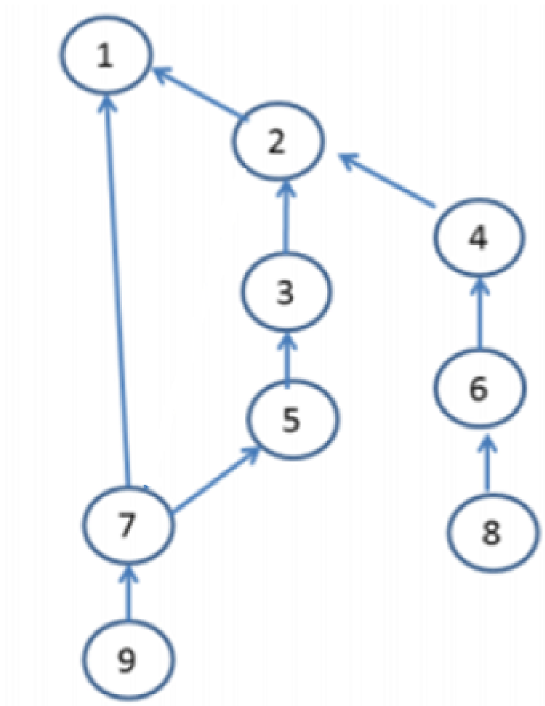
COS 226 Data Structures and Algorithms
Computer Science Department
Princeton University
Spring 2016

Week 8 Activity

1. Shortest Common Ancestor (SCA).

   (a) A common ancestral path between two vertices v and w in a directed graph is a directed path from v to a common ancestor x, together with a directed path from w to the same ancestor x. A shortest common ancestral path is a common ancestral path of minimum total length. We refer to the common ancestor in a shortest ancestral path as a shortest common ancestor (SCA). Given the digraph below, find the SCA of vertices 4 and 9 and minimum path length.

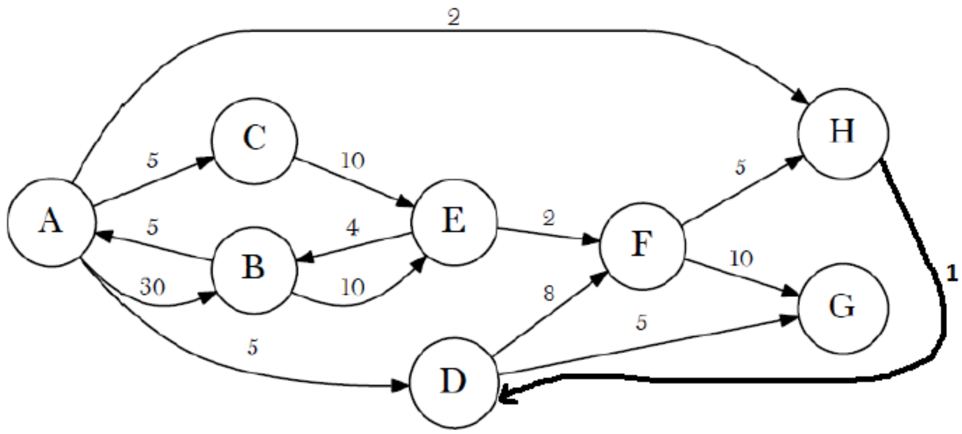


   (b) Assume that `sca` is an instance of the `ShortestCommonAncestor` class (as defined in the Wordnet assignment) Describe an algorithm for calculating `sca.ancestor(int v, int w)`. Your algorithm should run in linear time (proportional to $V + E$) and should work even if the digraph contains cycles.

   (c) (Homework) How would your algorithm differ if V and W are two sets of vertices (instead of a single vertex each)?

2. Shortest Paths.

(a) Consider the following weighted digraph below.



Dijkstra's algorithm - Complete the table below until the first 6 vertices are removed from the priority queue. Start from vertex A.

vertex	A	B	C	D	E	F	G	H
distTo								
edgeTo								

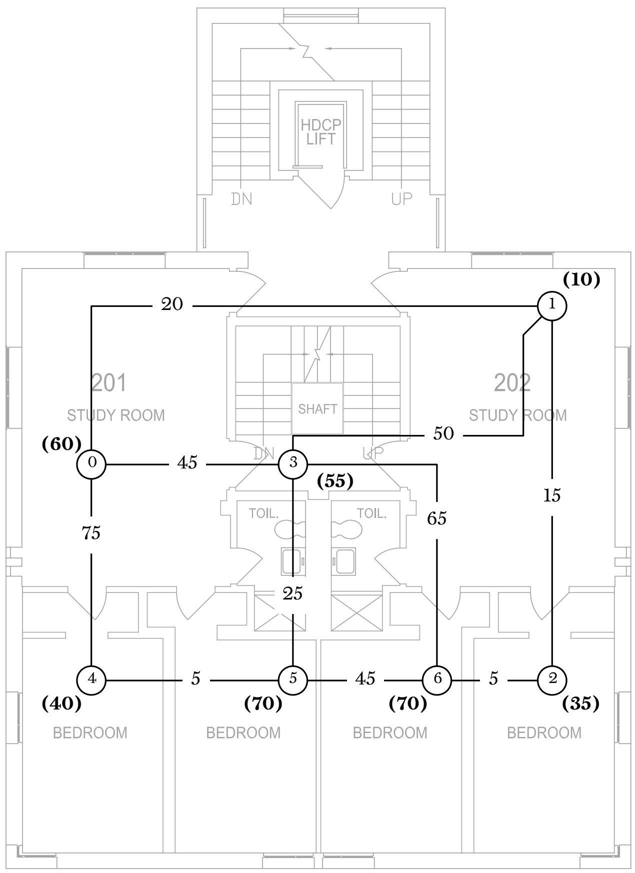
(b) This graph is different from the graph in part (a). Suppose you are using Dijkstras algorithm starting from some source vertex s. The table below shows the shortest paths tree (edgeTo[] and distTo[]) immediately after vertex 4 has been relaxed. Give the order in which the first 5 vertices were deleted from the priority queue and relaxed. Is it possible to know the next vertex that will be relaxed? and why?

v	distTo[]	edgeTo[]
0	∞	*null*
1	7.0	5
2	13.0	3
3	0.0	*null*
4	10.0	7
5	3.0	3
6	12.0	1
7	8.0	3

(c) Draw a graph with at least 3 nodes and one negative edge weight where Dijkstra's algorithm fails.

3. Algorithm Design - Dorm Room connectivity problem (bonus)

   Seven dorm rooms (numbered from 0 to 6) must be connected using a router or a hard wiring. The router cost for each room is shown in parentheses (for example, router cost for room 0 is 60). The wiring cost for connecting two rooms is shown as edge weights. (for example, wiring cost between rooms 0 and 3 is 45). Find the minimum cost to connect rooms to the internet using either a router or a hard wire between rooms.



   (a) Formulate the problem as a minimum spanning tree problem. To demonstrate your formulation, modify the figure above to show the MST problem that you would solve.

   (b) Using one of the MST algorithms, solve the problem and state the configuration (router/wire) that enable the optimal solution.