02 1.9200e+02 1.0000e+00 1.0000e+00 1.0000e-30  earth.gif
5.1200e+02 1.9200e+02 2.0000e+00 1.0000e+00 1.0000e-40  venus.gif
1.9200e+02 5.1200e+02 1.0000e+00 2.0000e+00 1.0000e-50  mars.gif
Drawing the universe

Step 5C.

- Draw background image.
- Write loop to display $n$ bodies.
- Call `StdDraw.show()` to display results on screen.
- Call `StdDraw.pause(20)` to control animation speed.

![planets.txt](planets.txt)

![kaleidoscope.txt](kaleidoscope.txt)
Calculating the force (between two bodies at time t)

**Step 5A.**
- Apply Newton’s law of gravity.
- A bit of high-school trig (formulas provided).

### Distance between two bodies

\[ r = \sqrt{\Delta x^2 + \Delta y^2} \]

### Force between two bodies

\[ F = \frac{G m_1 m_2}{r^2} \]

\[ F_x = F \cos \theta \]

\[ F_y = F \sin \theta \]

\[ \cos \theta = \frac{\Delta x}{r}, \quad \sin \theta = \frac{\Delta y}{r} \]
Calculating the force (between all pairs of bodies at time $t$)

Principle of superposition. Add all pairwise forces.

\[ \vec{F}_{\text{earth}} = \vec{F}_{\text{mars} \to \text{earth}} + \vec{F}_{\text{mercury} \to \text{earth}} + \vec{F}_{\text{sun} \to \text{earth}} + \vec{F}_{\text{venus} \to \text{earth}} \]

How to implement?

- Need two extra arrays $f_x[]$ and $f_y[]$. Why?
- Need to examine all pairs of bodies, ala Distinct.java.

Warmup. Enumerate all pairs of bodies.

<table>
<thead>
<tr>
<th>$n = 5$</th>
<th>$n = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1 0–2 0–3 0–4</td>
<td>0–1 0–2 0–3</td>
</tr>
<tr>
<td>1–0 1–2 1–3 1–4</td>
<td>1–0 1–2 1–3</td>
</tr>
<tr>
<td>2–0 2–1 2–3 2–4</td>
<td>2–0 2–1 2–3</td>
</tr>
<tr>
<td>3–0 3–1 3–2 3–4</td>
<td>3–0 3–1 3–2</td>
</tr>
<tr>
<td>4–0 4–1 4–2 4–3</td>
<td></td>
</tr>
</tbody>
</table>
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Advice

- Develop code incrementally; test after each step.
  - Test, test, test.
  - Take your time!
  - Start early!
  - Seek help if you get stuck.
  - Write outline of code (using comments) first; fill in code later.

key to becoming a good programmer
Command-line bug

% java-introcs NBody 157788000.0 25000.0 > planets.txt

<Ctrl-C>

% java-introcs NBody 157788000.0 25000.0 < planets.txt
Exception in thread "main" java.util.NoSuchElementException
  at java.util.Scanner.throwFor(Scanner.java:907)
  at java.util.Scanner.next(Scanner.java:1530)
  at java.util.Scanner.nextInt(Scanner.java:2160)
  at java.util.Scanner.nextInt(Scanner.java:2119)
  at StdIn.readInt(StdIn.java:319)
  at NBody.main(NBody.java:54)

% more planets.txt
[it's empty - you erased it!]
Visual bugs

no motion

no gravity
Visual bugs

no double buffering

planets repel one another
Visual bugs

wrong force loop

cut-and-paste error (x vs. y)
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Other universes

planetsparty.txt
(created by Mary Fan)

twinbinaries.txt
(David Costanzo)
Other universes

chaosblossum.txt
(created by Erik Keselica)

galaxy.txt
(created by Matt Tilghman)