Implicit Surfaces & Solid Representations

COS 426, Spring 2015
Princeton University
3D Object Representations

- Raw data
  - Range image
  - Point cloud

- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit

- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep

- High-level structures
  - Scene graph
  - Application specific
3D Object Representations

- Desirable properties of an object representation
  - Easy to acquire
  - Accurate
  - Concise
  - Intuitive editing
  - Efficient editing
  - Efficient display
  - Efficient intersections
  - Guaranteed validity
  - Guaranteed smoothness
  - etc.

Large Geometric Model Repository
Georgia Tech
3D Object Representations

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Large Geometric Model Repository
Georgia Tech
Implicit Surfaces

- Represent surface with function over all space
Implicit Surfaces

- Surface defined implicitly by function
Implicit Surfaces

- Surface defined implicitly by function:
  - $f(x, y, z) = 0$ (on surface)
  - $f(x, y, z) < 0$ (inside)
  - $f(x, y, z) > 0$ (outside)
Implicit Surfaces

- Normals defined by partial derivatives
  - \( \text{normal}(x, y, z) = \text{normalize}(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}) \)
Implicit Surface Properties

(1) Efficient check for whether point is inside
   ◦ Evaluate $f(x,y,z)$ to see if point is inside/outside/on
   ◦ Example: ellipsoid

$$f(x, y, z) = \left( \frac{x}{r_x} \right)^2 + \left( \frac{y}{r_y} \right)^2 + \left( \frac{z}{r_z} \right)^2 - 1$$
(2) Efficient surface intersections

- Substitute to find intersections

Ray: \( P = P_0 + tV \)
Sphere: \( |P - O|^2 - r^2 = 0 \)

Substituting for \( P \), we get:
\[ |P_0 + tV - O|^2 - r^2 = 0 \]

Solve quadratic equation:
\[ at^2 + bt + c = 0 \]
where:
\[ a = 1 \]
\[ b = 2 \ V \cdot (P_0 - O) \]
\[ c = |P_0 - O|^2 - r^2 = 0 \]
Implicit Surface Properties

(3) Efficient boolean operations (CSG)

- How would you implement: Union? Intersection? Difference?
Implicit Surface Properties

(4) Efficient topology changes

- Surface is not represented explicitly!
Implicit Surface Properties

(4) Efficient topology changes
   - Surface is not represented explicitly!

Bloomenthal
Comparison to Parametric Surfaces

- Implicit
  - Efficient intersections & topology changes

- Parametric
  - Efficient “marching” along surface & rendering
Implicit Surface Representations

• How do we define implicit function?
  ○ $f(x,y,z) = ?$
Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - Voxels
  - Basis functions
  - Others
Implicit Surface Representations

• How do we define implicit function?
  ➢ Algebraics
    ◦ Voxels
    ◦ Basis functions
    ◦ Others
Algebraic Surfaces

- Implicit function is polynomial
  \[ f(x,y,z) = ax^d + by^d + cz^d + dx^{d-1}y + dx^{d-1}z + dy^{d-1}x + \ldots \]

\[
f(x, y, z) = \left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 - 1
\]
Algebraic Surfaces

- Most common form: quadrics
  - \( f(x,y,z) = ax^2 + by^2 + cz^2 + 2dxy + 2eyz + 2fxz + 2gx + 2hy + 2jz + k \)

- Examples
  - Sphere
  - Ellipsoid
  - Paraboloid
  - Hyperboloid

Menon
Algebraic Surfaces

- Higher degree algebraics

Cubic

Quartic

Degree six
Algebraic Surfaces

- Equivalent parametric surface
  - Tensor product patch of degree m and n curves yields algebraic function with degree 2mn

Bicubic patch has degree 18!
Algebraic Surfaces

- Intersection
  - Intersection of degree $m$ and $n$ algebraic surfaces yields curve with degree $mn$

Intersection of bicubic patches has degree 324!
Algebraic Surfaces

• Function extends to infinity
  ◦ Must trim to get desired patch (this is difficult!)
Implicit Surface Representations

• How do we define implicit function?
  ◦ Algebraics
  ☢ Voxels
  ◦ Basis functions
Voxels

- Regular array of 3D samples (like image)
  - Samples are called **voxels** ("**volume pixels**")
Voxels

• Example isosurfaces

SUNY Stoney Brook

Princeton University
Voxels

- Regular array of 3D samples (like image)
  - Applying reconstruction filter (e.g. trilinear) yields $f(x,y,z)$
  - Isosurface at $f(x,y,z) = 0$ defines surface
Voxels

- Iso-surface extraction algorithm
  - e.g., Marching cubes
Voxels

- Iso-surface extraction algorithm
  - e.g., Marching cubes (15 cases)
Voxel Storage

- $O(n^3)$ storage for $n \times n \times n$ grid
  - 1 billion voxels for 1000 x 1000 x 1000
Implicit Surface Representations

• How do we define implicit function?
  ◦ Algebraics
  ◦ Voxels
  ➢ Basis functions
Basis functions

- Implicit function is sum of basis functions
  - Example:

    $$f(P) = a_0 e^{-b_0 d(P, P_0)^2} + a_1 e^{-b_1 d(P, P_1)^2} + \cdots - \tau$$

![Diagram showing basis functions and points P0, P1, and P]
Radial Basis Functions

- **Blobby models**
  \[ D(r) = ae^{-br^2} \]

- **Meta balls**
  \[ D(r) = \begin{cases} 
  a(1 - \frac{3r^2}{b^2}) & 0 \leq r \leq b/3 \\
  \frac{3a}{2} (1 - \frac{r}{b})^2 & b/3 \leq r \leq b \\
  0 & b \leq r 
\end{cases} \]

- **Soft objects**
  \[ D(r) = \begin{cases} 
  a(1 - \frac{4r^6}{9b^6}) + \frac{17r^4}{9b^4} - \frac{22r^2}{9b^2} & r \leq b \\
  0 & r \geq b 
\end{cases} \]
Blobby Models

- Implicit function is sum of Gaussians

\[ f(P) = a_0 e^{-b_0 d(P, P_0)^2} + a_1 e^{-b_1 d(P, P_1)^2} + \cdots - \tau \]
Blobby Models

• Sum of two blobs
Blobby Models

- Sum of four blobs
Blobby Model of Face

(a) $N = 1$

(b) $N = 2$
Blobby Model of Face

(c) $N = 10$

(d) $N = 35$
Blobby Model of Face

(e) $N = 70$

(f) $N = 243$
Blobby Model of Head

(a) $N = 1$

(b) $N = 2$
Blobby Model of Head

(c) $N = 20$

(d) $N = 60$
Blobby Model of Head

(e) $N = 120$

(f) $N = 451$
Blobby Models

Objects resulting from CSG of implicit soft objects and other primitives

Menon
Variational Implicit Surfaces
Variational Implicit Surfaces
Implicit Surface Summary

- **Advantages:**
  - Easy to test if point is on surface
  - Easy to compute intersections/unions/differences
  - Easy to handle topological changes

- **Disadvantages:**
  - Indirect specification of surface
  - Hard to describe sharp features
  - Hard to enumerate points on surface
    - Slow rendering
## Summary

<table>
<thead>
<tr>
<th>Feature</th>
<th>Polygonal Mesh</th>
<th>Implicit Surface</th>
<th>Parametric Surface</th>
<th>Subdivision Surface</th>
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<tbody>
<tr>
<td>Accurate</td>
<td>No</td>
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<td>Intuitive specification</td>
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</table>
3D Object Representations

- **Raw data**
  - Range image
  - Point cloud

- **Surfaces**
  - Polygonal mesh
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  - Parametric
  - Implicit

- **Solids**
  - Voxels
  - BSP tree
  - CSG
  - Sweep

- **High-level structures**
  - Scene graph
  - Application specific
Solid Modeling

- Represent solid interiors of objects

www.volumegraphics.com
Motivation 1

- Some acquisition methods generate solids

Airflow Inside a Thunderstorm

(Bob Wilhelmson, University of Illinois at Urbana-Champaign)

Visible Human

(National Library of Medicine)
Motivation 2

- Some applications require solids
  - Examples: medicine, CAD/CAM
Motivation 3

• Some operations are easier with solids
  ◦ Example: union, difference, intersection

![Union](image1)
![Difference](image2)
3D Object Representations

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Return to Voxels

- Regular array of 3D samples (like image)

www.volumegraphics.com
Voxels

- Store properties of solid object with each voxel
  - Occupancy
  - Color
  - Density
  - Temperature
  - etc.

Engine Block
Stanford University

Visible Human
(National Library of Medicine)
Voxel Processing

• Signal processing (just like images)
  ◦ Reconstruction
  ◦ Resampling

• Typical operations
  ◦ Blur
  ◦ Edge detect
  ◦ Warp
  ◦ etc.

• Often fully analogous to image processing
Voxel Boolean Operations

• Compare objects voxel by voxel
  - Trivial

\[ \begin{align*}
\text{Object 1} & \quad \cup \quad \text{Object 2} & = & \quad \text{Result 1} \\
\text{Object 3} & \quad \cap \quad \text{Object 4} & = & \quad \text{Result 2}
\end{align*} \]
Voxel Display

- **Isosurface rendering**
  - Interpolate samples stored on regular grid
  - Isosurface at \( f(x,y,z) = 0 \) defines surface

\[
\begin{align*}
2.3 & \quad 1.7 & \quad 0.9 & \quad 0.2 \\
1.2 & \quad 0.4 & \quad 0.1 & \quad -0.8 \\
0.3 & \quad -0.5 & \quad -0.7 & \quad -1.4 \\
0.2 & \quad -0.9 & \quad -1.7 & \quad -2.5
\end{align*}
\]
Voxel Display

- Slicing
  - Draw 2D image resulting from intersecting voxels with a plane

Visible Human
(National Library of Medicine)
Voxel Display

• Ray casting
  ◦ Integrate density along rays: compositing!
Voxel Display

- Extended ray-casting
  - *Transfer functions*: Map voxel values to opacity and material
  - Normals (for lighting) from density gradient

Bruckner et al. 2007
Voxels

• Advantages
  ◦ Simple, intuitive, unambiguous
  ◦ Same complexity for all objects
  ◦ Natural acquisition for some applications
  ◦ Trivial boolean operations

• Disadvantages
  ◦ Approximate
  ◦ Not affine invariant
  ◦ Expensive display
  ◦ Large storage requirements
Voxels

- What resolution should be used?
Quadtrees & Octrees

- Refine resolution of voxels hierarchically
  - More concise and efficient for non-uniform objects

Uniform Voxels  Quadtree (Octree in 3D)

FvDFH Figure 12.21
Quadtree Processing

• Hierarchical versions of voxel methods
  ◦ Finding neighbor cell requires traversal of hierarchy:
    expected/amortized $O(1)$
Quadtree Boolean Operations

\[ A \cup B \]

\[ A \cap B \]

FvDFH Figure 12.24
3D Object Representations

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• Solids
  ○ Voxels
  ➢ BSP tree
  ○ CSG
  ○ Sweep

• High-level structures
  ○ Scene graph
  ○ Application specific
BSP Trees

Object

Binary Spatial Partition

Binary Tree

Naylor
BSP Trees

• Key properties
  ◦ visibility ordering (later)
  ◦ hierarchy of convex regions (useful for collision)
3D Object Representations

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  - Scene graph
  - Application specific
Constructive Solid Geometry (CSG)

- Represent solid object as hierarchy of boolean operations
  - Union
  - Intersection
  - Difference

FvDFH Figure 12.27
CSG Acquisition

- Interactive modeling programs
  - Intuitive way to design objects
CSG Acquisition

- Interactive modeling programs
  - Intuitive way to design objects
CSG Boolean Operations

• Create a new CSG node joining subtrees
  ○ Union
  ○ Intersection
  ○ Difference
CSG Display & Analysis

- Ray casting
3D Object Representations

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Sweeps

- Swept volume
  - Sweep one curve along path of another curve
Sweeps

• Surface of revolution
  ◦ Take a curve and rotate it about an axis
Sweeps

- Surface of revolution
  - Take a curve and rotate it about an axis
## Summary

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<tr>
<th>Feature</th>
<th>Voxels</th>
<th>Octree</th>
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<td>Efficient boolean operations</td>
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