This test is 9 questions, weighted as indicated. The exam is closed book, except that you are allowed to use a one page cheatsheet. No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. Put your name, login ID, and precept number on this page (now), and write out and sign the Honor Code pledge before turning in the test. You have 50 minutes to complete the test.

"I pledge my honor that I have not violated the Honor Code during this examination."

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1. **Binary Search Trees** (8 points).

Suppose that you are searching for the key 70 in a binary search tree. In the following list, circle the letters corresponding to sequences that could *not* be the sequence of keys examined.

A. 70
B. 99 10 80 20 60 70
C. 10 80 40 32 50 70
D. 10 26 30 48 50 62 70
E. **41 99 20 85 70**
F. 44 80 55 61 70
G. 99 11 53 86 70
H. 22 58 81 70
2. **REs/DFAs** (10 points). For this problem the alphabet consists of the two letters a and b and the two numbers 0 and 1. Consider the following DFAs (identified by letters) and REs (identified by numbers).

![DFAs and REs](image)

<table>
<thead>
<tr>
<th>DFA</th>
<th>RE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>none</td>
</tr>
<tr>
<td>B</td>
<td>All strings with at least one number</td>
</tr>
<tr>
<td>C</td>
<td>All strings of even length in which letters and numbers alternate</td>
</tr>
<tr>
<td>D</td>
<td>All strings that end in a number</td>
</tr>
<tr>
<td>E</td>
<td>All strings that begin and end with letters and have at least one number</td>
</tr>
</tbody>
</table>

1. \((a|b)(a|b)*(0|1)(a|b|0|1)*(a|b)\)

2. \((a|b)(a|b)*(0|1)(a|b)*(a|b)\)

3. \((a|b)*(0|1)(a|b|0|1)\)

4. \(((a|b)(0|1))\)* | \(((0|1)(a|b))\)*

For each of the descriptions below write a letter in the first column if the corresponding DFA recognizes all the strings in the given set (and only those strings!) and write a number in the second column if the corresponding RE is an equivalent description of the set. If no RE or DFA does the job write "none" (blanks are considered to be incorrect).

<table>
<thead>
<tr>
<th>DFA</th>
<th>RE</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>none</td>
<td>4</td>
</tr>
<tr>
<td>A</td>
<td>none</td>
</tr>
<tr>
<td>none</td>
<td>1</td>
</tr>
</tbody>
</table>
3. **Linked structures** (10 points). Consider the following Java class, which implements a linked data structure, and then answer the questions on the next page.

```java
public class Forest{
    private Node[] links;

    private class Node{
        private Node next;
    }

    public Forest(int N){
        links = new Node[N];
        for (int i = 0; i < N; i++)
            links[i] = new Node();
    }

    private Node root(int i){
        Node x = links[i];
        while (x.next != null) x = x.next;
        return x;
    }

    public void merge(int i, int j){
        root(i).next = root(j);
    }

    public boolean merged(int i, int j){
        return root(i) == root(j);
    }
}
```
3 (continued). Forest builds general structures known as forests, where roots have null links and every other node points to its parent. To begin to understand how it works, verify that the client code at left produces the forest drawn below (each node is labeled with the index in the links array that has a reference to it).

```
Forest t = new Forest(6);
t.merge(0, 1);
t.merge(2, 1);
t.merge(4, 5);
```

A. (6 points) Draw the forest that is created by the following client code:

```
Forest t = new Forest(8);
t.merge(0, 3);
t.merge(1, 2);
t.merge(1, 4);
t.merge(5, 6);
t.merge(3, 4);
t.merge(7, 5);
```

B. (4 points) Now suppose that the following calls to `t.merged()` follow the client code of part A. In the blank to the left of each call, write `true` or `false` to give the value returned.

__ **true**__  t.merged(0, 3);

__ **false**__  t.merged(0, 7);

__ **true**__  t.merged(1, 3);

__ **false**__  t.merged(4, 5);
4. **ADTs and APIs** (10 points).

A. (3 points) Give an API for the `Forest` class in question 3. For full credit, include comments.

```
Forest(int N)  Create a forest of items 0 to N-1.
void merge(int i, int j)  Put i and j into the same tree.
boolean merged(int i, int j)  Are i and j in the same tree?
```

B. (7 points) In the blanks to the left of each of the descriptions at the bottom of the page write the letters corresponding to the identifiers from `Forest` that match the description. In your answer, some blanks may be empty and others may have multiple letters.

A. **links**

B. **Node**

C. **Forest**

D. **root**

E. **merge**

F. **x**

```
__E C__  Any client can refer to it.
__ A __  Instance variable.
__ D E _ Instance method.
F D B A  No client can refer to it.
__B C__  Data type.
__A F__  Reference variable.
__ B __ Inner class.
```
5. **Divide and conquer** (8 points). Consider the following class:

```java
public class Mystery
{
    public static String twist(String s)
    {
        if (s.length() < 2) return s;
        int m = s.length()/2;
        String lh = twist(s.substring(0, m));
        String rh = twist(s.substring(m, s.length()));
        s = rh + lh;
        System.out.println(s);
        return s;
    }

    public static void main(String[] args)
    {
        String s = args[0];
        s = twist(s);
    }
}
```

Recall that `s.substring(i, j)` returns the substring of `s` from indices `i` to `j-1`. For example, if `s` is the string "stressed" then `s.substring(0, 6)` is "stress".

A. Give the one line of output produced by the command

```
% java Mystery ab
___ba___
```

B. Give the seven lines of output produced by the command

```
% java Mystery stressed
___ts___
___er___
___erts___
___ss___
___de___
___dess___
desserts
```
6. **Gates** (5 points). Identify the gates below by writing AND, OR, NOT, NOR, or XOR above each of them.

   ![Gates Diagram]

7. **Components** (5 points). Identify the circuit components below by writing DECODER, MAJORITY, MULTIPLEXER, MEMORY BIT, or ODD PARITY above each of them.

   ![Components Diagram]
8. **Theory** (16 points). Match the theoretical results, conjectures, theses, and ideas (A-H) with their practical consequences, by writing a letter in the blank to the left of each. Assume that “fast” means “in polynomial time,” “TSP” stands for the traveling salesperson problem, and “SAT” stands for the boolean satisfiability problem. In this problem, each letter should be used once and only once.

A. Cook’s Theorem
B. Turing’s Entscheidungsproblem (decision problem) paper, 1937
C. The Church-Turing thesis
D. P = NP
E. There is a DFA that accepts the same language as any given RE
F. Karp’s reductions in his landmark 1972 paper
G. Factoring is hard
H. P is not equal to NP

_C_ My laptop can solve any problem that yours can.

_F_ If you can solve the TSP fast, then you can also solve SAT fast.

_D_ SAT can be solved fast.

_G_ Online banks can retain their current encryption programs.

_H_ The TSP cannot be solved fast.

_B_ Some problems cannot be solved by following a set of mechanical instructions.

_A_ If you can solve SAT fast, then you can also solve the TSP fast.

_E_ It is easy to write a program that searches for email addresses on a web page.
9. **Turing machine (8 points).** Consider the following Turing machine.

![Turing machine diagram]

**A.** (4 points) Give the contents of the tape after the machine halts when it is run for the given input tapes, with the head initially positioned at the leftmost 1. (*Reminder:* the tape head moves on entering a state.)

Input:   ...# # # 1 # # # ...

Result:  __ __ __ # # 1 0 # # # ...

Input:   ... # # # 1 0 1 # # # ...

Result:  __ __ __ # # # # # 1 0 1 0 # # # ...

**B.** (2 points) What function does this machine compute (as a mathematical function of $x$, where $x$ is the binary number on the tape)?

$$2x$$

**C.** (2 points) Suppose that your computer is not connected to any external (wireless or wired) source. What can this Turing machine do that your computer cannot do?

**Compute $2x$ for arbitrarily large $x$.**