This test has 10 questions worth a total of 50 points. You have 50 minutes. The exam is closed book, except that you are allowed to use a one page cheatsheet. No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. Write out and sign the Honor Code pledge before turning in the test.

“I pledge my honor that I have not violated the Honor Code during this examination.”

Name: ___________________________  NetID: ___________________________

Problem | Score | Problem | Score
---------|-------|---------|-------
0        |       | 5       |       |
1        |       | 6       |       |
2        |       | 7       |       |
3        |       | 8       |       |
4        |       | 9       |       |
Sub 1    |       | Sub 2   |       |

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Signature

P01  TTh 1:30  Keith
P01A TTh 1:30  Doug
P01B TTh 1:30  Victor
P01C TTh 1:30  Richard
P01D TTh 1:30  Gordon
P01E TTh 1:30  Arman
P02  TTh 2:30  Doug
P03  TTh 3:30  Gordon
P03A TTh 3:30  Keith
P04  TTh 7:30  Nick
P05  WF 10    Dmitry
P06  WF 1:30  Victor
P06A WF 1:30  Chris
P06B WF 1:30  Donna
P07  WF 12:30 Donna

Do not remove this exam from the room.
0. Miscellaneous. (1 point)

(a) Write your name and Princeton NetID in the space provided on the front of the exam, and circle your precept number.

(b) Write and sign the honor code on the front of the exam.

1. Data types. (7 points)

(a) Define what is a data type.

A set of ________________ and ________________

on those ________________.

(b) Suppose that the following four variables are initialized in the four statements below.

```java
String r = "Hello";
String s = "World";
String t = r + " " + s;
String u = "Hello World";
```

Give the type and value of each Java expression below. If it leads to a compile-time or runtime-error, specify that for the type (and leave the value column blank).

<table>
<thead>
<tr>
<th>Java expression</th>
<th>type</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>r.length()</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r.charAt(r.length())</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t == u)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>u.equals(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r.substring(0, r.length())</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Scientific computation. (4 points)

(a) The left plot below shows the function $f(x) = (\cos x - 1) / x^2$ in the interval $-4 \cdot 10^{-8} \leq x \leq 4 \cdot 10^{-8}$. The right plot below shows the result of attempting to plot $f(x)$ with the Matlab command `ezplot`.

\[
\text{>> ezplot('}(\cos(x) - 1) / x^2', [-4e-8, 4e-8]);
\]

Which of the following best explains why the Matlab plot of $f(x)$ is so inaccurate? Circle the best answer.

i. Matlab is not using enough sample points and/or the samples are not taken uniformly.

ii. Matlab uses the IEEE 754 floating-point standard and the algorithm for computing $f(x)$ is \textit{unstable} in the specified interval because of catastrophic cancellation.

iii. Matlab uses the IEEE 754 floating-point standard and the function $f(x)$ is \textit{ill-conditioned} in the specified interval because of catastrophic cancellation.

(b) Which of the following numbers are exactly representable as a Java \texttt{double}?

Circle \textit{all} such numbers.

\[
1/5 \quad 1/2 \quad 3/4 \quad 1 \quad \pi \quad 123
\]
3. Linked structures. (5 points)

Suppose that the Node data type is defined as

```java
private class Node {
    private int item;
    private Node next;
}
```

and that first is a variable of type Node that refers to one node in a circular linked list.

Let x be a variable that refers to a newly created node.

```java
Node x = new Node();
x.item = 4;
```

Consider the following three code fragments.

I. `x.next = first.next;
   first.next = x;`

II. `first.next = x;
    x.next = first.next;`

III. `Node z = first;
     x.next = first.next;
     z.next = x;`

Which of them inserts x into the circular linked list immediately after first? Assume that the initial circular linked list is not empty. Circle the best answer.

I only  I and III only
II only  I, II, and III
I and II only
4. **Data type design. (6 points)**

(a) Which of the following help *enforce immutability* in Java? Write the letter Y next to each description that helps enforce immutability and N if it does not.

- ___ declaring instance variables to be private
- ___ declaring instance variables to be immutable
- ___ declaring instance variables to be final
- ___ defensively copying instance variables
- ___ overloading instance methods

(b) Suppose that a client passes a reference to an object to a static method and that object’s value after the function call is *different from* its value before the call. Which of the following data types that you’ve encountered in this course could be the type of the object? Circle one or more answers.

- double[]
- String
- Stack<String>
- GuitarString
- Tour
- Complex
5. Analysis of algorithms. (6 points)

Given an integer array \( a[] \), the following method counts the number of distinct integers.

```java
public static int distinct(int[] a) {
    int N = a.length;
    int count = 0;
    for (int i = 0; i < N; i++) {
        boolean distinct = true;
        for (int j = i+1; j < N; j++) {
            if (a[i] == a[j]) distinct = false;
        }
        if (distinct) count++;
    }
    return count;
}
```

(a) What is the order of growth of the running time of `distinct()` as a function of \( N \)?
Circle the best answer.

\[
N \quad N \log N \quad N^2 \quad N^3 \quad 2^N \quad N!
\]

(b) Suppose that `distinct()` takes 15 seconds to process an array of size 100,000. Estimate how long (in seconds) it will take to process an array of size 400,000.

(c) Suppose that you declare the following two-dimensional array in Java:

```java
double[][] a = new double[N][N];
```

How much memory in bytes does `a[][]` consume as a function of \( N \)?
Circle the best answer. *Hint: a Java double is 64 bits.*

\[
\sim 4N \quad \sim 8N \quad \sim 32N \quad \sim 64N \quad \sim 4N^2 \quad \sim 8N^2 \quad \sim 32N^2 \quad \sim 64N^2
\]
6. **Symbol tables. (4 points)**

Consider the binary search tree at left. Put the same set of keys into the binary tree at right so that it is a binary search tree.

[Diagram of binary search trees]

7. **Regular expressions. (4 points)**

A *camelCase* string over the alphabet \{ a, b, A, B \} is characterized by:

- starts with a lowercase letter
- no two consecutive uppercase letters
- ends with a lowercase letter

<table>
<thead>
<tr>
<th>matches</th>
<th>does not match</th>
</tr>
</thead>
<tbody>
<tr>
<td>aaaa</td>
<td>Baaa</td>
</tr>
<tr>
<td>aaaBa</td>
<td>aaaaB</td>
</tr>
<tr>
<td>aBaaAaa</td>
<td>ABABBBA</td>
</tr>
<tr>
<td>baBabAaAbAbb</td>
<td>aBAAbaba</td>
</tr>
<tr>
<td>b</td>
<td>B</td>
</tr>
<tr>
<td>bb</td>
<td>$\epsilon$</td>
</tr>
</tbody>
</table>

Which regular expression below matches *camelCase* strings? Circle the best answer.

i. \((a|b) (.*) (a|b)*\)

ii. \((a|b) (a|b)* ((A|B) (a|b) (a|b))*\)

iii. \(((a|b)* (A|B) (a|b))\)*

iv. \((a|b) ((a|b)* (A|B) (a|b))\)* \((a|b)\)
8. **Theory of computation. (8 points)**

(a) For each statement on the left, pick the best matching description on the right. You may use each letter any number of times.

- There exists a mathematical function that can be computed in Java, but cannot be computed on a Turing machine.
  
  A. known to be true
  B. known to be false
  C. if true would falsify the Church-Turing thesis
  D. if true would falsify the extended Church-Turing thesis
  E. if true would prove the Church-Turing thesis

- There exists a mathematical function that can be computed in polynomial time on a quantum computer, but cannot be computed in polynomial time on a Turing machine. Assume that quantum computers can be built.
  
  A. known to be true
  B. known to be false
  C. if true would falsify the Church-Turing thesis
  D. if true would falsify the extended Church-Turing thesis
  E. if true would prove the Church-Turing thesis

- There exists a mathematical function that can be computed in polynomial time in Java, but cannot be computed in polynomial time on a Turing machine.

- There exists a Universal Turing machine that can simulate the behavior of any other Turing machine.

(b) For each statement on the left, pick the best matching description on the right. You may use each letter any number of times.

- Not all search problems can be solved in polynomial time.
  
  A. known to be true
  B. known to be false
  C. if true would imply \( P = NP \)
  D. if true would imply \( P \neq NP \)
  E. if true would prove the extended Church-Turing thesis

- There exists a search problem that can be solved in polynomial time.

- Both FACTOR and 3-Sat can be solved in polynomial time.

- Exactly one of 3-Sat and Tsp can be solved in polynomial time.
9. Circuits. (5 points)

(a) Fill in the truth table for a 3-bit palindrome XYZ (reads same forwards and backwards).

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tbody>
</table>

(b) Write the (unsimplified) sum-of-products boolean formula for 3-bit palindromes.

(c) As a function of \( N \), how many \( N \)-input AND gates are in the (unsimplified) sum-of-products circuit for an \( N \)-bit palindrome. Assume that \( N \) is an odd integer and \( N \geq 3 \).