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5.5 Data Compression

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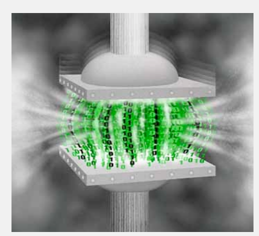
5.5 Data Compression

Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.



Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, …

> " *Everyday, we create 2.5 quintillion bytes of data—so much that 90% of the data in the world today has been created in the last two years alone.* " — *IBM report on big data (2011)*

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: GZIP, BZIP, 7z.
- Archivers: PKZIP.
- File systems: NTFS, ZFS, HFS+, ReFS, GFS.

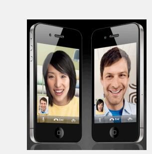
Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.



Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.
- Skype, Google hangout.

Databases. Google, Facebook, NSA, ...

Lossless compression and expansion

Message. Bitstream B we want to compress.

Compress. Generates a "compressed" representation $C(B)$.

Expand. Reconstructs original bitstream B.

uses fewer bits (you hope)

Basic model for data compression

Compression ratio. Bits in $C(B)$ / bits in B.

Ex. 50–75% or better compression ratio for natural language.

Food for thought

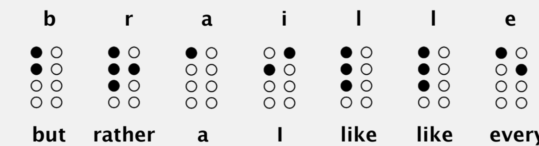
Data compression has been omnipresent since antiquity:

- Number systems.
- Natural languages.
- Mathematical notation.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

has played a central role in communications technology,

- Grade 2 Braille.
- Morse code.
- Telephone system.



and is part of modern life.

- JPEG.
- MP3.
- MPEG.

Q. What role will it play in the future?

Data representation: genomic code

Genome. String over the alphabet { A, T, C, G }.

Goal. Encode an N-character genome: A T A G A T G C A T A G...

Standard ASCII encoding.

- 8 bits per char.
- $8N$ bits.

char	hex	binary
'A'	41	01000001
'T'	54	01010100
'C'	43	01000011
'G'	47	01000111

Two-bit encoding.

- 2 bits per char.
- $2N$ bits (25% compression ratio).

char	binary
'A'	00
'T'	01
'C'	10
'G'	11

Fixed-length code. k-bit code supports alphabet of size 2^k.

Amazing but true. Some genomic databases in 1990s used ASCII.

Reading and writing binary data

Binary standard input. Read bits from standard input.

public class BinaryStdIn

boolean	readBoolean()	*read 1 bit of data and return as a boolean value*
char	readChar()	*read 8 bits of data and return as a char value*
char	readChar(int r)	*read r bits of data and return as a char value*
[similar methods for byte (8 bits); short (16 bits); int (32 bits); long and double (64 bits)]		
boolean	isEmpty()	*is the bitstream empty?*
void	close()	*close the bitstream*

Binary standard output. Write bits to standard output

public class BinaryStdOut

void	write(boolean b)	*write the specified bit*
void	write(char c)	*write the specified 8-bit char*
void	write(char c, int r)	*write the r least significant bits of the specified char*
[similar methods for byte (8 bits); short (16 bits); int (32 bits); long and double (64 bits)]		
void	close()	*close the bitstream*

Writing binary data

Date representation. Three different ways to represent 12/31/1999.

A character stream (StdOut)

```
StdOut.print(month + "/" + day + "/" + year);
```

```
00110001|00110010|00101111|00110111|00110001|00101111|00110001|00111001|00111001|00111001
   1        2        /        3        1        /        1        9        9        9
```

80 bits

Three ints (BinaryStdOut)

```
BinaryStdOut.write(month);
BinaryStdOut.write(day);
BinaryStdOut.write(year);
```

```
00000000000000000000000000001100|00000000000000000000000000011111|00000000000000000000011111001111
              12                                31                                1999
```

96 bits

A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)

```
BinaryStdOut.write(month, 4);
BinaryStdOut.write(day, 5);
BinaryStdOut.write(year, 12);
```

```
110011111011111001111000
12  31   1999
```

21 bits (+ 3 bits for byte alignment at close)

Binary dumps

Q. How to examine the contents of a bitstream?

Standard character stream

```
% more abra.txt
ABRACADABRA!
```

Bitstream represented as 0 and 1 characters

```
% java BinaryDump 16 < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

Bitstream represented with hex digits

```
%  java HexDump 4 < abra.txt
41 42 52 41
43 41 44 41
42 52 41 21
12 bytes
```

Bitstream represented as pixels in a Picture

```
% java PictureDump 16 6 < abra.txt
```

16-by-6 pixel window, magnified

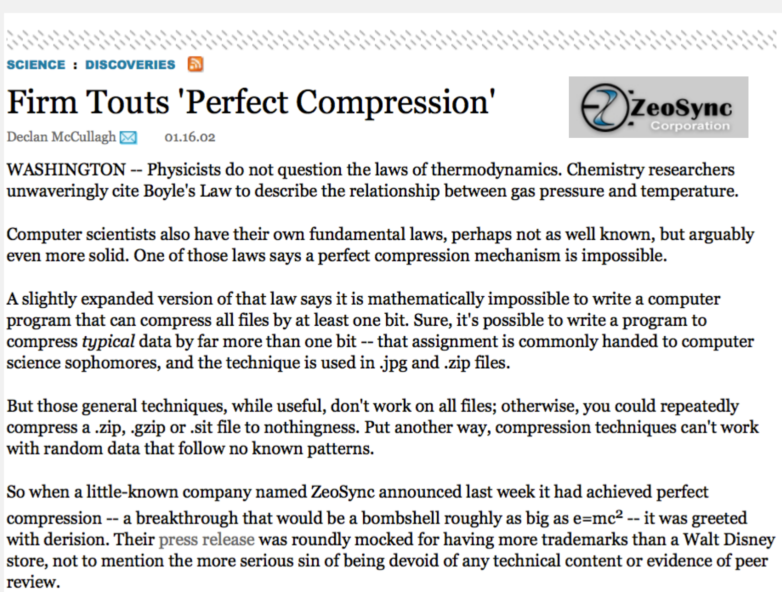
```
96 bits
```

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	“	#	$	%	&	‘	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

Hexadecimal to ASCII conversion table

Universal data compression

ZeoSync. Announced 100:1 lossless compression of random data using Zero Space Tuner™ and BinaryAccelerator™ technology.



SCIENCE : DISCOVERIES

Firm Touts 'Perfect Compression'

Declan McCullagh 01.16.02

WASHINGTON -- Physicists do not question the laws of thermodynamics. Chemistry researchers unwaveringly cite Boyle's Law to describe the relationship between gas pressure and temperature.

Computer scientists also have their own fundamental laws, perhaps not as well known, but arguably even more solid. One of those laws says a perfect compression mechanism is impossible.

A slightly expanded version of that law says it is mathematically impossible to write a computer program that can compress all files by at least one bit. Sure, it's possible to write a program to compress *typical* data by far more than one bit -- that assignment is commonly handed to computer science sophomores, and the technique is used in .jpg and .zip files.

But those general techniques, while useful, don't work on all files; otherwise, you could repeatedly compress a .zip, .gzip or .sit file to nothingness. Put another way, compression techniques can't work with random data that follow no known patterns.

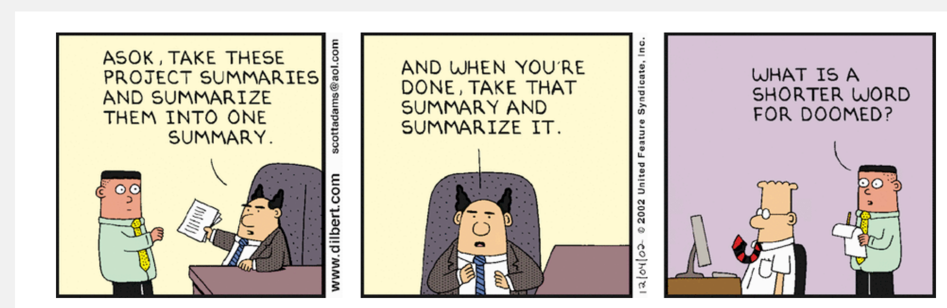
So when a little-known company named ZeoSync announced last week it had achieved perfect compression -- a breakthrough that would be a bombshell roughly as big as $e=mc^2$ -- it was greeted with derision. Their press release was roundly mocked for having more trademarks than a Walt Disney store, not to mention the more serious sin of being devoid of any technical content or evidence of peer review.

ZeoSync corporation folds after issuing $40 million in private stock

Universal data compression

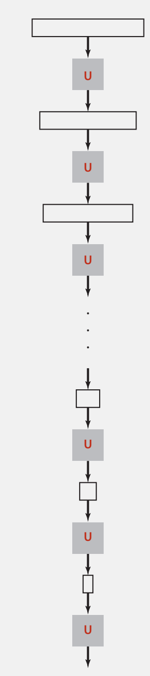
Proposition. No algorithm can compress every bitstring.

Pf 1. [by contradiction]



Pf 2. [by counting]

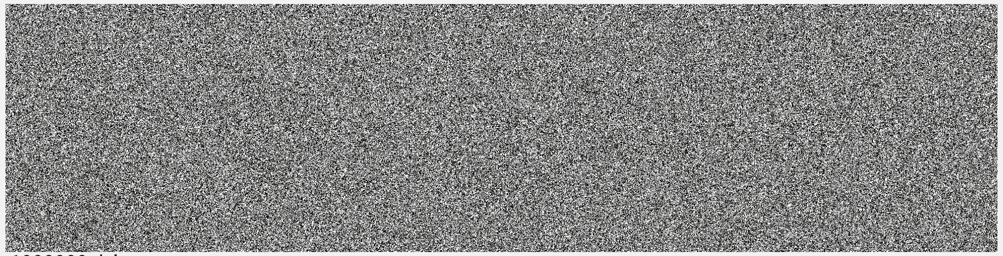
- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- 2^{1000} possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999}$ can be encoded with ≤ 999 bits.
- Similarly, only 1 in 2^{499} bitstrings can be encoded with ≤ 500 bits!



Universal data compression?

Undecidability

```
% java RandomBits | java PictureDump 2000 500
```



1000000 bits

A difficult file to compress: one million (pseudo-) random bits

```
public class RandomBits
{
   public static void main(String[] args)
   {
      int x = 11111;
      for (int i = 0; i < 1000000; i++)
      {
         x = x * 314159 + 218281;
         BinaryStdOut.write(x > 0);
      }
      BinaryStdOut.close();
   }
}
```


Rdenudcany in Enlgsih lnagugae

Q. How much redundancy in the English language?

A. Quite a bit.

> *" ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang. "* — Graham Rawlinson

The gaol of data cmperisoson is to inetdify rdenudcany and epxloit it.

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5.5 Data Compression

- introduction
- *run-length coding*
- Huffman compression
- LZW compression

Run-length encoding

Simple type of redundancy in a bitstream. Long runs of repeated bits.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 ← 40 bits

Representation. 4-bit counts to represent alternating runs of 0s and 1s: 15 0s, then 7 1s, then 7 0s, then 11 1s.

1 1 1 1 0 1 1 1 0 1 1 1 1 0 1 1 ← 16 bits (instead of 40)

(15, 7, 7, 11)

Q. How many bits to store the counts?

A. We typically use 8 (but 4 in the example above for brevity).

Q. What to do when run length exceeds max count?

A. Intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

Run-length encoding: Java implementation

```
public class RunLength
{
   private final static int R   = 256;     // maximum run-length count
   private final static int lgR = 8;       // number of bits per count

   public static void compress()
   {  /* see textbook */  }

   public static void expand()
   {
      boolean bit = false;
      while (!BinaryStdIn.isEmpty())
      {
         int run = BinaryStdIn.readInt(lgR);    // read 8-bit count from standard input
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(bit);            // write 1 bit to standard output
         bit = !bit;
      }
      BinaryStdOut.close();                     // pad 0s for byte alignment
   }

}
```


Data compression: quiz 1

What is the best compression ratio achievable from run-length coding when using 8-bit counts?

A. 1 / 256

B. 1 / 16

C. 8 / 255

D. 24 / 510 = 4 / 85

E. *I don't know.*

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5.5 Data Compression

David Huffman

Variable-length codes

Use different number of bits to encode different chars.

Ex. Morse code: • • • – – – • • •

Issue. Ambiguity.

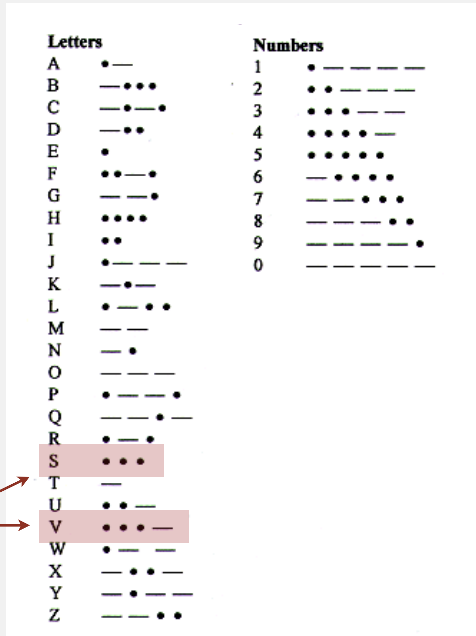
S O S ?

V 7 ?

I A M I E ?

E E W N I ?

In practice. Use a medium gap to separate codewords.



codeword for S is a prefix of codeword for V

Variable-length codes

Q. How do we avoid ambiguity?

A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special stop characater to each codeword.

Ex 3. General prefix-free code.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Compressed bitstring

011111110011001000111111100101 ← *30 bits*
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Compressed bitstring

11000111101011100110001111101 ← *29 bits*
A B R A C A D A B R A !

Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

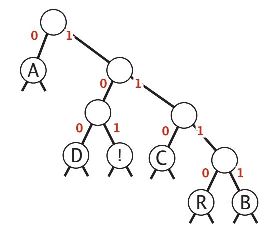
A. A binary trie!

- Characters in leaves.
- Codeword is path from root to leaf.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Trie representation



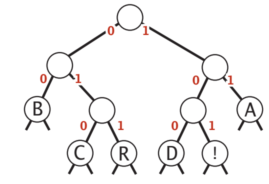
Compressed bitstring

011111110011001000111111100101 ← *30 bits*
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Trie representation



Compressed bitstring

11000111101011100110001111101 ← *29 bits*
A B R A C A D A B R A !

Prefix-free codes: expansion

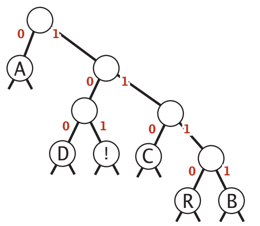
Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, write character; return to root node; repeat.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Trie representation



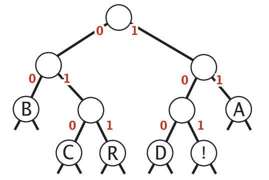
Compressed bitstring

011111110011001000111111100101 ← *30 bits*
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Trie representation



Compressed bitstring

11000111101011100110001111101 ← *29 bits*
A B R A C A D A B R A !

Prefix-free codes: compression

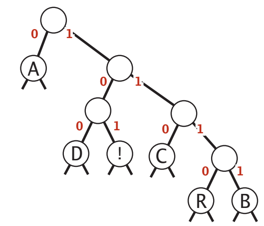
Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.
- Method 2: create ST of key-value pairs.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Trie representation



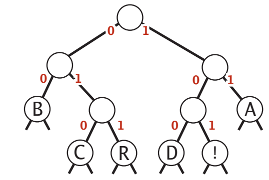
Compressed bitstring

011111110011001000111111100101 ← *30 bits*
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Trie representation



Compressed bitstring

11000111101011100110001111101 ← *29 bits*
A B R A C A D A B R A !

Data compression: quiz 2

Consider the following trie representation of a prefix-free code.
Expand the compressed bitstring 100101000111011 ?

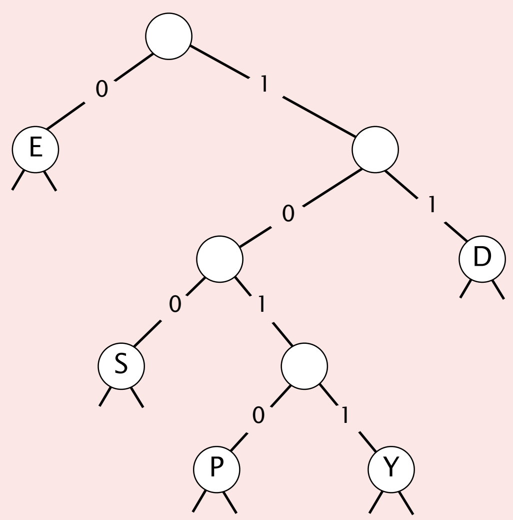
A. PEED

B. PESDEY

C. SPED

D. SPEEDY

E. *I don't know.*



Huffman coding overview

Static model. Use the same prefix-free code for all messages.
Dynamic model. Use a custom prefix-free code for each message.

Compression.

- Read message.
- Build best prefix-free code for message. How? [ahead]
- Write prefix-free code (as a trie).
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code (as a trie) from file.
- Read compressed message and expand using trie.

Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
   private final char ch;   // used only for leaf nodes
   private final int freq;  // used only by compress()
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }
}
```

initializing constructor

is Node a leaf?

compare Nodes by frequency (stay tuned)

Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();
   int N = BinaryStdIn.readInt();

   for (int i = 0; i < N; i++)
   {
      Node x = root;
      while (!x.isLeaf())
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch, 8);
   }
   BinaryStdOut.close();
}
```

read in encoding trie

read in number of chars

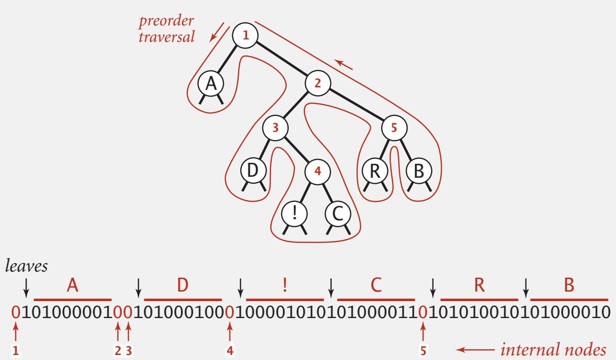
expand codeword for i^{th} char

Running time. Linear in input size N.

Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal of trie; mark leaf and internal nodes with a bit.



Using preorder traversal to encode a trie as a bitstream

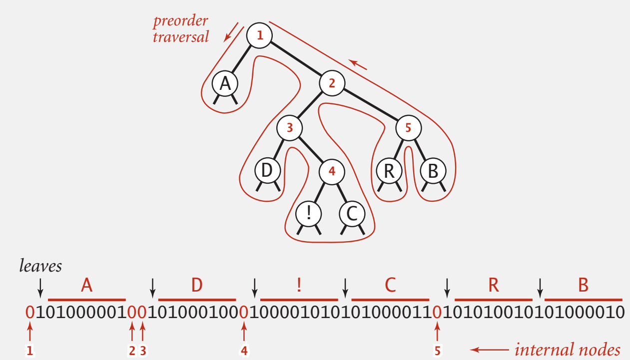
```
private static void writeTrie(Node x)
{
   if (x.isLeaf())
   {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch, 8);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

Prefix-free codes: how to transmit

Q. How to read in the trie?

A. Reconstruct from preorder traversal of trie.



Using preorder traversal to encode a trie as a bitstream

```
private static Node readTrie()
{
   if (BinaryStdIn.readBoolean())
   {
      char c = BinaryStdIn.readChar(8);
      return new Node(c, 0, null, null);
   }
   Node x = readTrie();
   Node y = readTrie();
   return new Node('\0', 0, x, y);
}
```

arbitrary value (value not used with internal nodes)

Huffman codes

Q. How to find best prefix-free code?

Huffman algorithm:

- Count frequency `freq[i]` for each char `i` in input.
- Start with one node corresponding to each char `i` (with weight `freq[i]`).
- Repeat until single trie formed:
  - select two tries with min weight `freq[i]` and `freq[j]`
  - merge into single trie with weight `freq[i] + freq[j]`

Applications:

JPEG, PDF, mp3, DivX MPEG4, gzip

Constructing a Huffman encoding trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
   MinPQ<Node> pq = new MinPQ<Node>();
   for (char i = 0; i < R; i++)
      if (freq[i] > 0)
         pq.insert(new Node(i, freq[i], null, null));

   while (pq.size() > 1)
   {
      Node x = pq.delMin();
      Node y = pq.delMin();
      Node parent = new Node('\0', x.freq + y.freq, x, y);
      pq.insert(parent);
   }

   return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used for internal nodes

total frequency

two subtries

Huffman compression summary

Proposition. Huffman's algorithm produces an optimal prefix-free code.
Pf. See textbook.

(no prefix-free code uses fewer bits)

Two-pass implementation (for compression).
- Pass 1: tabulate character frequencies; build trie.
- Pass 2: encode file by traversing trie (or symbol table).

Running time (for compression). Using a binary heap $\Rightarrow$ $N + R \log R$.
Running time (for expansion). Using a binary trie $\Rightarrow$ N.

(input size) (alphabet size)

Q. Can we do better? [stay tuned]

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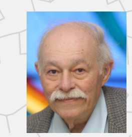
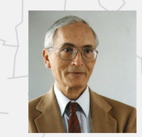
5.5 Data Compression

- introduction
- run-length coding
- Huffman compression
- LZW compression


Abraham Lempel


Jacob Ziv

Statistical methods

Static model. Same model for all texts.
- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.
- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.
- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

LZW compression demo

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D	A B		R A		B R		A B R			A
value	41	42	52	41	43	41	44	81		83		82		88			41 80

LZW compression for A B R A C A D A B R A B R A B R A

key	value	key	value	key	value
⋮	⋮	AB	81	DA	87
A	41	BR	82	ABR	88
B	42	RA	83	RAB	89
C	43	AC	84	BRA	8A
D	44	CA	85	ABRA	8B
⋮	⋮	AD	86		

codeword table

Lempel-Ziv-Welch compression

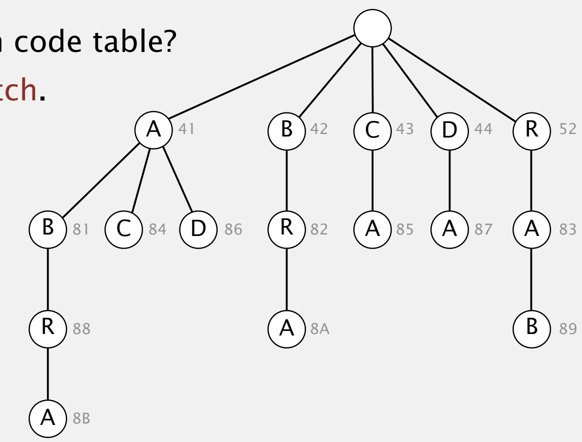
LZW compression.

- Create ST associating W-bit codewords with string keys.
- Initialize ST with codewords for single-character keys.
- Find longest string s in ST that is a prefix of unscanned part of input.
- Write the W-bit codeword associated with s.
- Add $s + c$ to ST, where c is next character in the input.

longest prefix match

Q. How to represent LZW compression code table?

A. A trie to support longest prefix match.



LZW expansion demo

value	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	

LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80

key	value	key	value	key	value
⋮	⋮	81	AB	87	DA
41	A	82	BR	88	ABR
42	B	83	RA	89	RAB
43	C	84	AC	8A	BRA
44	D	85	CA	8B	ABRA
⋮	⋮	86	AD		

codeword table

LZW expansion

LZW expansion.

- Create ST associating string values with W-bit keys.
- Initialize ST to contain single-character values.
- Read a W-bit key.
- Find associated string value in ST and write it out.
- Update ST.

Q. How to represent LZW expansion code table?

A. An array of length 2^W.

key	value
⋮	⋮
65	A
66	B
67	C
68	D
⋮	⋮
129	AB
130	BR
131	RA
132	AC
133	CA
134	AD
135	DA
136	ABR
137	RAB
138	BRA
139	ABRA
⋮	⋮

Data compression: quiz 3

What is the LZW compression of A B A B A B A ?

A. 41 42 41 42 41 42 80

B. 41 42 41 81 81

C. 41 42 81 81 41

D. 41 42 81 83 80

E. *I don't know.*

LZW tricky case: compression

input	A	B	A	B	A	B	A
matches	A	B	A B		A B A		
value	41	42	81		83		80

LZW compression for ABABABA

key	value
⋮	⋮
A	41
B	42
C	43
D	44
⋮	⋮

key	value
AB	81
BA	82
ABA	83

codeword table

LZW tricky case: expansion

value	41	42	81	83	80
output	A	B	A B	A B A	

LZW expansion for 41 42 81 83 80

need to know code for 83 before it is in codeword table!

we can deduce that the code for 83 is ABx for some character x

now, we have deduced x!

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BA
83	ABA

codeword table

LZW implementation details

How big to make ST?

- How long is message?
- Whole message similar model?
- [many other variations]

What to do when ST fills up?

- Throw away and start over. [GIF]
- Throw away when not effective. [Unix compress]
- [many other variations]

Why not put longer substrings in ST?

- [many variations have been developed]

LZW in the real world

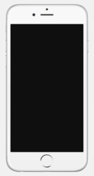
Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

Unix compress, GIF, TIFF, V.42bis modem: LZW. ← previously under patent

zip, 7zip, gzip, jar, png, pdf: deflate / zlib. ← not patented (widely used in open source)

iPhone, Wii, Apache HTTP server: deflate / zlib.



Apache HTTP SERVER PROJECT

Lossless data compression benchmarks

year	scheme	bits / char
1967	**ASCII**	7.00
1950	**Huffman**	4.70
1977	**LZ77**	3.94
1984	**LZMW**	3.32
1987	**LZH**	3.30
1987	**move-to-front**	3.24
1987	**LZB**	3.18
1987	**gzip**	2.71
1988	**PPMC**	2.48
1994	**SAKDC**	2.47
1994	**PPM**	2.34
1995	**Burrows-Wheeler**	2.29
1997	**BOA**	1.99
1999	**RK**	1.89

← next programming assignment (Burrows-Wheeler)

data compression using Calgary corpus

Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes. [Huffman]
- Represent variable-length symbols with fixed-length codes. [LZW]

Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3, …
- FFT/DCT, wavelets, fractals, …

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}\left(n+\frac{1}{2}\right)k\right]$$

Theoretical limits on compression. Shannon entropy: $H(X) = -\sum_i^n p(x_i) \lg p(x_i)$

Practical compression. Exploit extra knowledge whenever possible.