Flipped lecture – L01

- Why flip?
  - Why not?
  - Find the most efficient way to deliver content
  - Improve the outcomes
  - Focus more on conceptual and conventional
  - Quick poll
- Format of the flipped Lecture
  - Mini lecture – a quick overview of concepts (30 mins) - Directed Graphs, MST's
  - Group Worksheets (30 minutes)
    - Work with 2-3 students next to you
  - Discussion of Solutions (20 mins)

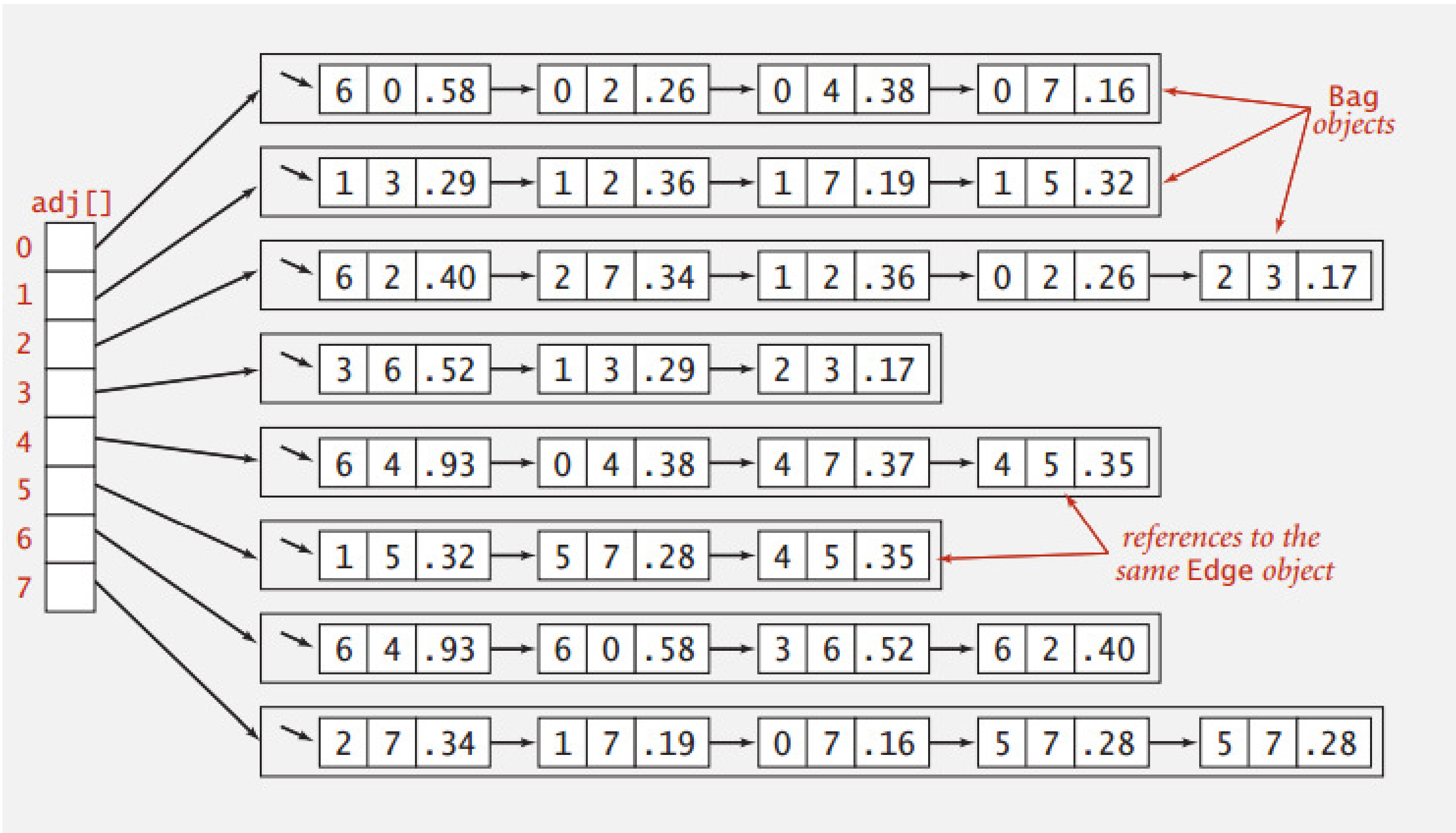
Time spent on videos

7.14.5 Directed Graphs - Strong Components	100%	Total Spend Time : 197 hours 57 minutes 3 seconds
7.14.4 Directed Graphs - Topological Sort	45.7%	Total Spend Time : 90 hours 33 minutes 10 seconds
7.14.1 Directed Graphs - Introduction	44%	Total Spend Time : 87 hours 3 minutes 58 seconds
7.14.3 Directed Graphs - digraph Search	40.4%	Total Spend Time : 79 hours 55 minutes 6 seconds
8.15.3 Minimum Spanning Trees - Edge Weighted Gr...	29.7%	Total Spend Time : 58 hours 50 minutes 53 seconds
8.15.5 Minimum Spanning Trees - Prim's Algorithm	27.1%	Total Spend Time : 53 hours 40 minutes 58 seconds
8.15.4 Minimum Spanning Trees - Kruskals Algorithm	24.2%	
8.15.2 Minimum Spanning Trees - Greedy Algorithms	18.3%	Total Spend Time : 36 hours 19 minutes 26 seconds
8.15.1 Minimum Spanning Trees - Introduction	18.3%	Total Spend Time : 36 hours 15 minutes 21 seconds

API

public class Digraph	
	Digraph(int V)
	Digraph(In in)
void	addEdge(int v, int w)
Iterable<Integer>	adj(int v)
int	V()
int	E()
Digraph	reverse()
String	toString()

Implementation



Implementation of a weighted digraph using Adjacency List represented by **Array of Bags** (flexible list length)

Order of Growth

representation	space	insert edge from v to w	edge from v to w?	iterate over vertices adjacent from v?
list of edges	E	1	E	E
adjacency matrix	V^2	$1^{\dagger}$	1	V
adjacency lists	$E + V$	1	$outdegree(v)$	$outdegree(v)$

Indegree and outdegree

- Indegree of a vertex v
  - Number of edges directed at the vertex v
  - Order of growth
    - adjacency list - E+V
    - adjacency matrix - V
- Outdegree of vertex v
  - Number of edges from the vertex v to other vertices
  - Order of growth
    - Adjacency list - outdegree(v)
    - Adjacency matrix - V

Directed graphs: quiz 1

Which is order of growth of running time to iterate over all vertices adjacent from v in a digraph using the adjacency-lists representation?

A. *indegree*(v)

B. *outdegree*(v)

C. *degree*(v)

D. V

E. *I don't know.*

Topological Order

Proposition. A digraph has a topological order iff no directed cycle.

Pf.

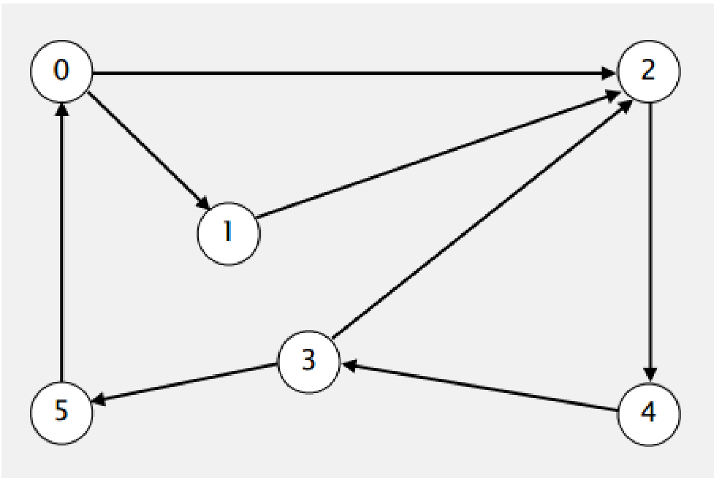
- If directed cycle, topological order impossible.
- If no directed cycle, DFS-based algorithm finds a topological order.

orderings

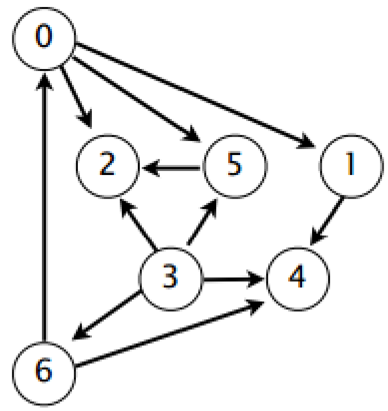
Orderings.

- Preorder: order in which `dfs()` is called.
- Postorder: order in which `dfs()` returns.
- Reverse postorder: reverse order in which `dfs()` returns.

Topological order?



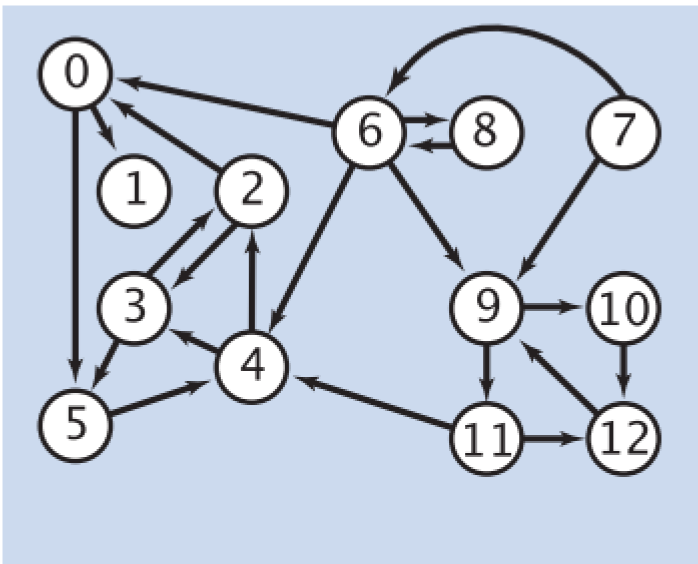
Graph 1



Graph 2

If there is a topological sort, does it matter which node we start with in DFS?

Code tracing




```
private void foo(Graph G, int s) {
      Queue<Integer> q = new Queue<Integer>();
      for (int v = 0; v < G.V(); v++)
          distTo[v] = INFINITY;
      distTo[s] = 0;
      marked[s] = true;
      q.enqueue(s);

      while (!q.isEmpty()) {
          int v = q.dequeue();
          for (int w : G.adj(v)) {
              if (!marked[w]) {
                  edgeTo[w] = v;
                  distTo[w] = distTo[v] + 1;
                  marked[w] = true;
                  q.enqueue(w);
              }
          }
      }
   }
```

replace by DiGraph

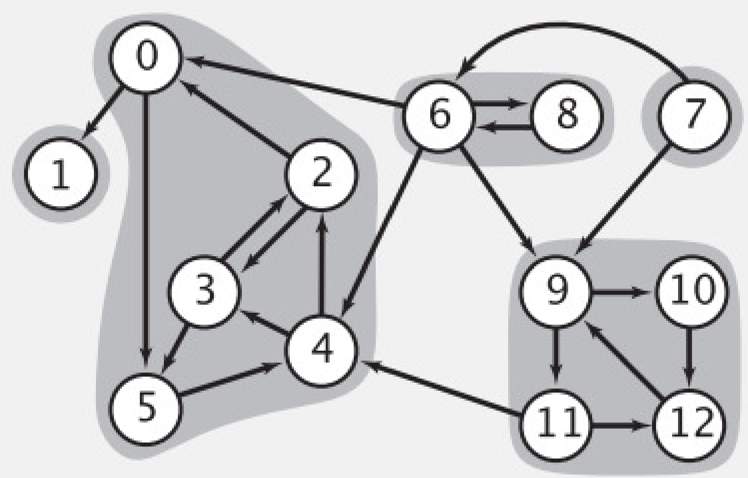
What does the code do if vertex 0 is marked as s?

Strong Components

Def. Vertices v and w are strongly connected if there is both a directed path from v to w and a directed path from w to v.

Def. A strong component is a maximal subset of strongly-connected vertices.

Key property. Strong connectivity is an equivalence relation:



5 strongly-connected components

How to find strong components?

How to Find SCC's

Kosaraju-Sharir algorithm: intuition

Reverse graph. Strong components in G are same as in G^R.

Kernel DAG. Contract each strong component into a single vertex.

Idea.

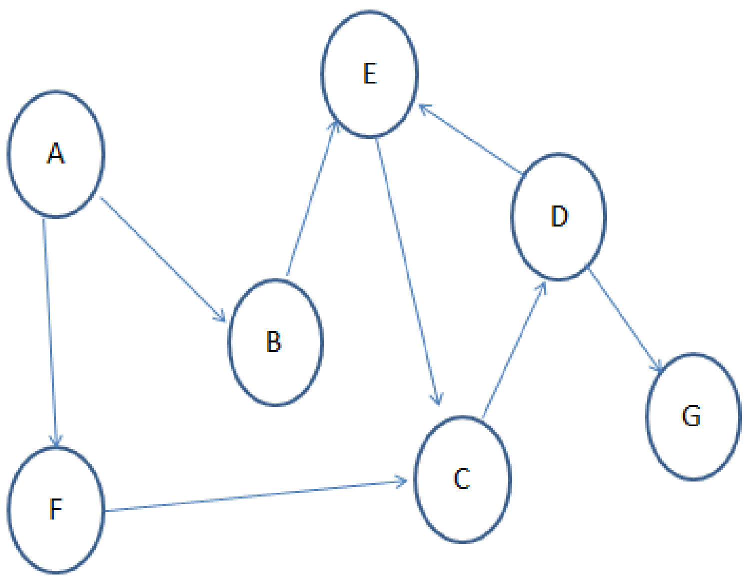
how to compute?

- Compute topological order (reverse postorder) in kernel DAG.
- Run DFS, considering vertices in reverse topological order.

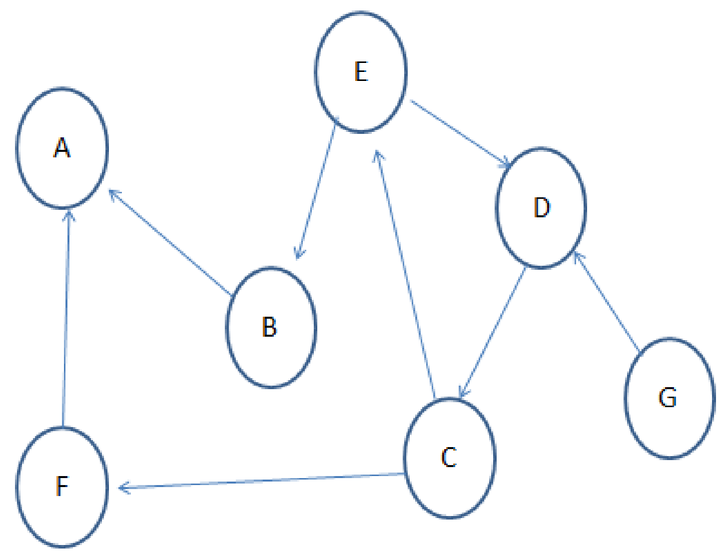
Proposition. Kosaraju-Sharir algorithm computes the strong components of a digraph in time proportional to $E + V$.

Computing SCC’s

- Step 1: Compute the reverse post order of G^R
- Step 2: Visit G in the order of reverse post order found in step 1



G



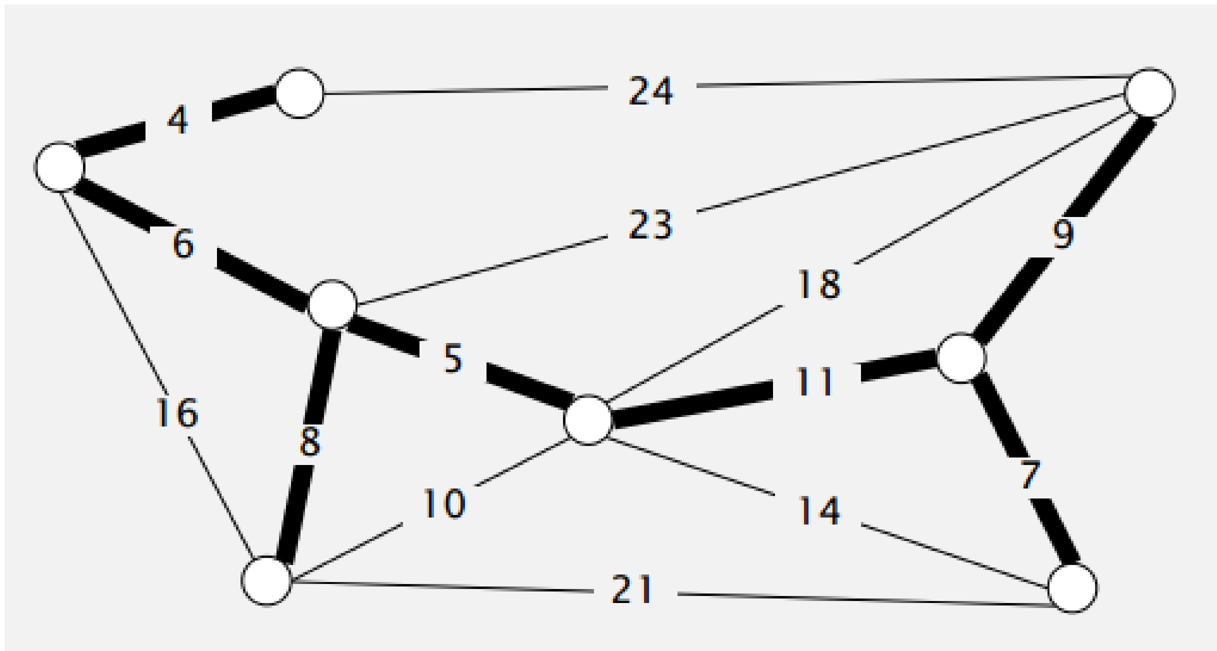
G^R

Minimum Spanning Tree (MST)

Facts and Questions

1. An undirected weighted graph
2. Connected
3. Find a subgraph that minimizes the total weight of edges
4. How many edges are in a MST?

Minimum Spanning Tree



Proposition: A connected graph with distinct edge weights has a unique MST

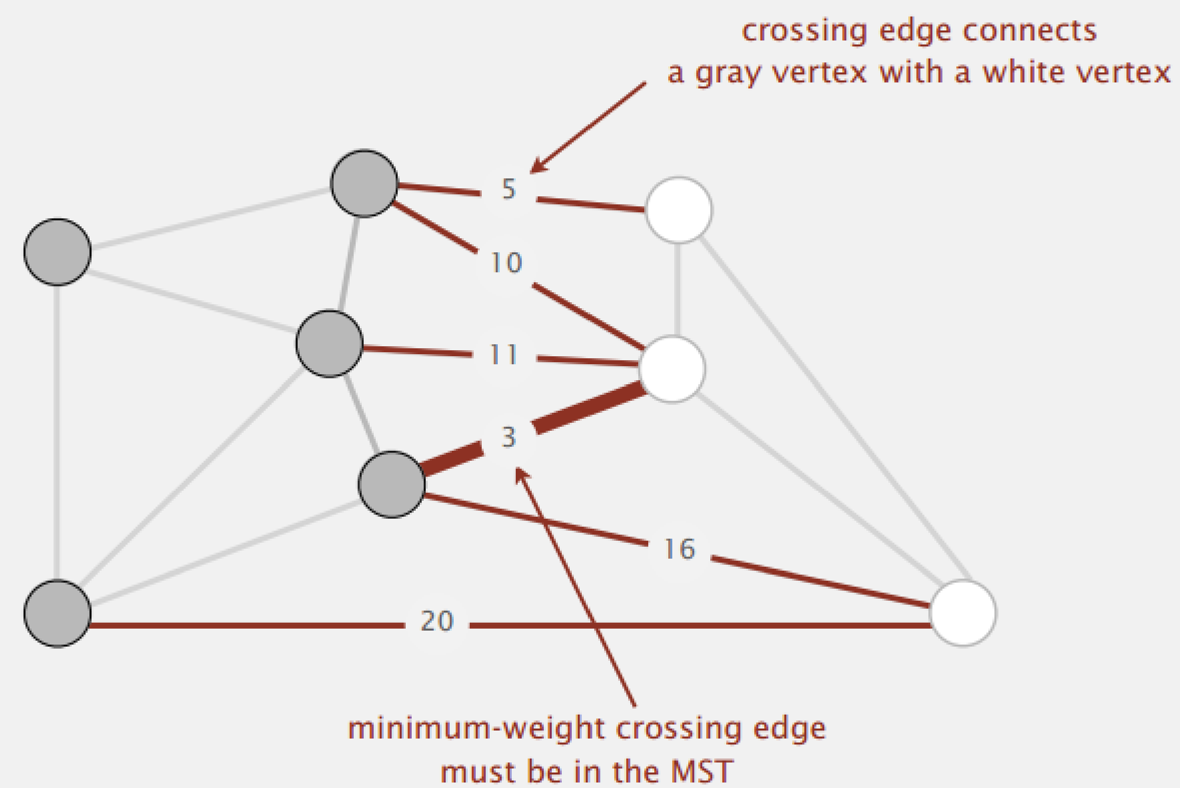
How to find it?

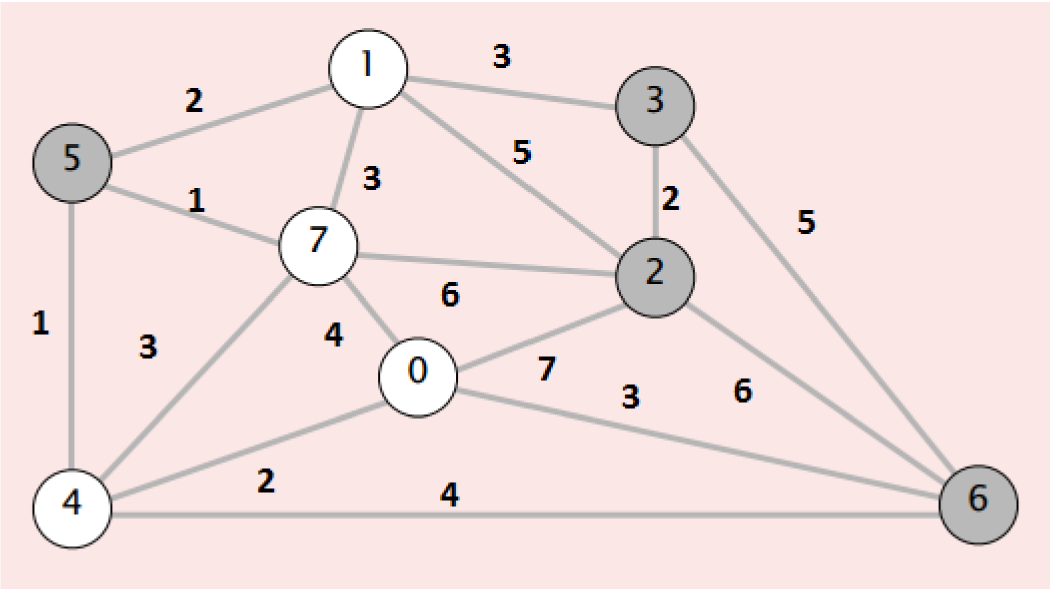
1. Sort all the edges - E log E **or** build a minPQ of edges
2. Find V-1 smallest edges **or** do delMin , V-1 times

Cut property

Def. A cut in a graph is a partition of its vertices into two (nonempty) sets.
Def. A crossing edge connects a vertex in one set with a vertex in the other.

Cut property. Given any cut, the crossing edge of min weight is in the MST.





What is the min weighted edge crossing the cut {2,3,5,6}?

Greedy MST algorithm

- Start with all edges colored gray.
- Find cut with no black crossing edges; color its min-weight edge black.
- Repeat until $V - 1$ edges are colored black.

API's

public class Edge implements Comparable<Edge>	
	Edge(int v, int w, double weight)
int	either()
int	other(int v)
int	compareTo(Edge that)
double	weight()
String	toString()

public class MST		
	MST(EdgeWeightedGraph G)	*constructor*
Iterable<Edge>	edges()	*edges in MST*
double	weight()	*weight of MST*

Kruskal's

```
private Queue<Edge> mst = new Queue<Edge>();

public KruskalMST(EdgeWeightedGraph G)
{
   MinPQ<Edge> pq = new MinPQ<Edge>(G.edges());

   UF uf = new UF(G.V());
   while (!pq.isEmpty() && mst.size() < G.V()-1)
   {
      Edge e = pq.delMin();
      int v = e.either(), w = e.other(v);
      if (!uf.connected(v, w))
      {
         uf.union(v, w);
         mst.enqueue(e);
      }
   }
```

operation	frequency	time per op
build pq	1	E
delete-min	E	$\log E$
union	V	$\log^* V$ †
connected	E	$\log^* V$ †

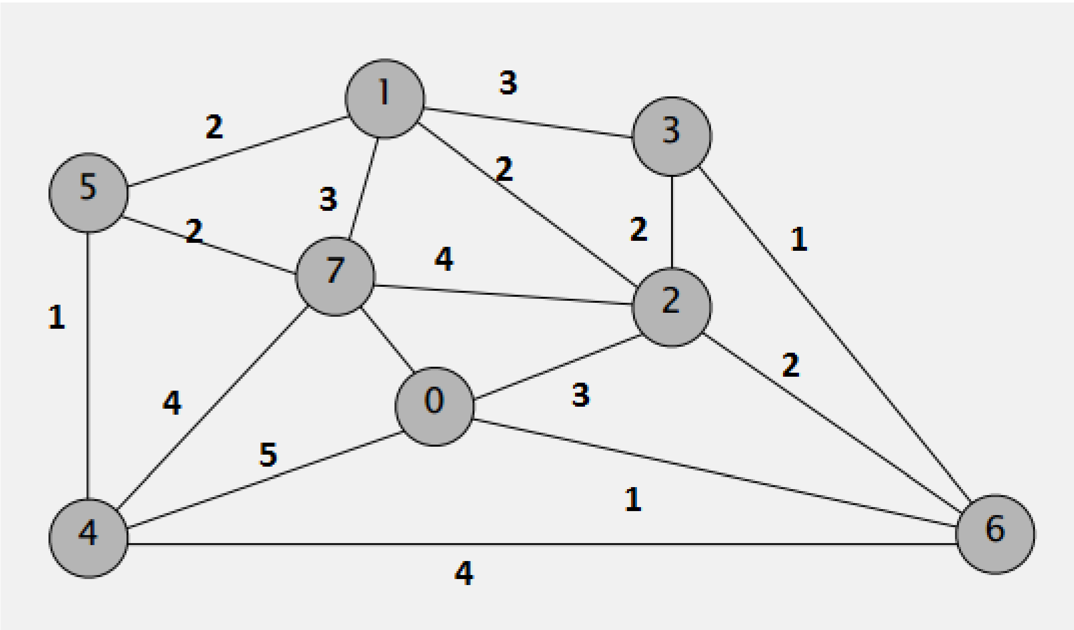
Kruskal code

```
public KruskalMST(EdgeWeightedGraph G)
{
   MinPQ<Edge> pq = new MinPQ<Edge>(G.edges());

   UF uf = new UF(G.V());
   while (!pq.isEmpty() && mst.size() < G.V()-1)
   {
      Edge e = pq.delMin();
      int v = e.either(), w = e.other(v);
      if (!uf.connected(v, w))
      {
         uf.union(v, w);
         mst.enqueue(e);
      }
   }
}
```

Prim's Algorithm (lazy)

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V - 1 edges.



Lazy Prim's

```
public LazyPrimMST(WeightedGraph G)
{
    pq = new MinPQ<Edge>();
    mst = new Queue<Edge>();
    marked = new boolean[G.V()];
    visit(G, 0);

    while (!pq.isEmpty() && mst.size() < G.V() - 1)
    {
       Edge e = pq.delMin();
       int v = e.either(), w = e.other(v);
       if (marked[v] && marked[w]) continue;
       mst.enqueue(e);
       if (!marked[v]) visit(G, v);
       if (!marked[w]) visit(G, w);
    }
```

```
private void visit(WeightedGraph G, int v)
{
   marked[v] = true;
   for (Edge e : G.adj(v))
      if (!marked[e.other(v)])
         pq.insert(e);
}
```

Proposition. Lazy Prim's algorithm computes the MST in time proportional to $E \log E$ and extra space proportional to E (in the worst case).

Pf.

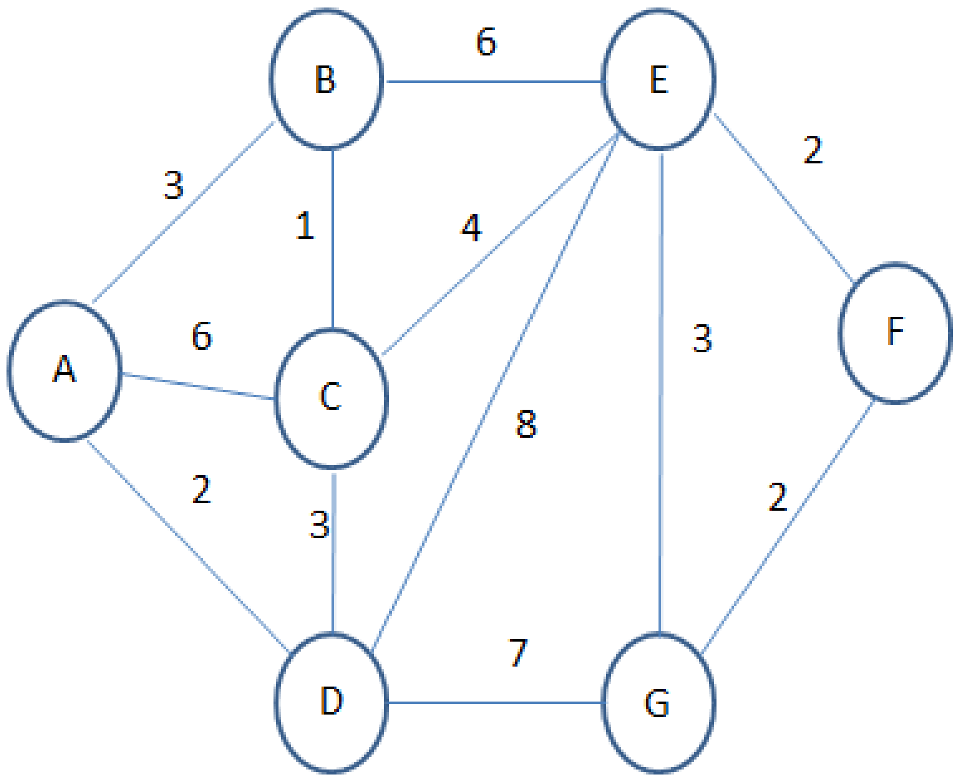
operation	frequency	binary heap
delete min	E	$\log E$
insert	E	$\log E$

Prim's (eager implementation)

Eager solution. Maintain a PQ of vertices connected by an edge to T, where priority of vertex v = weight of shortest edge connecting v to T.

- Delete min vertex v and add its associated edge $e = v–w$ to T.
- Update PQ by considering all edges $e = v–x$ incident to v
  - ignore if x is already in T
  - add x to PQ if not already on it
  - decrease priority of x if $v–x$ becomes shortest edge connecting x to T

Prims Eager Trace



V	Edges	weight
A		
B		
C		
D		
E		
F		
G		

Implementing a PQ with decreaseKey

```
public class IndexMinPQ<Key extends Comparable<Key>>
```

	`IndexMinPQ(int N)`	*create indexed priority queue with indices 0, 1, ..., N – 1*
`void`	`insert(int i, Key key)`	*associate key with index i*
`void`	`decreaseKey(int i, Key key)`	*decrease the key associated with index i*
`boolean`	`contains(int i)`	*is i an index on the priority queue?*
`int`	`delMin()`	*remove a minimal key and return its associated index*
`boolean`	`isEmpty()`	*is the priority queue empty?*
`int`	`size()`	*number of keys in the priority queue*

The idea of decrease key

- Maintain parallel arrays `keys[]`, `pq[]`, and `qp[]` so that:
  - `keys[i]` is the priority of `i`
  - `pq[i]` is the index of the key in heap position `i`
  - `qp[i]` is the heap position of the key with index `i`
- Use `swim(qp[i])` to implement `decreaseKey(i, key)`.

i	0	1	2	3	4	5	6	7	8
keys[i]	A	S	O	R	T	I	N	G	-
pq[i]	-	0	6	7	2	1	5	4	3
qp[i]	1	5	4	8	7	6	2	3	-