Global Illumination

COS 426, Spring 2014
Princeton University

Overview

- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - Inter-object reflections
  - Rendering equation
  - Recursive ray tracing
  - More advanced ray tracing
  - Radiosity

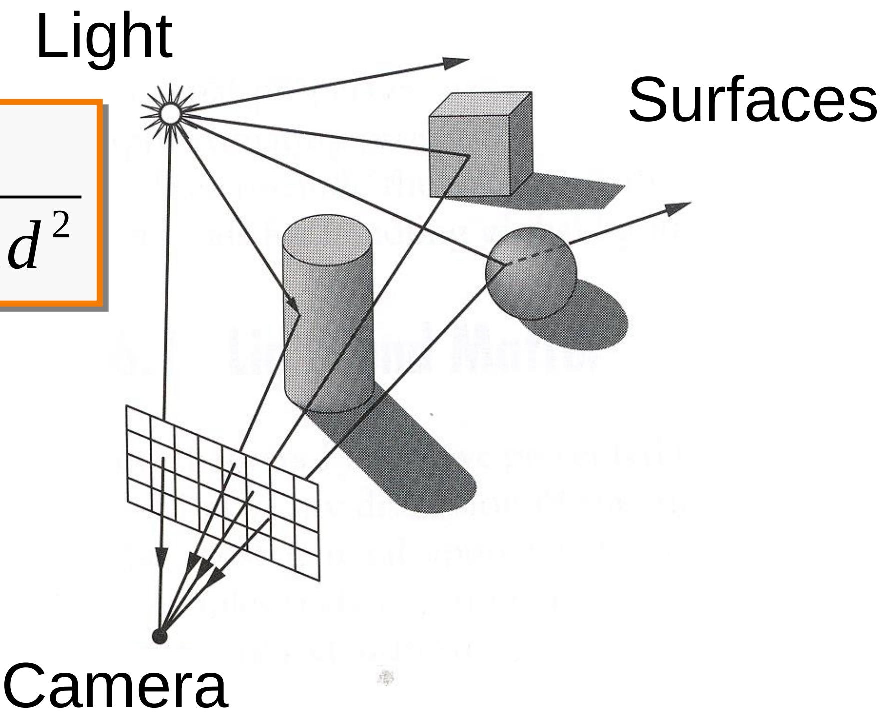
Kajiya 1986

Greg Ward

Direct Illumination (last lecture)

- For each ray traced from camera
  - Sum radiance reflected from each light

$$I_i = \frac{I_0}{c_a + l_a d + q_a d^2}$$



$$I = I_E + K_A I_{AL} + \sum_i \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) I_i$$

Example

Red’s Dream (Pixar Animation Studios)

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Greg Ward

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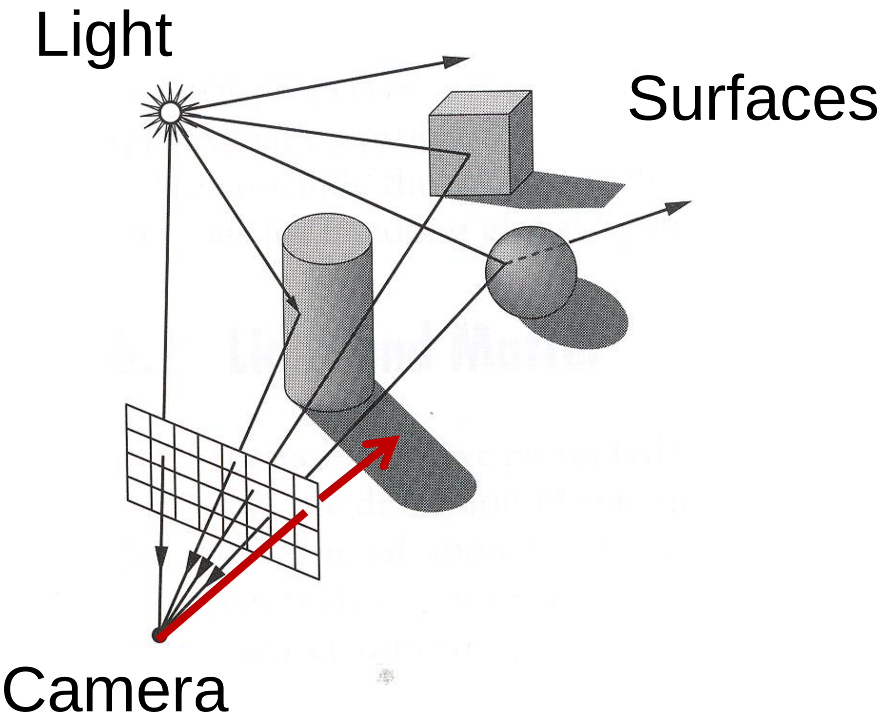
Greg Ward

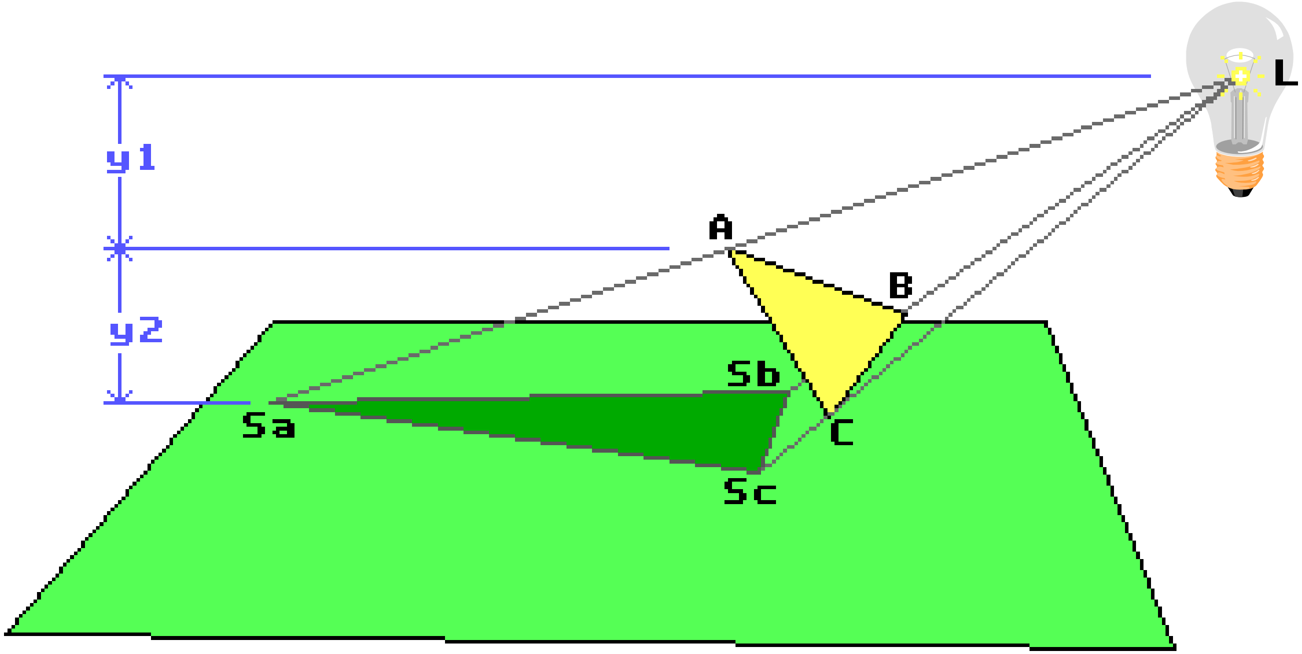
Shadows

- Hard shadows from point light sources



Shadows

- Hard shadows from point light sources



Shadows

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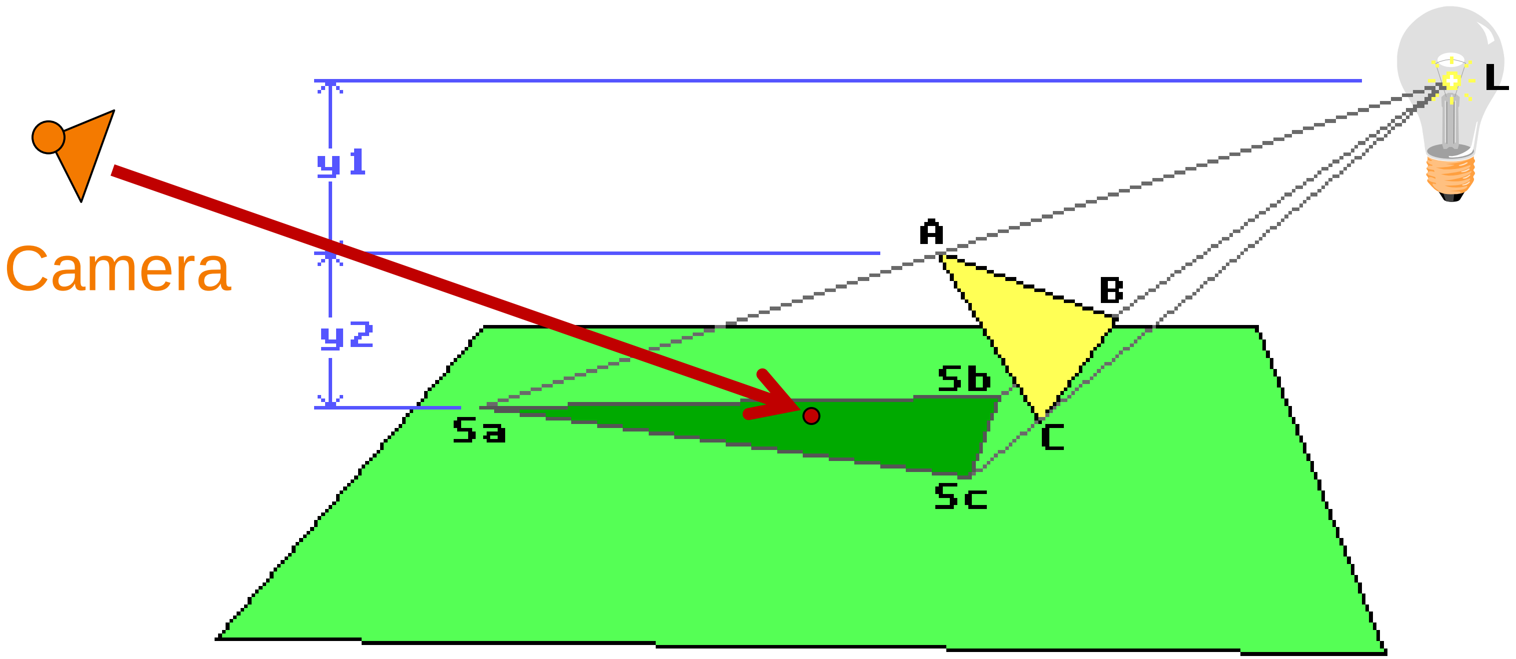
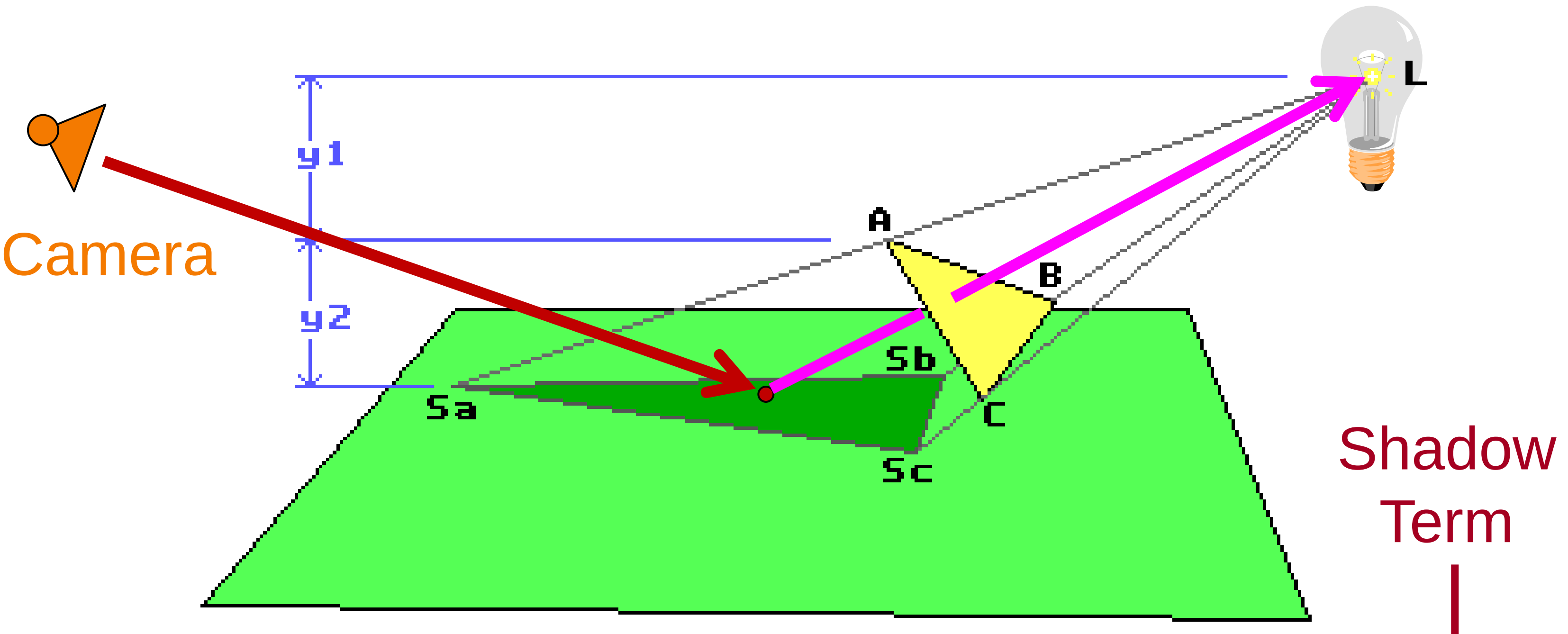


Shadows

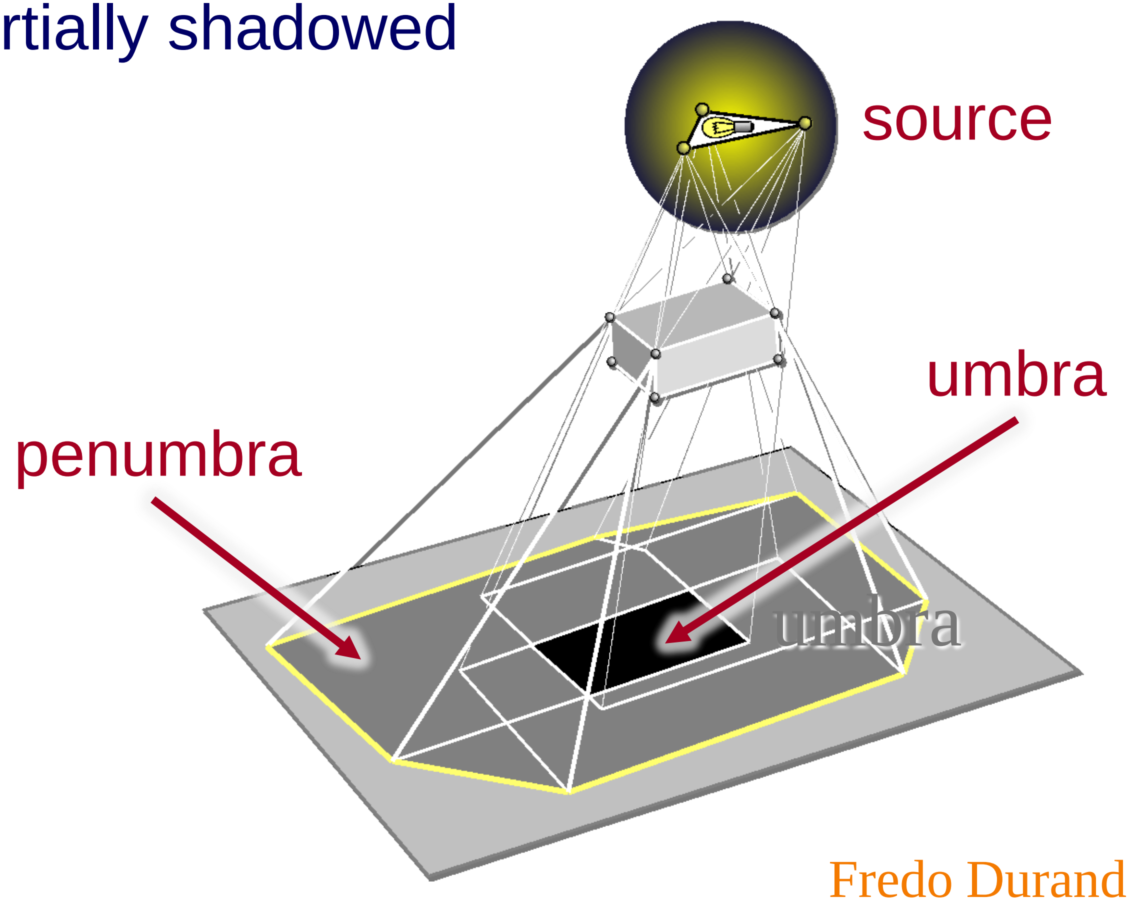
- Hard shadows from point light sources
  - Cast ray towards light; S_L=0 if blocked, S_L=1 otherwise



$$I = I_E + K_A I_{AL} + \sum_{i \in \text{lights}} \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_i I_i$$

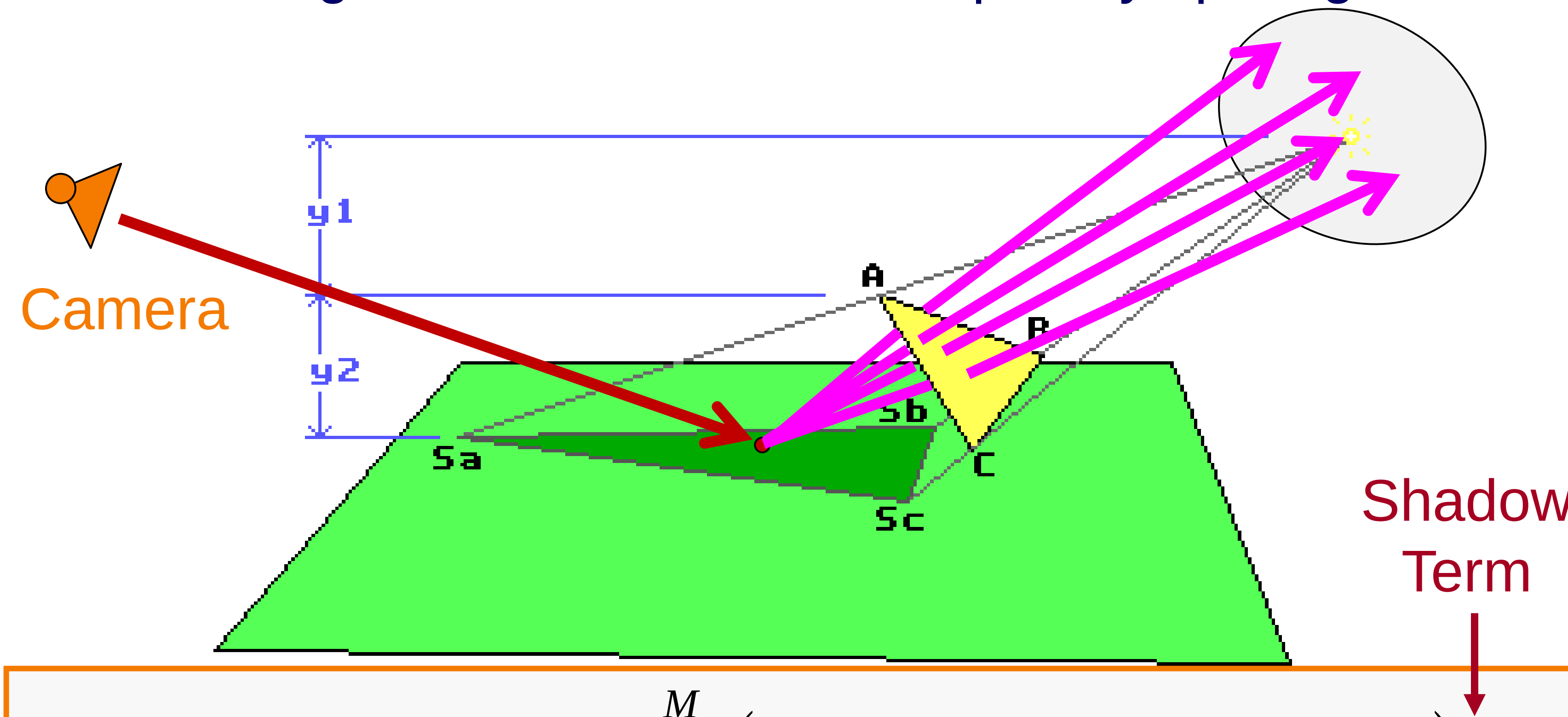
Shadows

- Soft shadows from area light sources
  - Umbra = fully shadowed
  - Penumbra = partially shadowed



Shadows

- Soft shadows from area light sources
  - Average illumination for M sample rays per light

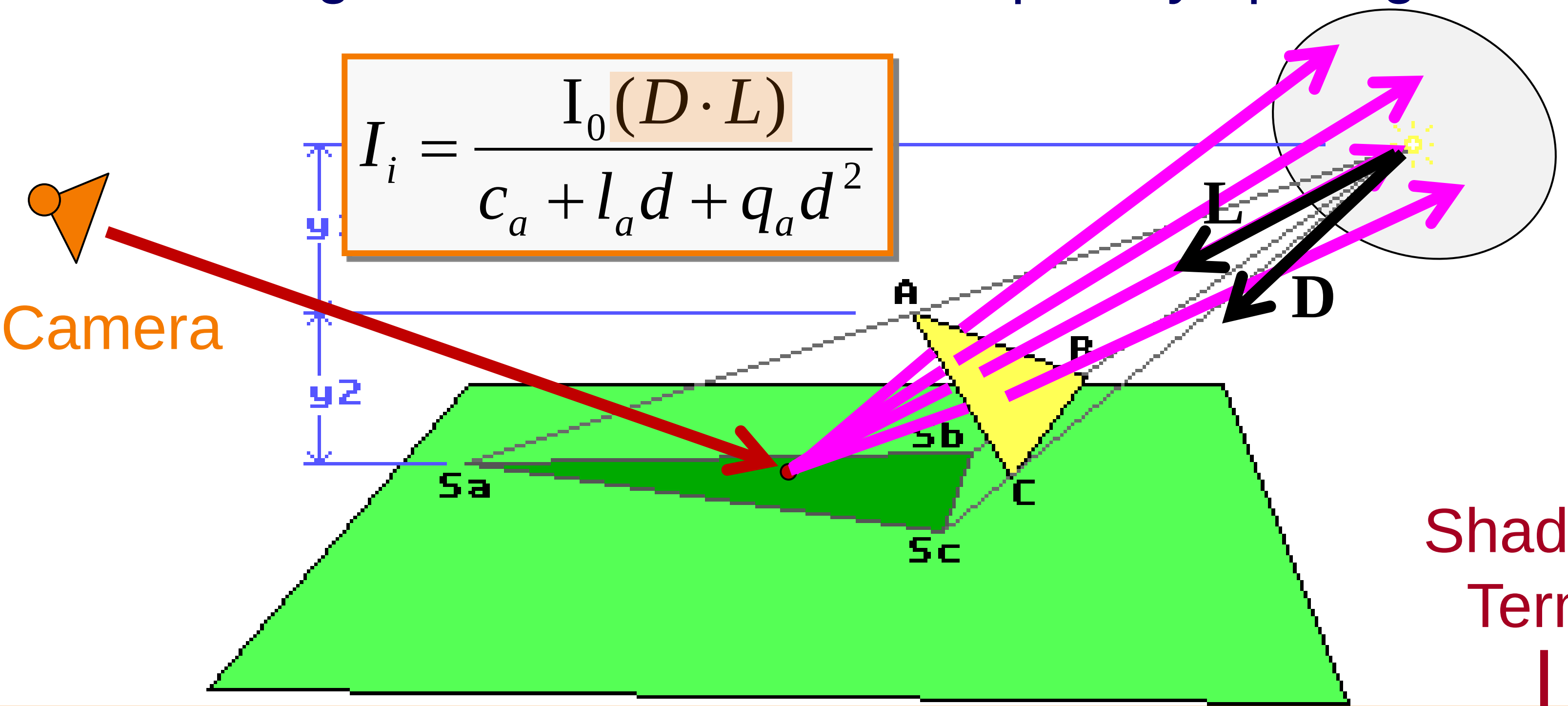


$$I = \cdots + \sum_{i \in \text{AreaLights}} \frac{1}{M} \sum_{j \in \text{samples}}^{M} \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

Shadows

- Soft shadows from circular area light sources
  - Average illumination for M sample rays per light

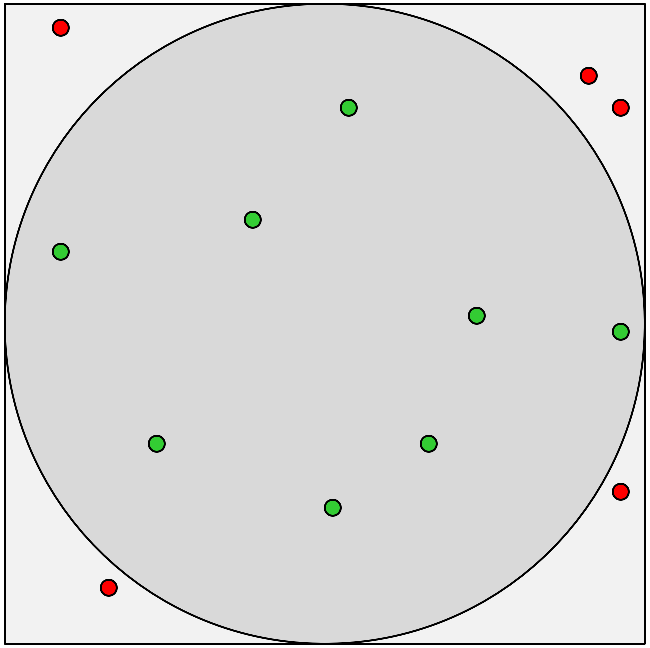
$$I_i = \frac{I_0 (D \cdot L)}{c_a + l_a d + q_a d^2}$$



$$I = \cdots + \sum_{i \in \text{AreaLights}} \tfrac{1}{M} \sum_{j \in \text{samples}}^{M} \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

Shadows

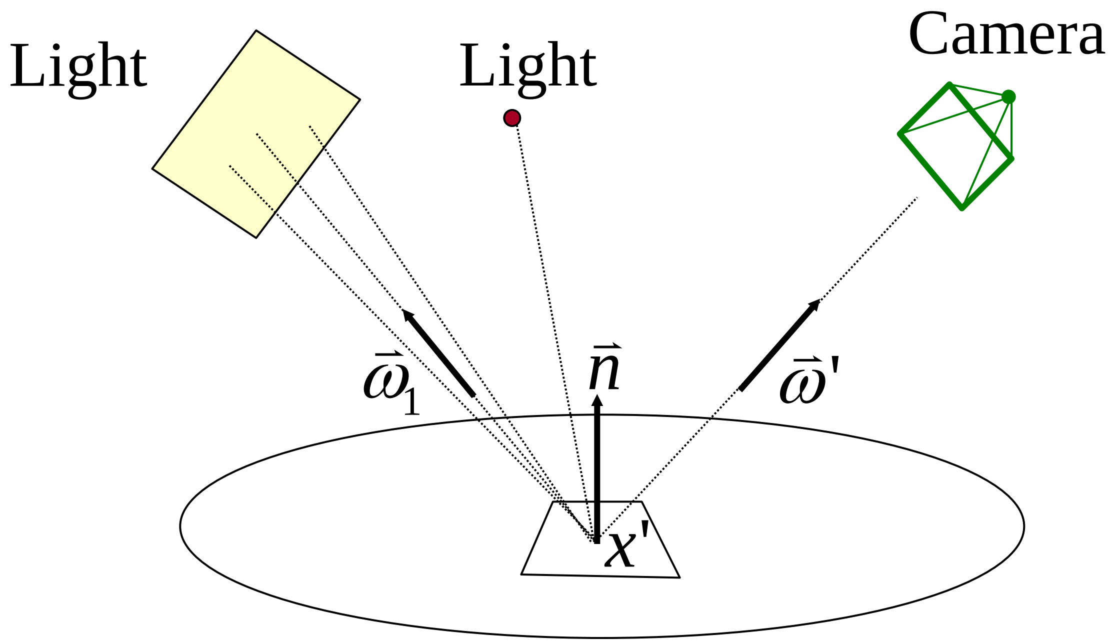
- Soft shadows from circular area light sources
  - Average illumination for M sample rays per light
    - Generate M random sample points on area light (e.g., with rejection sampling)
    - Compute illumination for every sample
    - Average



$$I = \cdots + \sum_{i \in \text{AreaLights}} \tfrac{1}{M} \sum_{j \in \text{samples}}^{M} \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

Direct Illumination

- Illumination from polygonal area light sources
  - Average illumination for M sample rays per light



$$I = \cdots + \sum_{i \in \text{AreaLights}} \frac{1}{M} \sum_{j \in \text{samples}}^{M} \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

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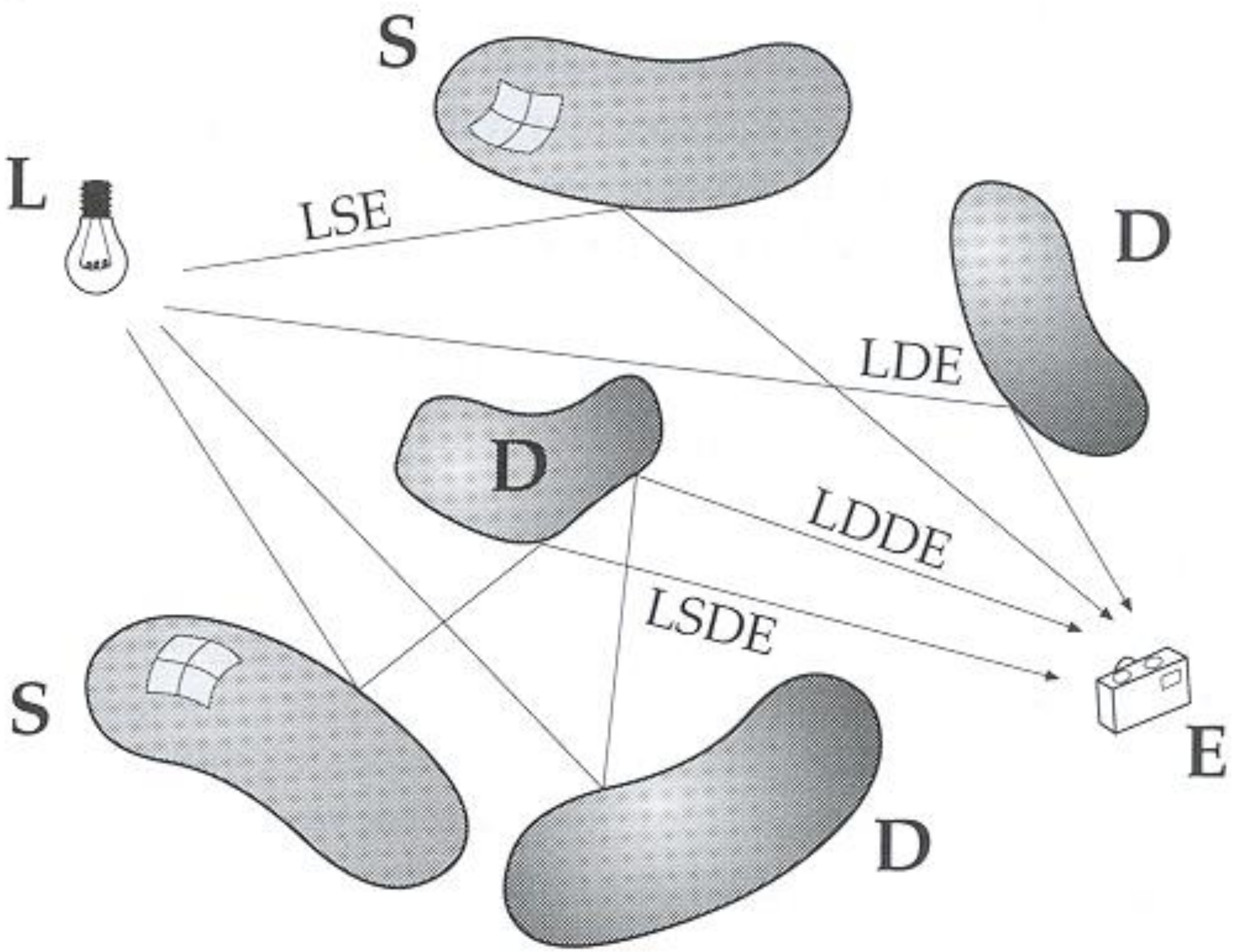
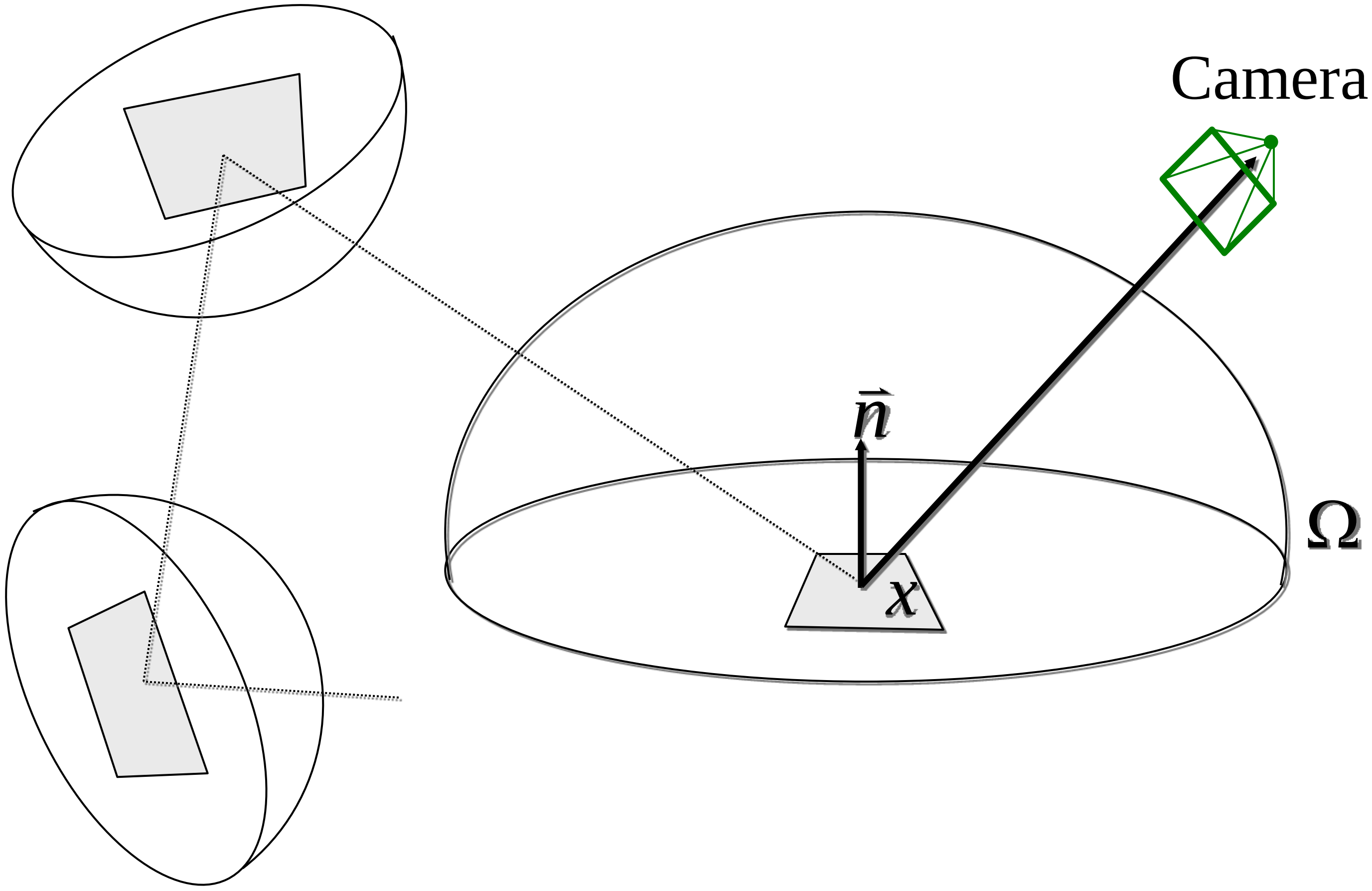
Greg Ward

Inter-Object Reflection



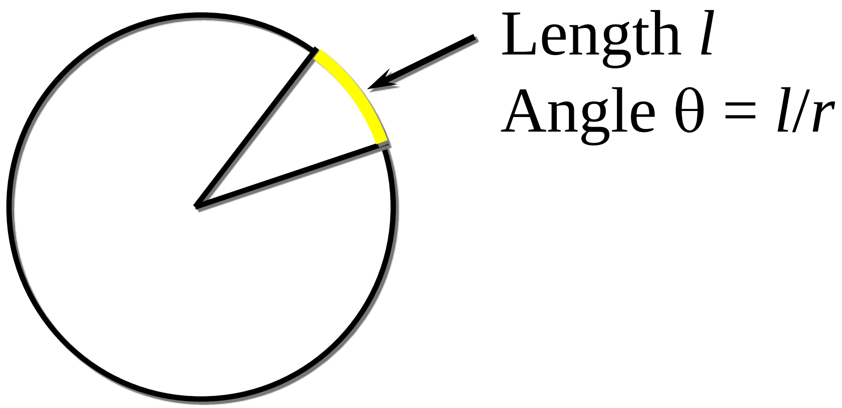
Inter-Object Reflection

- Radiance leaving point x on surface is sum of reflected irradiance arriving from other surfaces

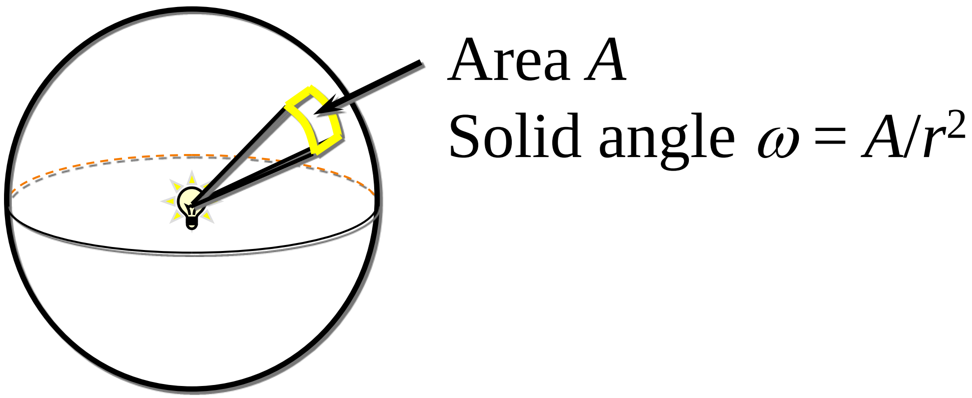


Solid Angle

- Angle in radians

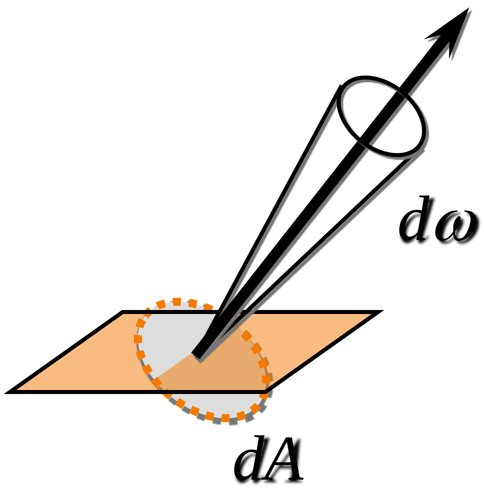


- Solid angle in steradians



Light Emitted from a Surface

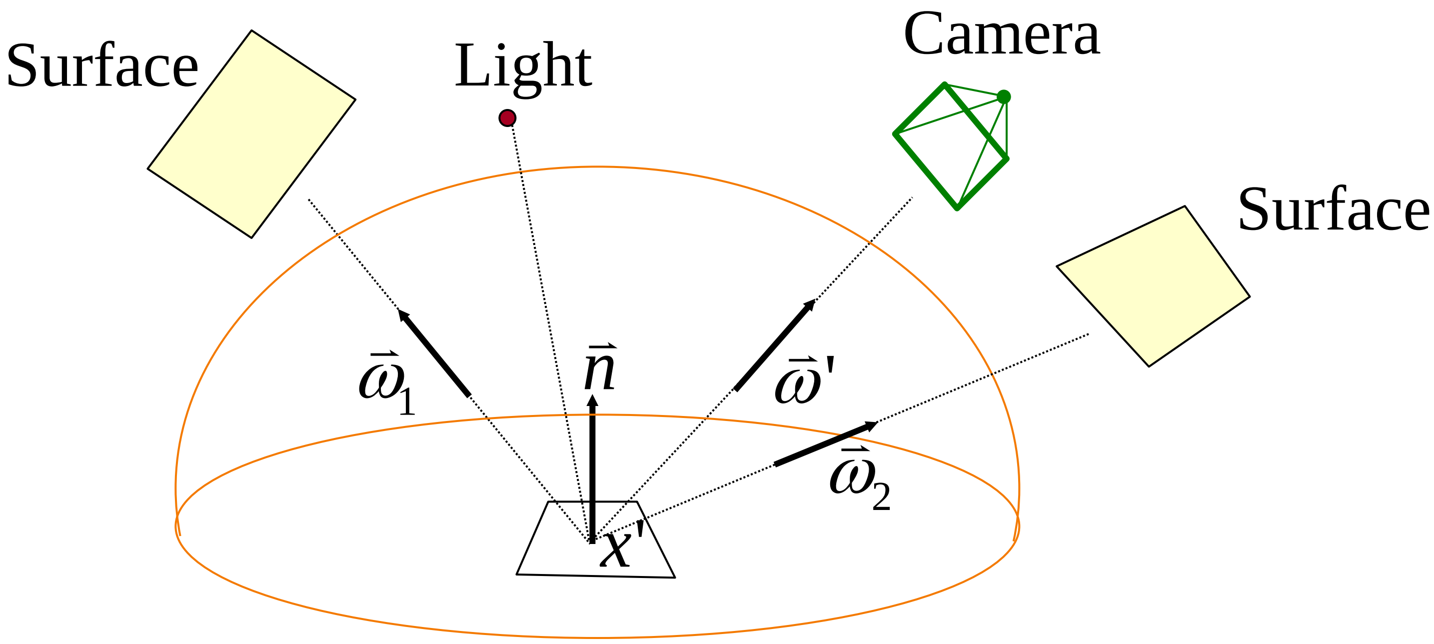
- Power per unit area per unit solid angle – *Radiance* (L)
  - Measured in $W/m^2/sr$



$$L = \frac{d\Phi}{dA\,d\omega}$$

Rendering Equation [Kajiya 86]

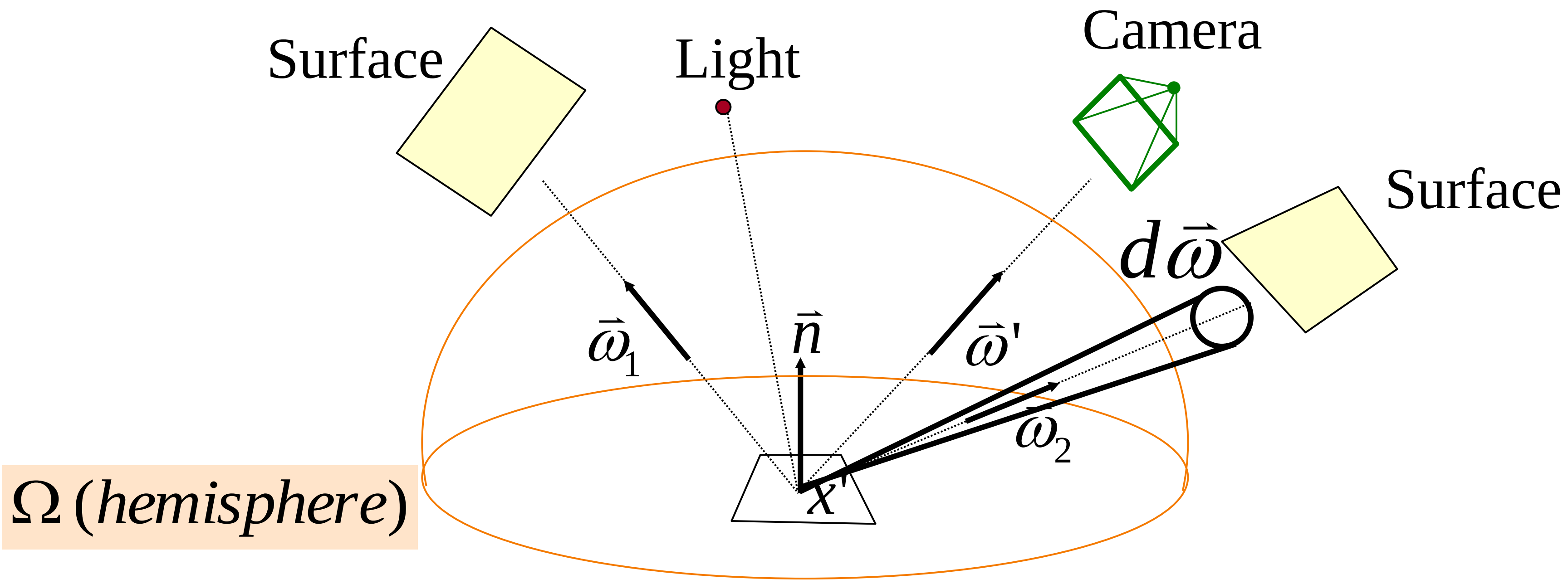
- Compute radiance in outgoing direction by integrating reflections over all incoming directions



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')(\vec{\omega}\cdot\vec{n})\,L_i(x',\vec{\omega})\,d\vec{\omega}$$

Rendering Equation [Kajiya 86]

- Compute radiance in outgoing direction by integrating reflections over all incoming directions



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')(\vec{\omega}\cdot\vec{n})\,L_i(x',\vec{\omega})\,d\vec{\omega}$$

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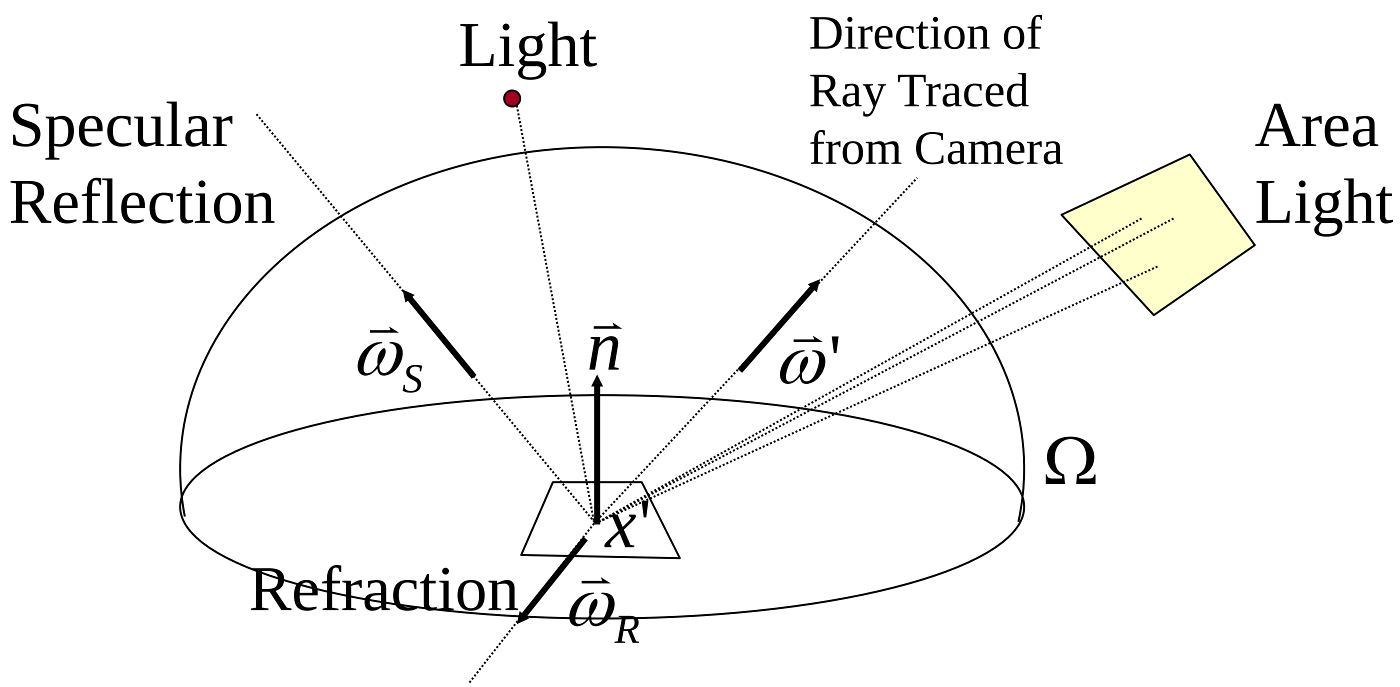
Greg Ward

Recursive Ray Tracing

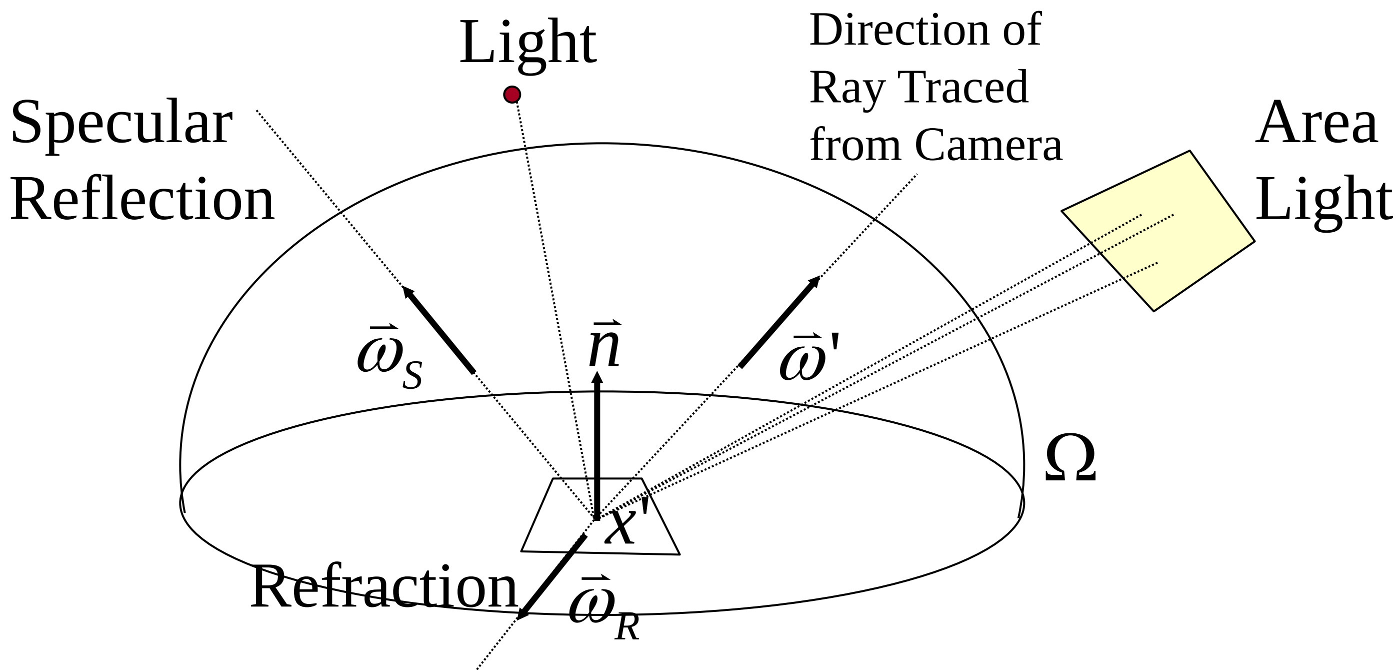
- Assume only significant irradiance is in directions of light sources, specular reflection, and refraction



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')(\vec{\omega}\cdot\vec{n})L_i(x',\vec{\omega})\,d\vec{\omega}$$

Recursive Ray Tracing

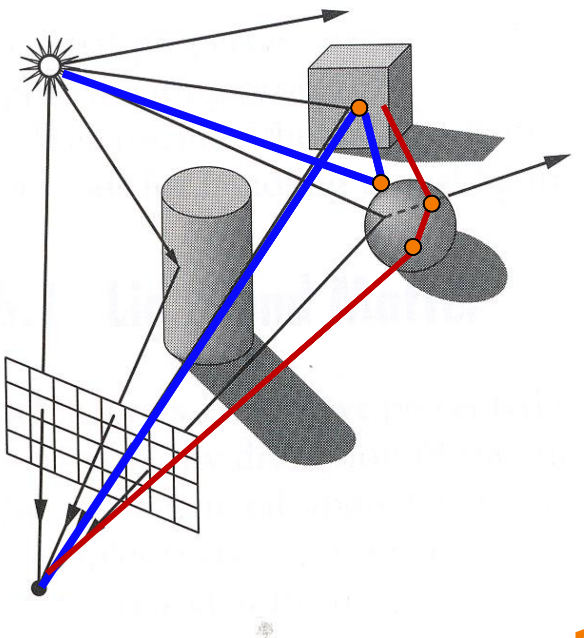
- Compute radiance in outgoing direction by summing reflections from directions of lights specular reflections, and refractions



$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Recursive Ray Tracing

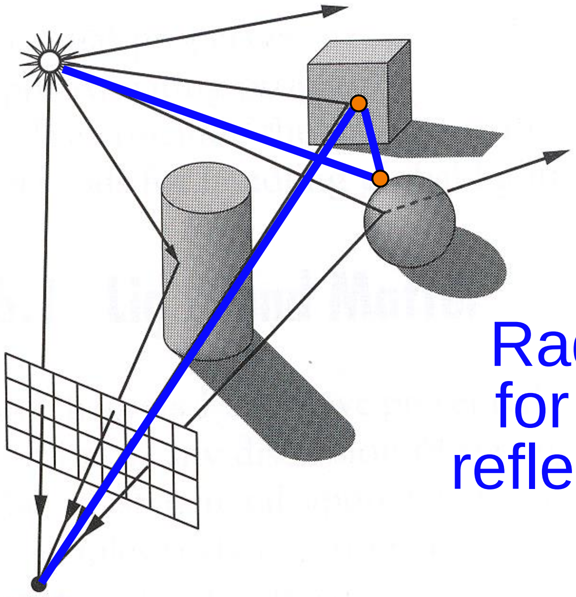
- Same as ray casting, but trace secondary rays for specular (mirror) reflection and refraction



$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Specular Reflection

- Trace secondary ray in direction of mirror reflection
  - Evaluate radiance along secondary ray and include it into illumination model

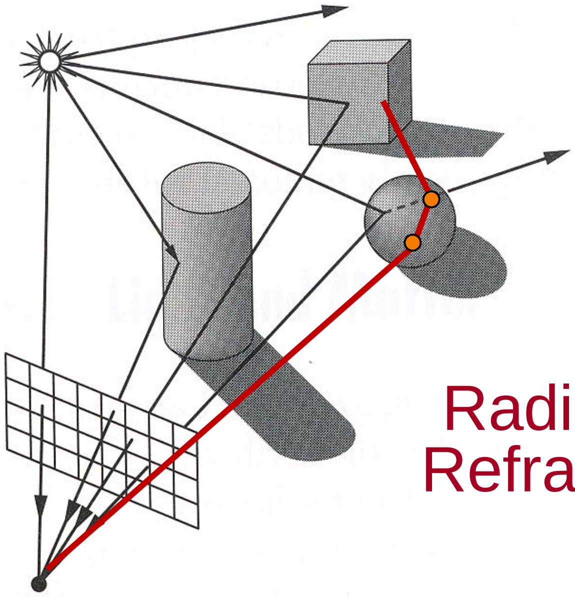


Radiance for mirror reflection ray

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Refraction

- Trace secondary ray in direction of refraction
  - Evaluate radiance along secondary ray and include it into illumination model



Radiance for Refraction Ray

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T \boldsymbol{I_T}$$

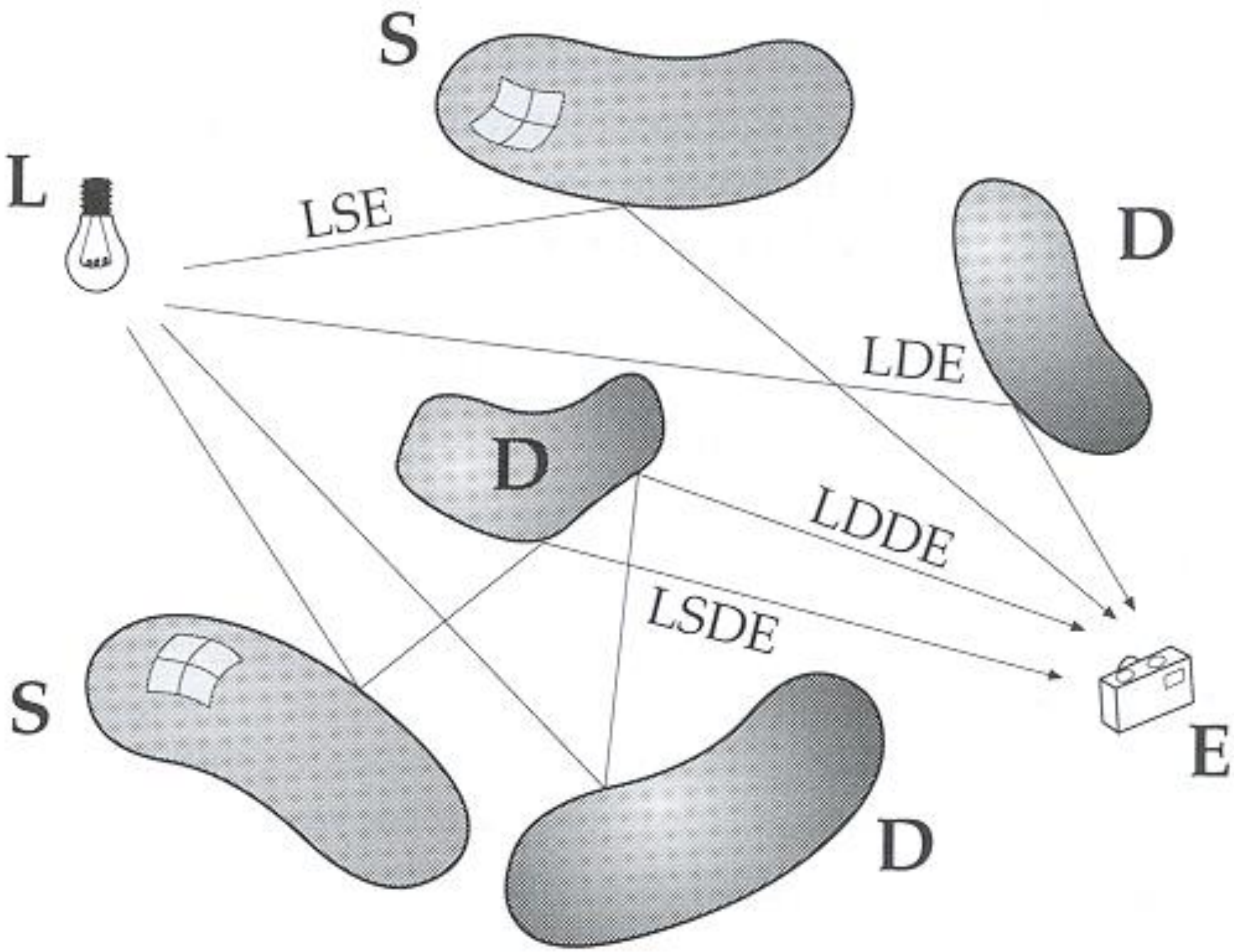
Recursive Ray Tracing

- ComputeRadiance is called recursively

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray, R3Intersection& hit)
{
    R3Ray specular_ray = SpecularRay(ray, hit);
    R3Ray refractive_ray = RefractiveRay(ray, hit);
    R3Rgb radiance = Phong(scene, ray, hit) +
                     Ks * ComputeRadiance(scene, specular_ray) +
                     Kt * ComputeRadiance(scene, refractive_ray);
    return radiance;
}
```

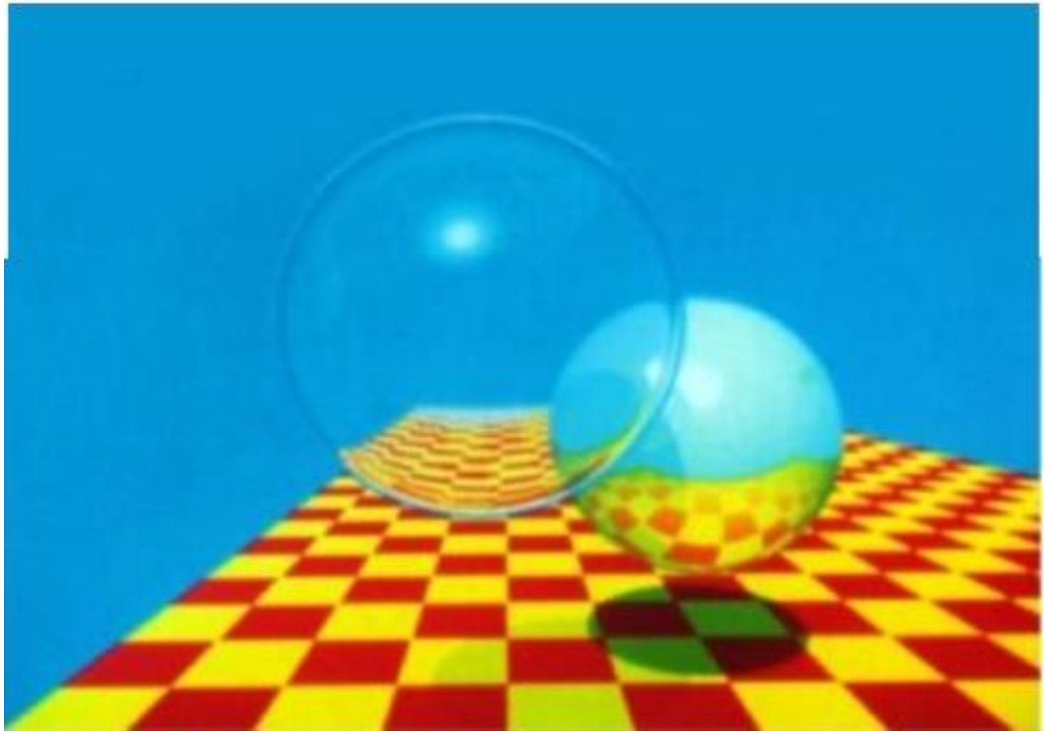
Recursive Ray Tracing

- Which paths?



Recursive Ray Tracing

- Specular reflection and refraction -- LD(S|R)*E



Whitted

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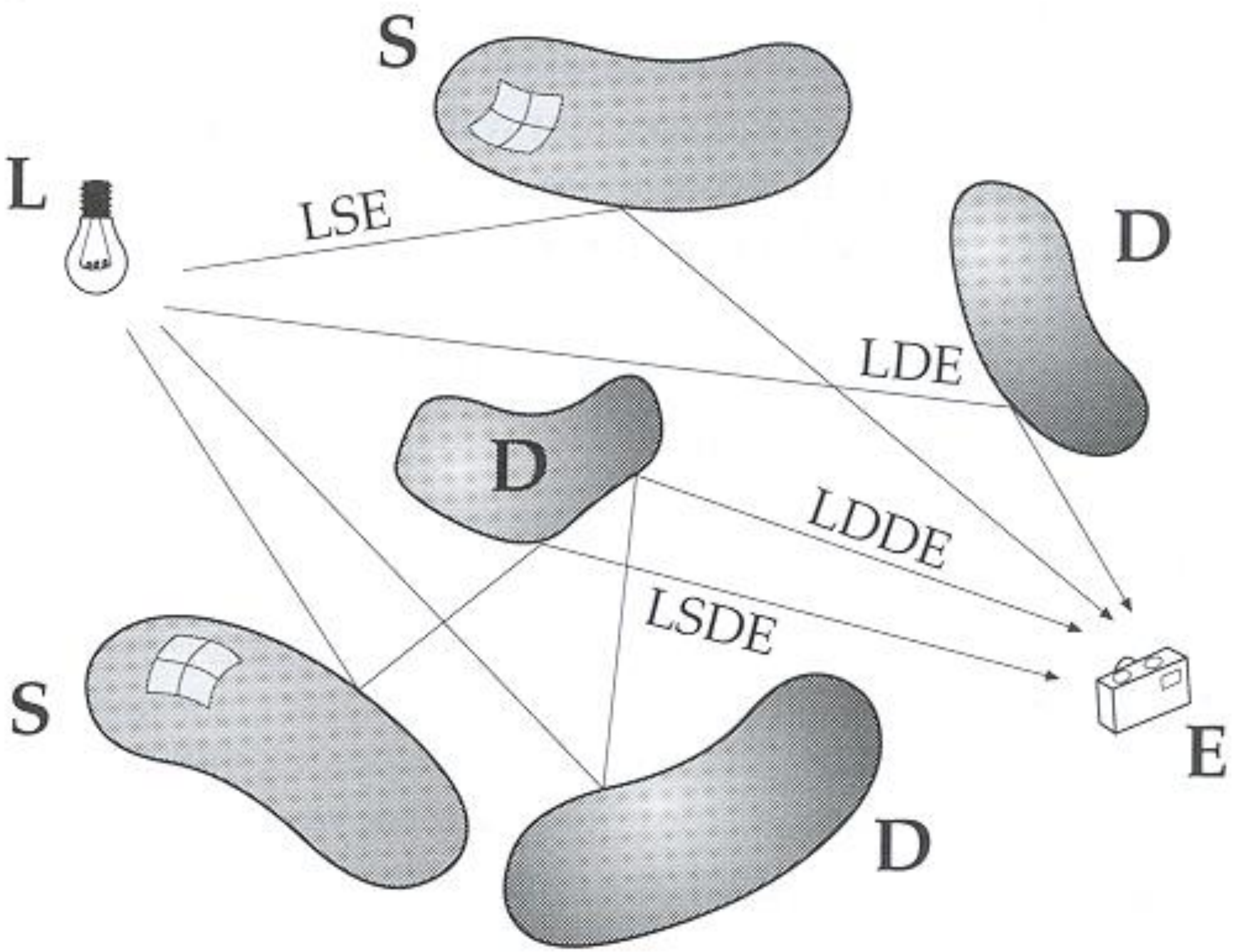
Kajiya 1986

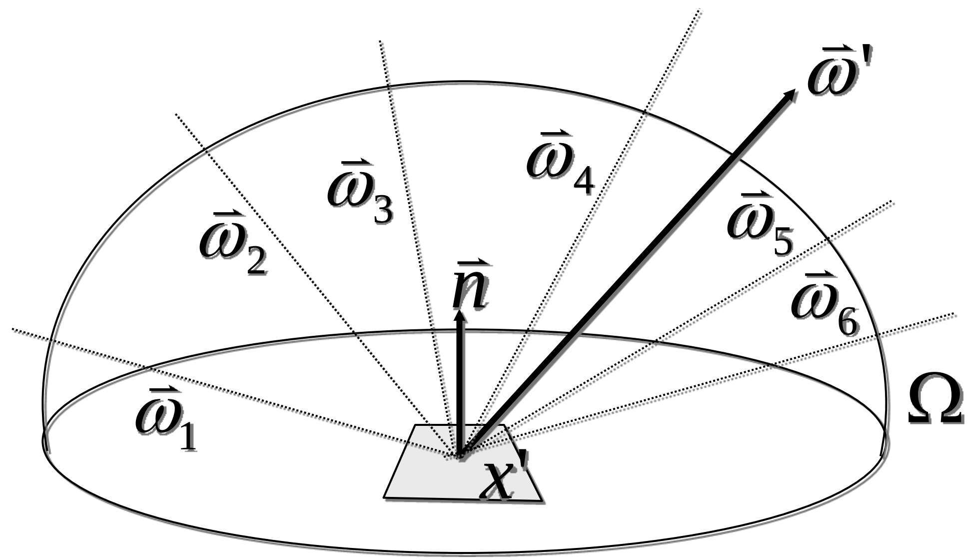
Greg Ward

Beyond Recursive Ray Tracing



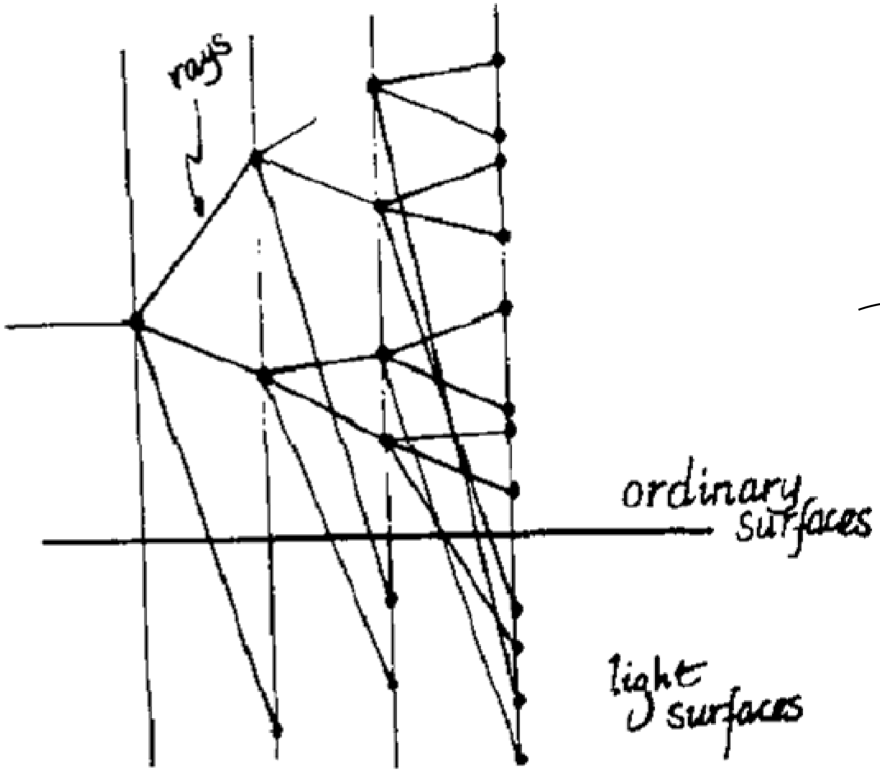
Distributed Ray Tracing

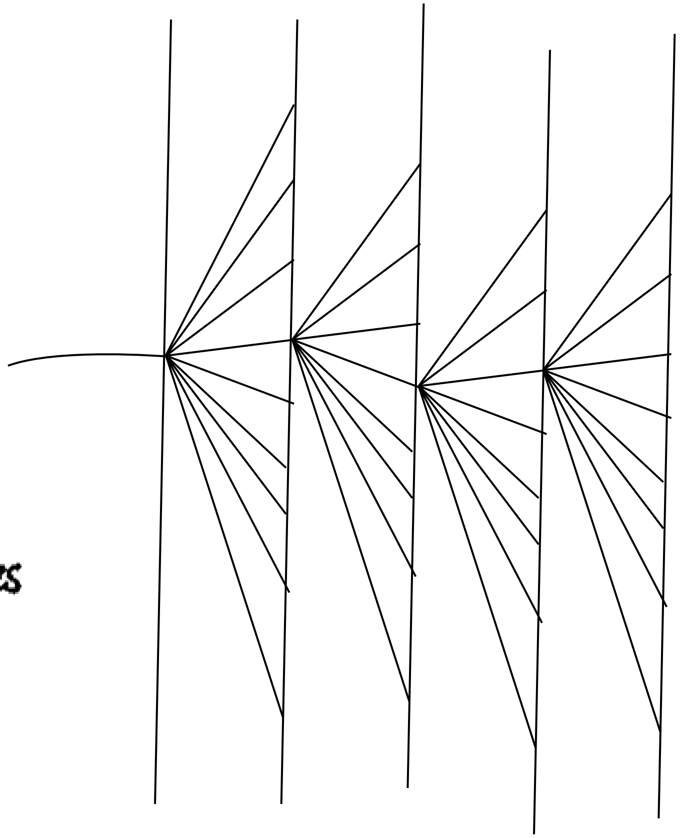
- Estimate integral for each reflection by sampling incoming directions



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{\text{samples}} f_r(x',\vec{\omega},\vec{\omega}')(\vec{\omega}\cdot\vec{n})\,L_i(x',\vec{\omega})\,d\vec{\omega}$$

Ordinary Ray Tracing vs. Distribution Ray Tracing





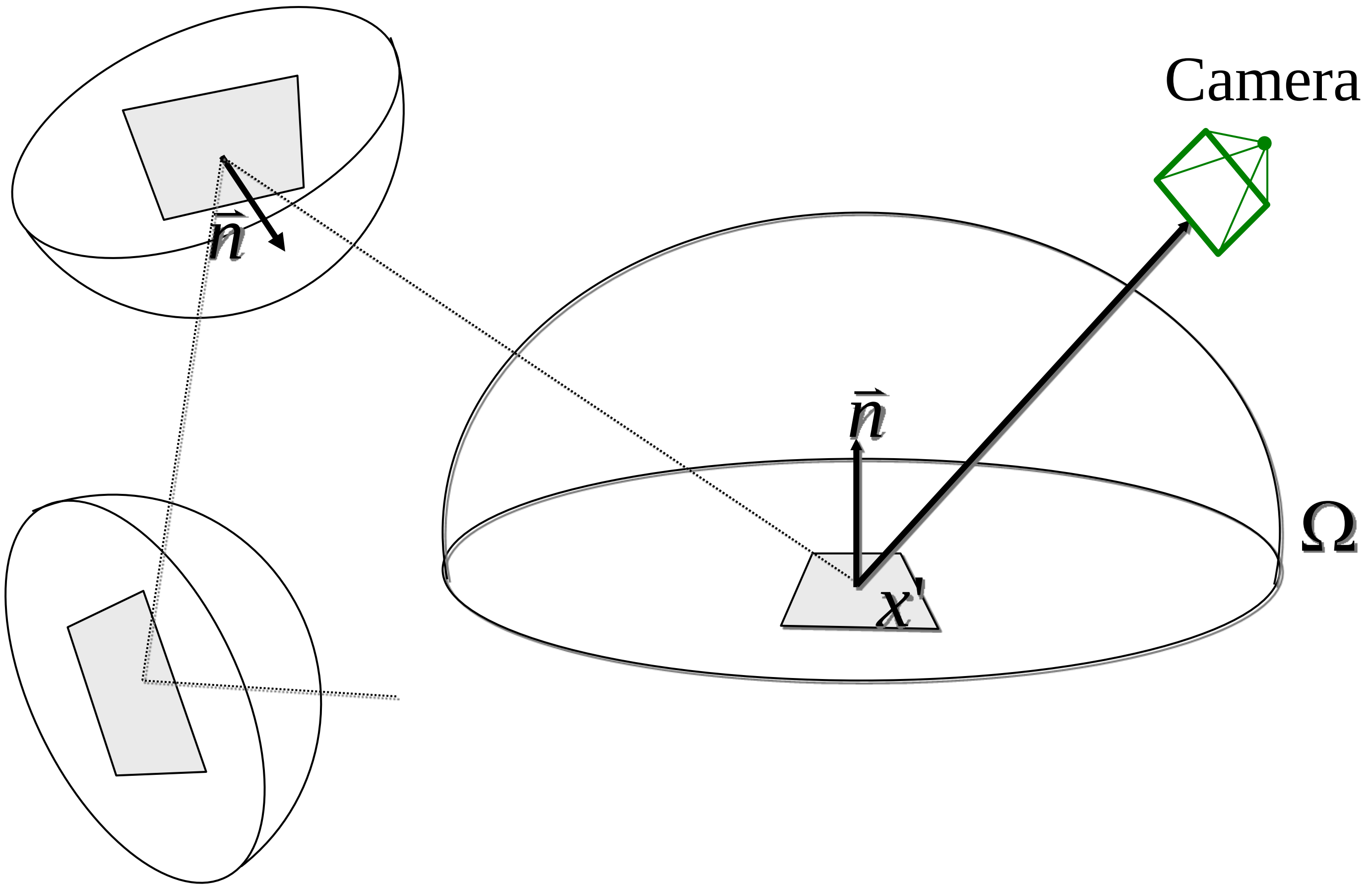
Ray tracing

Distributed ray tracing

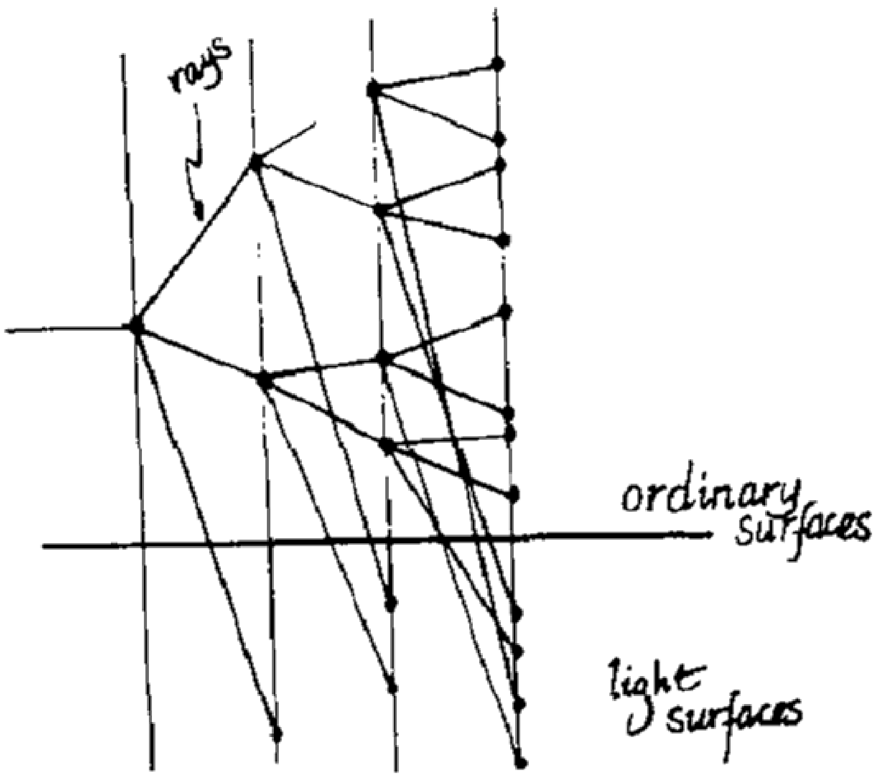
Kajiya

Monte Carlo Path Tracing

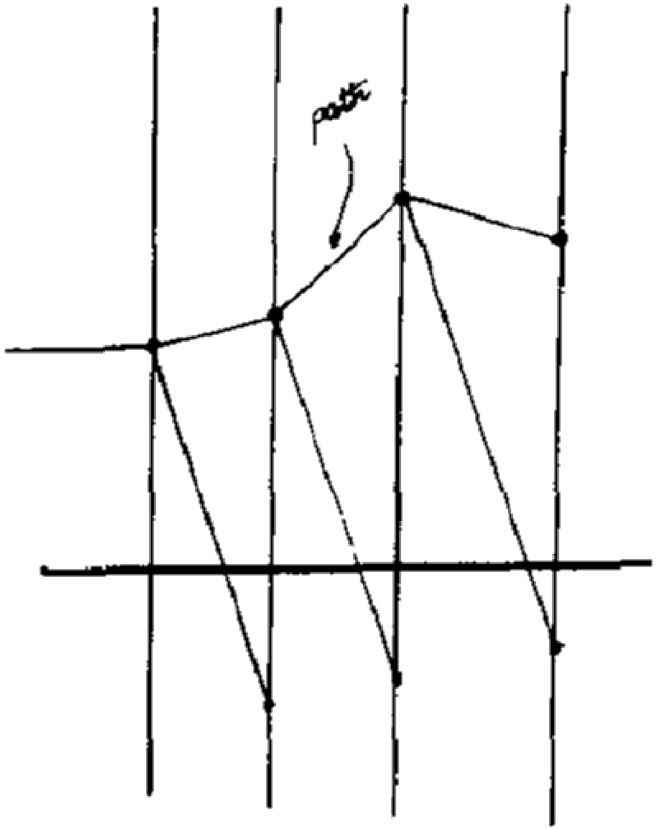
- Estimate integral for each pixel by sampling paths from camera



Ray Tracing vs. Path Tracing



Ray tracing



Path tracing

Kajiya

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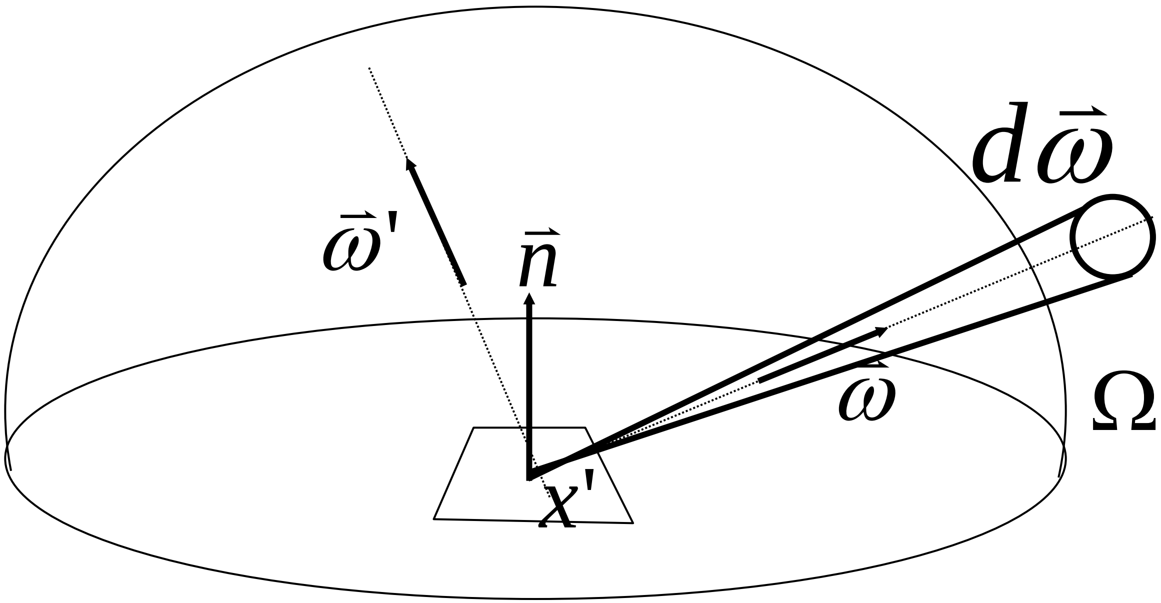
Greg Ward

Radiosity

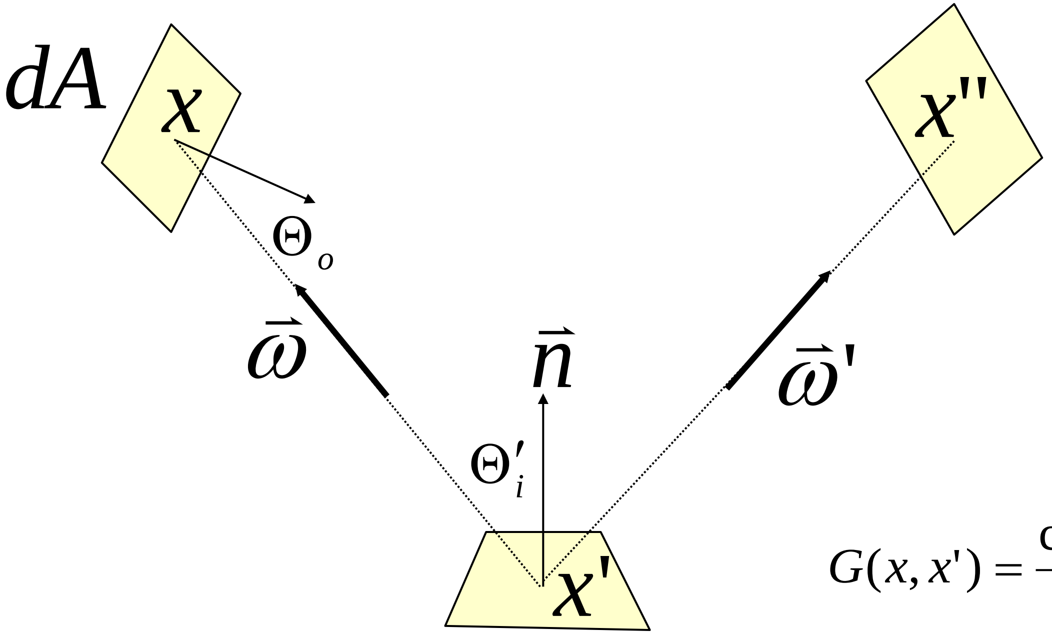
- Indirect diffuse illumination – LD*E

Rendering Equation (1)



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')(\vec{\omega}\cdot\vec{n})\,L_i(x',\vec{\omega})\,d\vec{\omega}$$

Rendering Equation (2)



$$G(x,x') = \frac{\cos\Theta_i' \cos\Theta_o}{\|x - x'\|^2}$$

$$L(x' \to x'') = L_e(x' \to x'') + \int_S f_r(x \to x' \to x'')\, L(x \to x')\, V(x,x')\, G(x,x')\, dA$$

Kajiya 1986

Radiosity Equation

$$L(x'\rightarrow x'')=L_e(x'\rightarrow x'')+\int_S f_r(x\rightarrow x'\rightarrow x'')\,L(x\rightarrow x')\,V(x,x')\,G(x,x')\,dA$$

Assume everything is Lambertian

$$\rho(x')=f_r(x\rightarrow x'\rightarrow x'')\pi$$

$$L(x')=L_e(x')+\frac{\rho(x')}{\pi}\int_S L(x)V(x,x')\,G(x,x')\,dA$$

Convert to Radiosities

$$B=\int_\Omega L_o\cos\theta\,d\omega$$

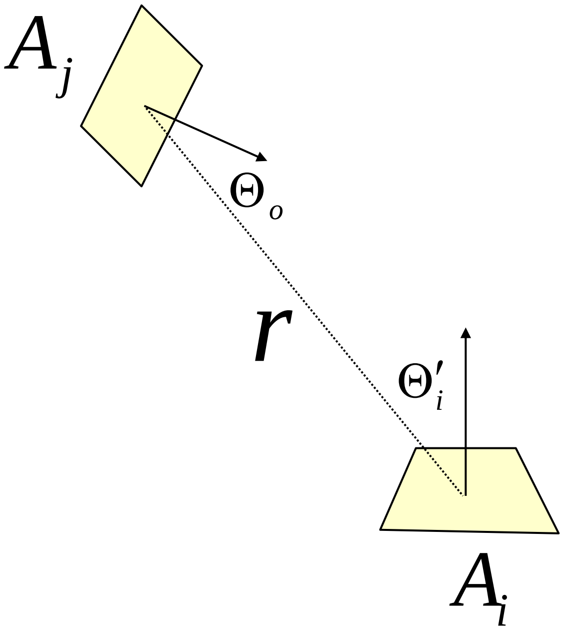
$$L=\frac{B}{\pi}$$

$$B(x')=B_e(x')+\frac{\rho(x')}{\pi}\int_S B(x)V(x,x')\,G(x,x')\,dA$$

Radiosity Approximation

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi} \int_S B(x) V(x,x') G(x,x') \, dA$$

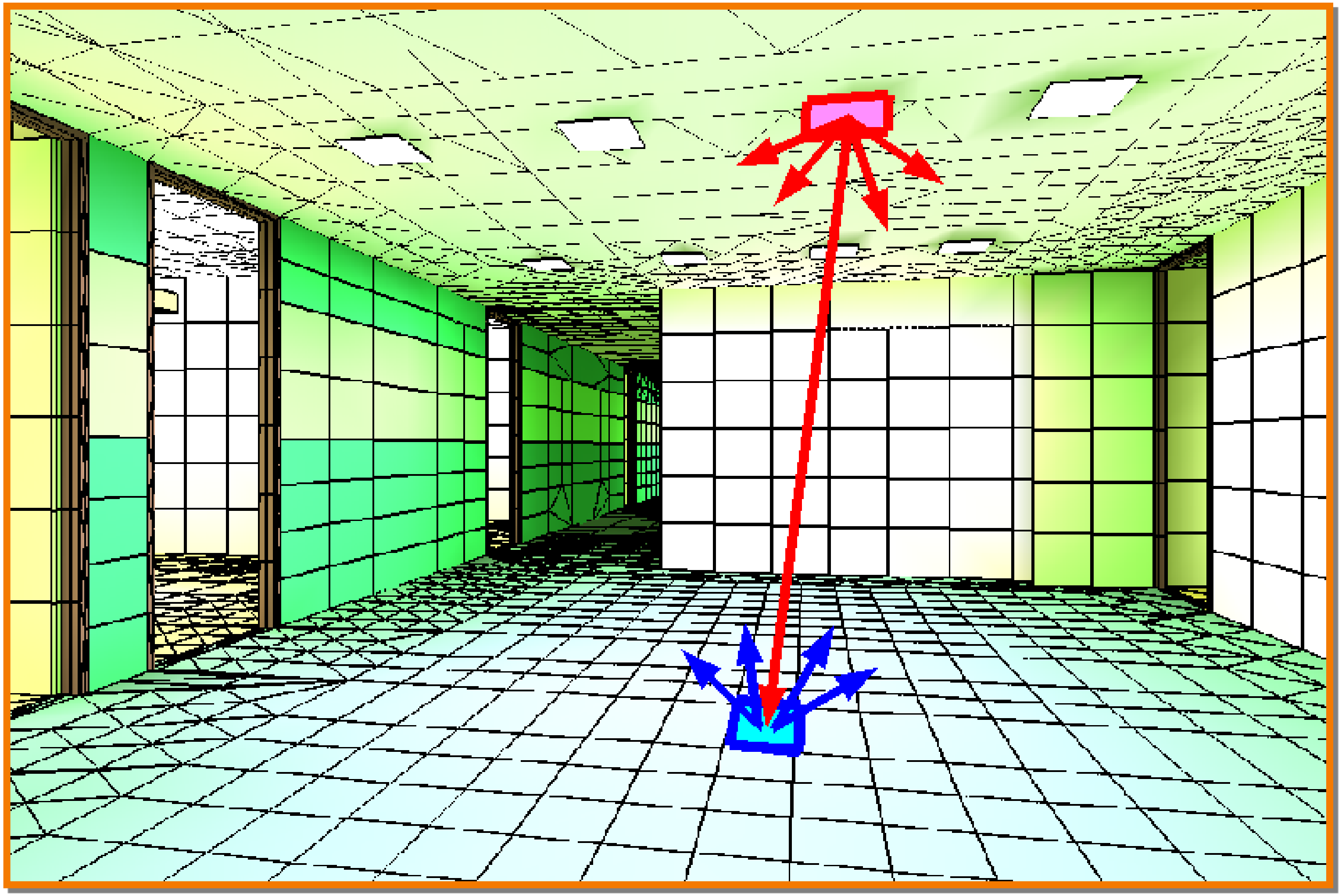
Discretize the surfaces
into "elements"



$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

where $F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{V_{ij} \cos\Theta_i' \cos\Theta_o}{\pi r^2} dA_j dA_i$

Radiosity Approximation



System of Equations

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$E_i = B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij} = E_i$$

$$\left(1 - \rho_i \sum_{j=1}^{N} F_{ii}\right) B_i - \rho_i \sum_{j=1}^{N} F_{ij} B_j = E_i$$

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{N} F_{ji} B_j A_j$$

← This is an energy balance equation

Radiosity

- Application
  - Interior lighting design
  - LD*E
- Issues
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving large linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Summary

- Global illumination
  - Rendering equation
- Solution methods
  - Sampling
    - Ray tracing
    - Distributed ray tracing
    - Monte Carlo path tracing
  - Discretization
    - Radiosity

Photorealistic rendering with global illumination is an integration problem