This test has 1 question. You have 50 minutes. The exam is open book, open note, and open web. You may use code from your programming assignments or the Introduction to Programming in Java booksite. No communication with any non-staff members is permitted. Submit your solution via Dropbox. Write out and sign the Honor Code pledge before turning in the test.

“I pledge my honor that I have not violated the Honor Code during this examination.”

Name: 

NetID: 

Total

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<td>Kevin</td>
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</table>

Do not remove this exam from the room.
Problem. Write a program Exam1.java that computes the *t-statistic* to test whether two paired samples \((x_1, x_2, \ldots, x_N)\) and \((y_1, y_2, \ldots, y_N)\) have the same mean. The test was first proposed by William Sealy Gosset, who was a chemist working for the Guinness brewing company in Dublin, Ireland.

- The *sample means* \(\mu_1\) and \(\mu_2\) of the two samples are defined as:

\[
\mu_1 = \frac{x_1 + x_2 + \cdots + x_N}{N}, \quad \mu_2 = \frac{y_1 + y_2 + \cdots + y_N}{N}
\]

- The *sample variances* \(\sigma_1^2\) and \(\sigma_2^2\) of the two samples are defined as:

\[
\sigma_1^2 = \frac{(x_1 - \mu_1)^2 + (x_2 - \mu_1)^2 + \cdots (x_N - \mu_1)^2}{N - 1}
\]

\[
\sigma_2^2 = \frac{(y_1 - \mu_2)^2 + (y_2 - \mu_2)^2 + \cdots (y_N - \mu_2)^2}{N - 1}
\]

- The *t-statistic* of the paired samples is defined as:

\[
t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{N} + \frac{\sigma_2^2}{N}}}
\]

A t-statistic value far from zero indicates greater statistical significance that the samples have different means.

Example. For example, suppose that \(N = 4\) and the paired samples are:

<table>
<thead>
<tr>
<th>(i)</th>
<th>(x_i)</th>
<th>(y_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.0</td>
<td>14.0</td>
</tr>
<tr>
<td>2</td>
<td>12.0</td>
<td>14.0</td>
</tr>
<tr>
<td>3</td>
<td>9.0</td>
<td>14.0</td>
</tr>
<tr>
<td>4</td>
<td>9.0</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Then,

\[
\mu_1 = \frac{10 + 12 + 9 + 9}{4} = 10, \quad \mu_2 = \frac{14 + 14 + 14 + 18}{4} = 15
\]

\[
\sigma_1^2 = \frac{(10 - 10)^2 + (12 - 10)^2 + (9 - 10)^2 + (9 - 10)^2}{3} = 2
\]

\[
\sigma_2^2 = \frac{(14 - 15)^2 + (14 - 15)^2 + (14 - 15)^2 + (18 - 15)^2}{3} = 4
\]

\[
t = \frac{10.0 - 15.0}{\sqrt{2/4 + 4/4}} = -4.08248290463863
\]
**API specification.** Your program `Exam1.java` must be organized as a library of functions with the following API:

```java
class Exam1 {
    public double mean(double[] x) {
        // sample mean of array elements
    }
    public double variance(double[] x) {
        // sample variance of array elements
    }
    public double tstat(double[] x, double[] y) {
        // t-statistic of paired array elements
    }
    public void main(String[] args) {
        // read paired samples from stdin, write t-statistic to stdout
    }
}
```

You may assume that the array arguments to `mean()`, `variance()`, and `tstat()` contain at least two elements (and that the two array arguments to `tstat()` have the same size).

**Input and output specifications.** The input from standard input consists of an integer \( N \) followed by \( N \) pairs of real numbers (the paired samples). The output to standard output is a single real number—the t-statistic of the paired samples.

```plaintext
% more input4.txt
4
10.0 14.0
12.0 14.0
9.0 14.0
9.0 18.0

% javac Exam1.java
% java Exam1 < input4.txt
-4.08248290463863
```

For convenience, the file `input4.txt` is available at

```
http://www.cs.princeton.edu/introcs/input4.txt
```

**Submission.** Submit the single file `Exam1.java` via Dropbox at

```
https://dropbox.cs.princeton.edu/COS126_S2010/Exam1
```

Be sure to click the `Check All Submitted Files` button to verify your submission.

**Grading.** Your program will be graded on correctness and clarity (including comments). You will receive partial credit for a program that correctly implements some of the functions in the API. You will receive a substantial penalty if your program does not compile or if you do not follow the proscribed API or input/output specifications.