Sampling, Resampling, and Warping

COS 426, Spring 2014

Tom Funkhouser

Image Processing

Goal: read an image, process it, write the result

input.jpg

output.jpg

```
imgpro input.jpg output.jpg -histogram_equalization
```

Image Processing Operations I

- Luminance
  - Brightness
  - Contrast.
  - Gamma
  - Histogram equalization
- Color
  - Black & white
  - Saturation
  - White balance
- Linear filtering
  - Blur & sharpen
  - Edge detect
  - Convolution
- Non-linear filtering
  - Median
  - Bilateral filter
- Dithering
  - Quantization
  - Ordered dither
  - Floyd-Steinberg

Image Processing Operations II

- Transformation
  - Scale
  - Rotate
  - Warp

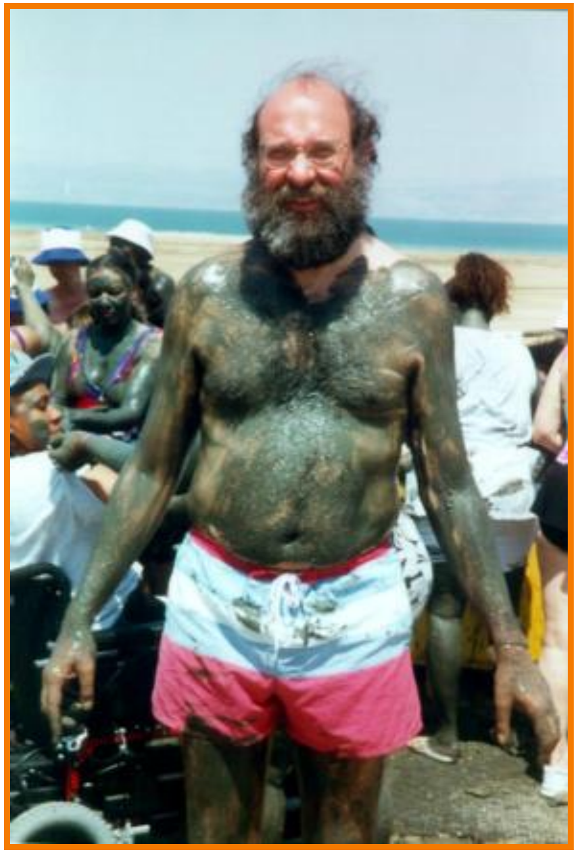
$\Big\}$ Today

- Combining images
  - Composite
  - Morph
  - Comp photo

$\Big\}$ Thursday

Image Transformation

- Move pixels of an image



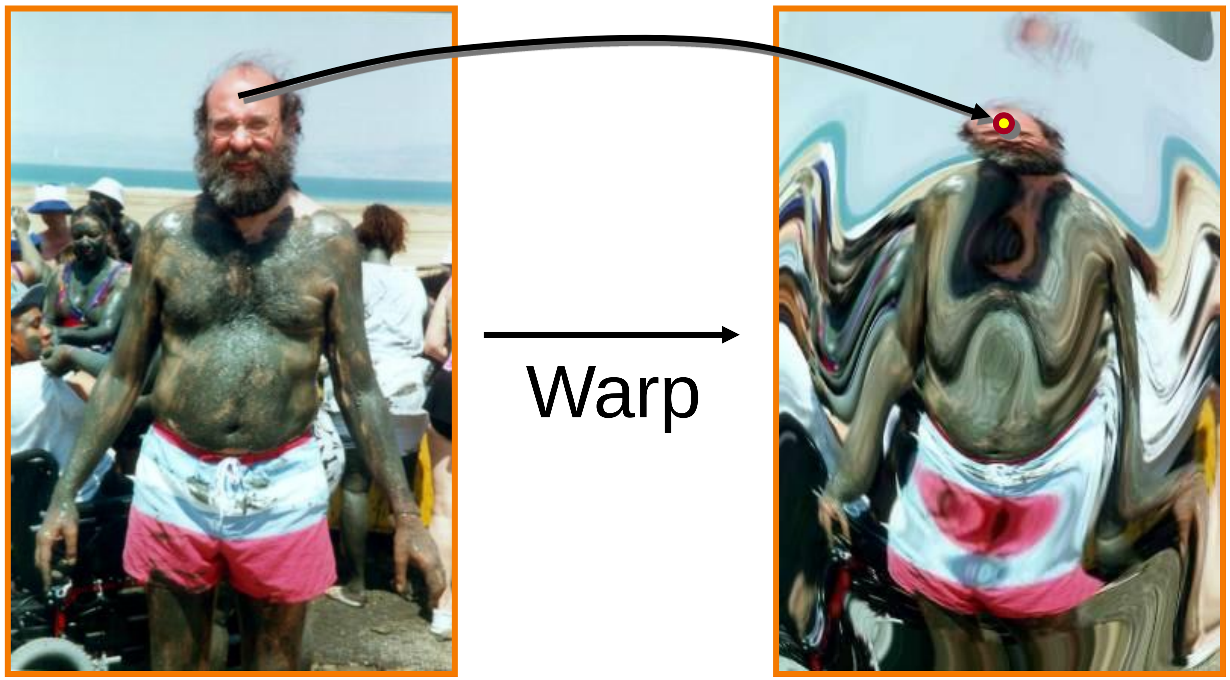
Source image

Warp

Destination image

Image Transformation

- Issues:
  - 1) Specifying where every pixel goes (mapping)

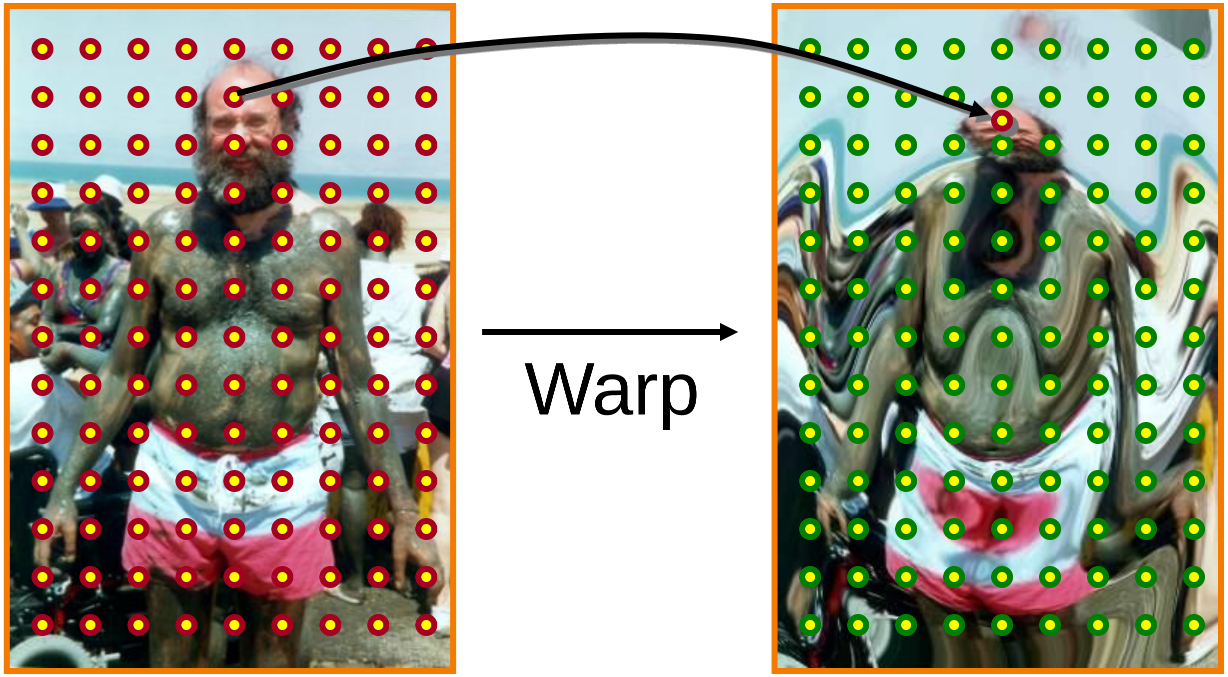


Source image

Destination image

Image Transformation

- Issues:
  1) Specifying where every pixel goes (mapping)
  2) Computing colors at destination pixels (resampling)

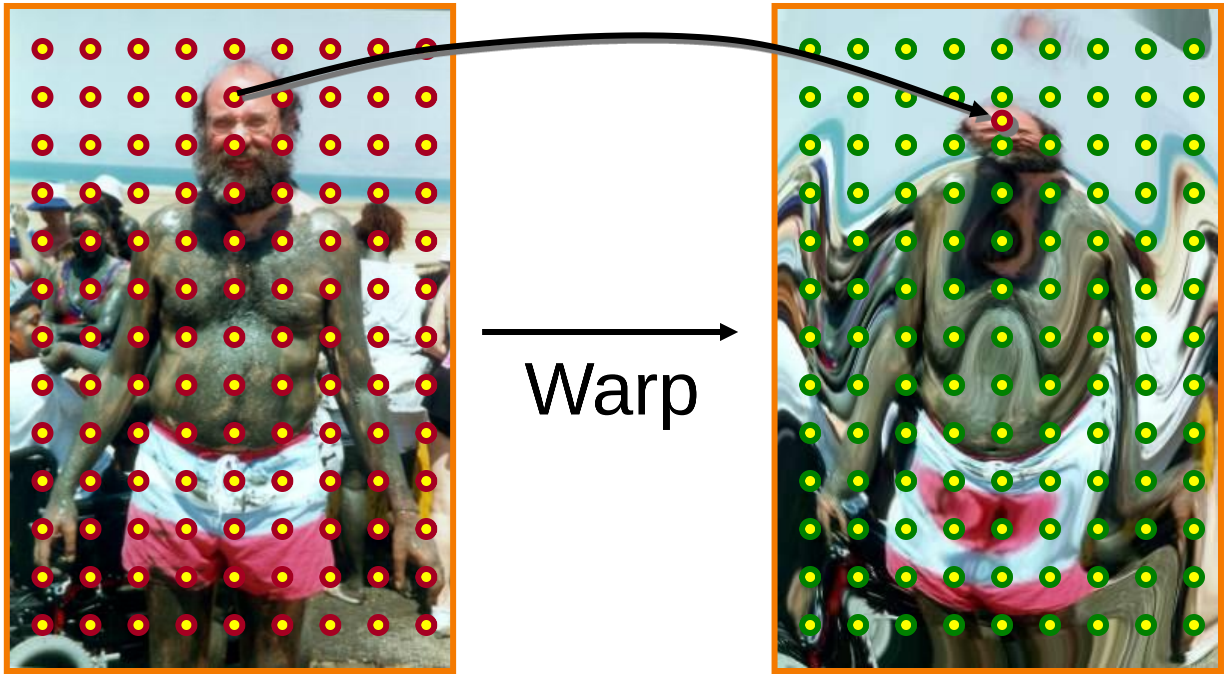


Source image

Destination image

Image Transformation

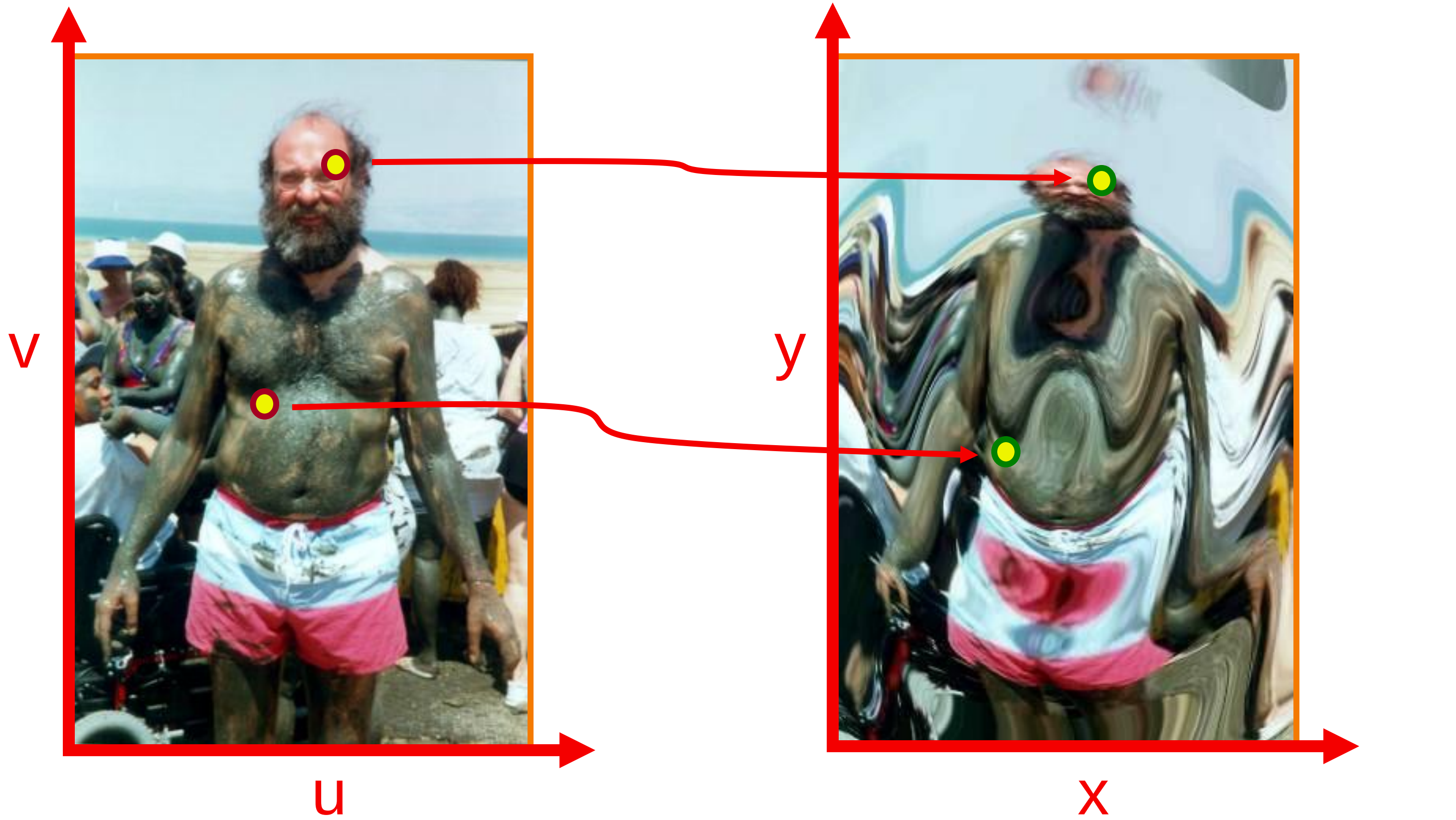
- Issues:
  1) Specifying where every pixel goes (mapping)
  2) Computing colors at destination pixels (resampling)



Source image Destination image

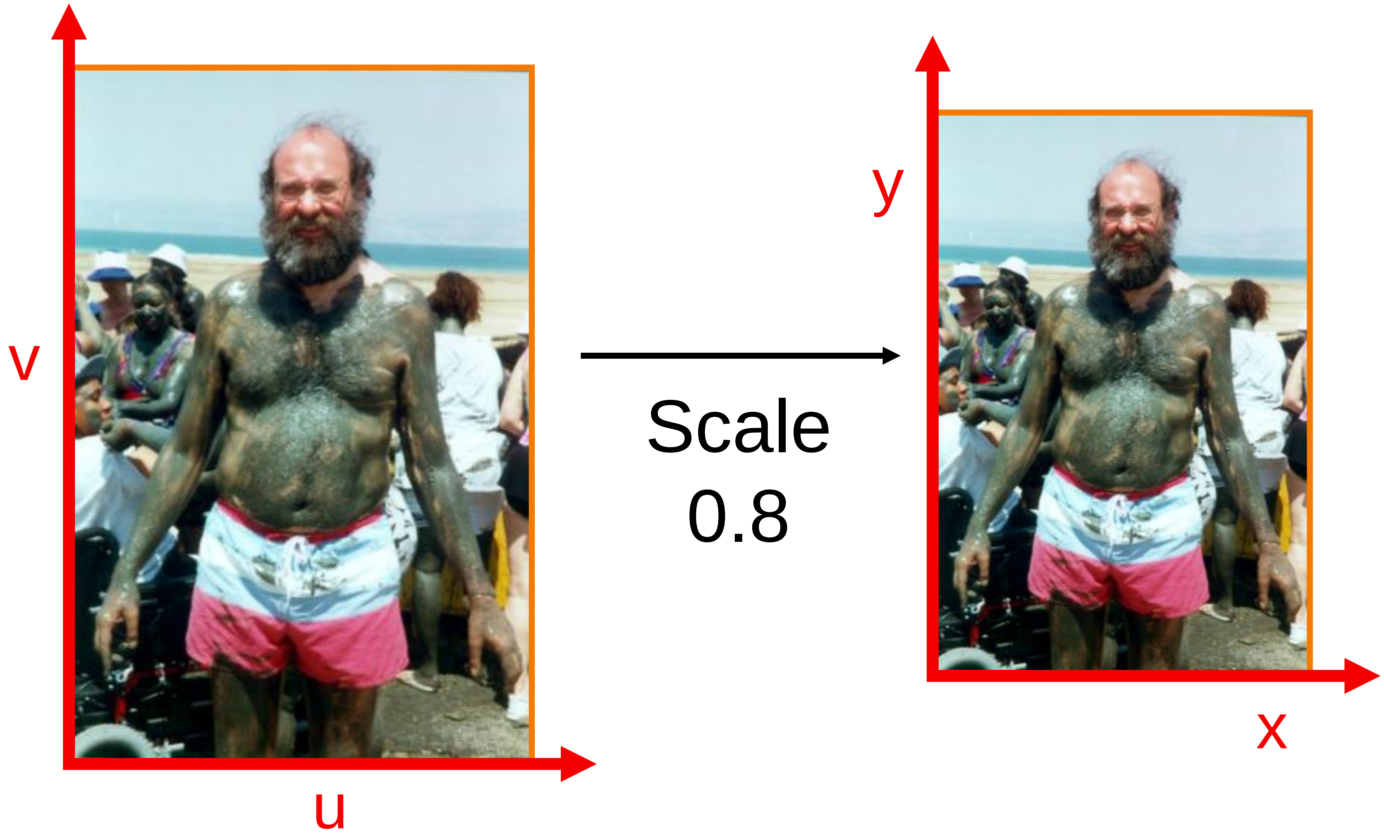
Mapping

- Define transformation
  - Describe the destination (x,y) for every source (u,v)



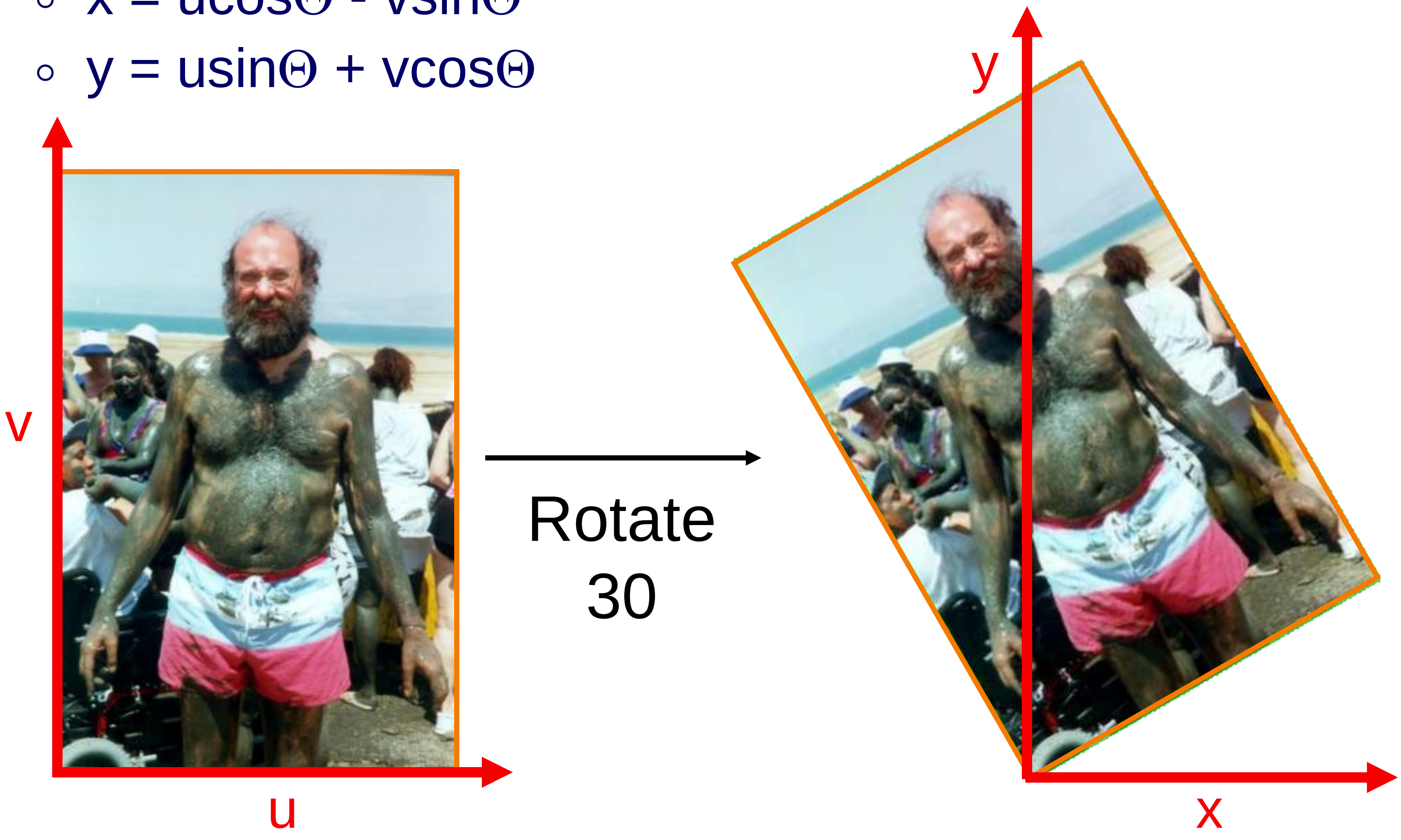
Parametric Mappings

- Scale by *factor*:
  - x = *factor* * u
  - y = *factor* * v



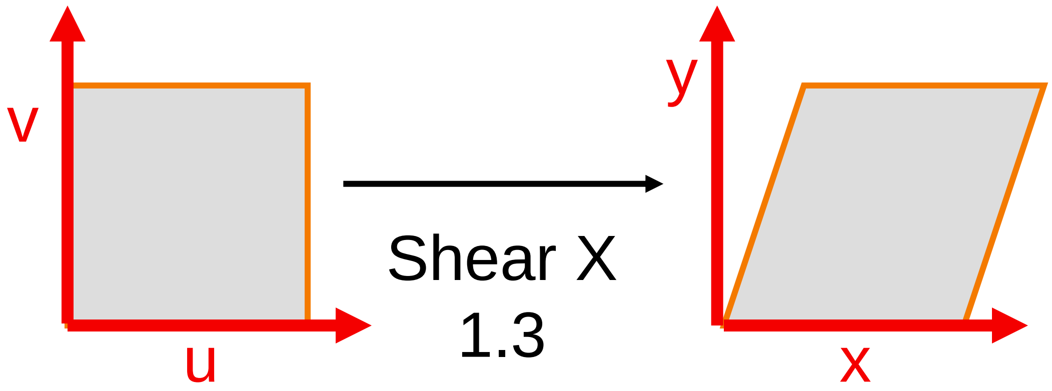
Parametric Mappings

- Rotate by Θ degrees:
  - $x = u\cos\Theta - v\sin\Theta$
  - $y = u\sin\Theta + v\cos\Theta$

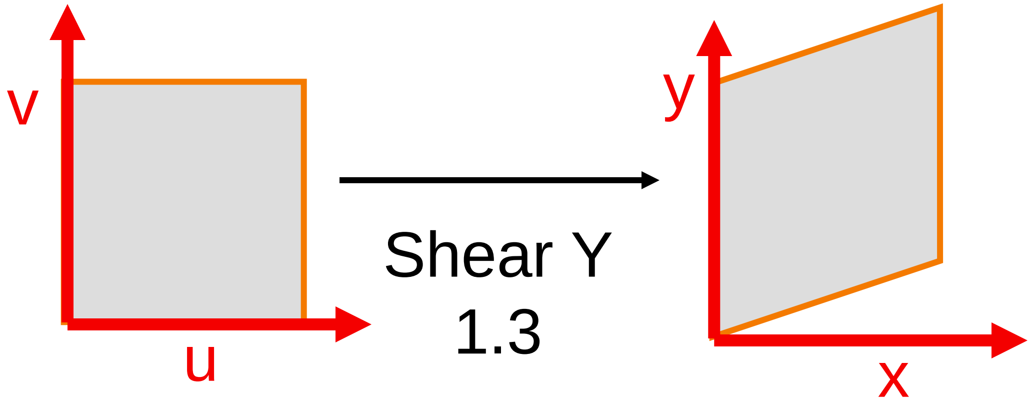


Parametric Mappings

- Shear in X by *factor:*
  - x = u + *factor* * v
  - y = v

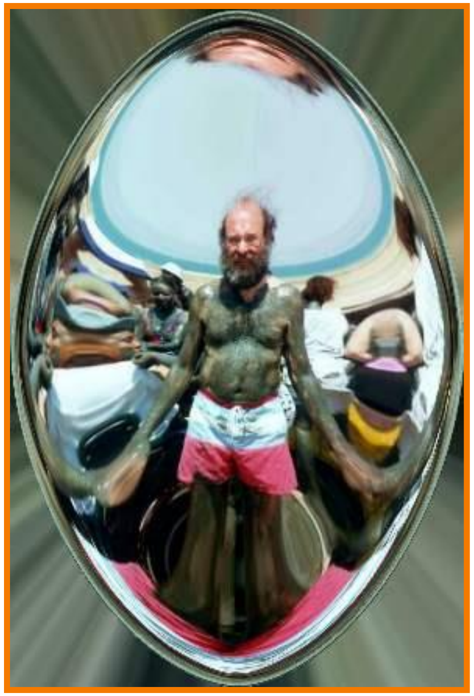


- Shear in Y by *factor:*
  - x = u
  - y = v + *factor* * u



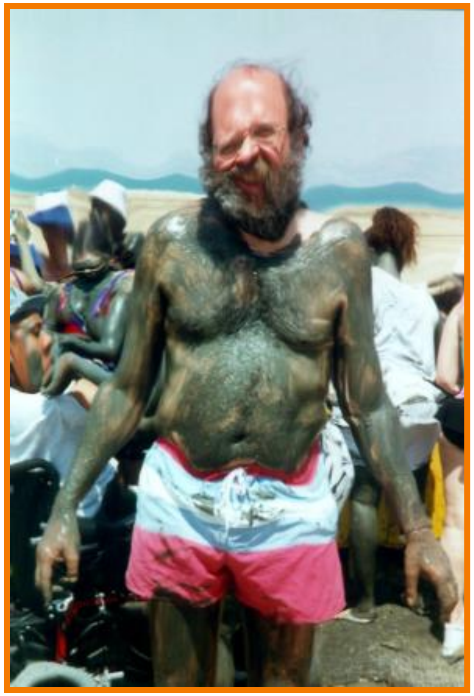
Other Parametric Mappings

- Any function of u and v:
  - $x = f_x(u,v)$
  - $y = f_y(u,v)$



Fish-eye

“Swirl”



“Rain”

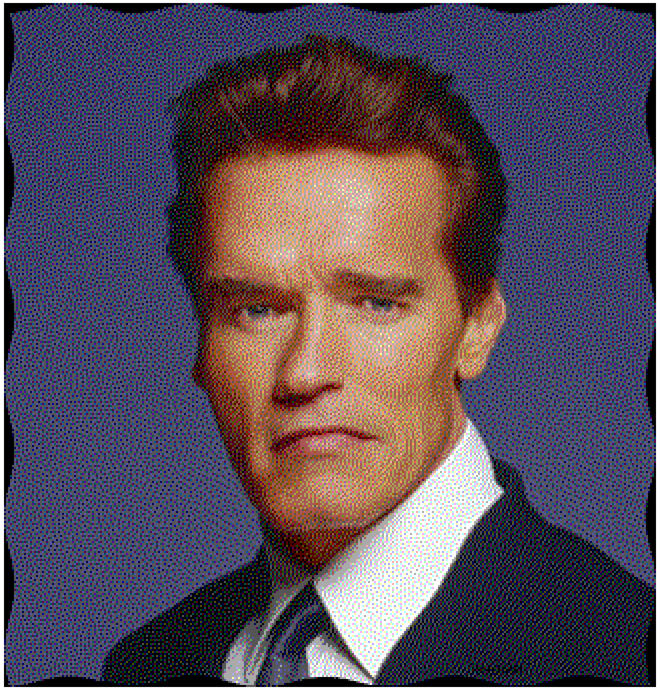
COS426 Examples

Aditya Bhaskara

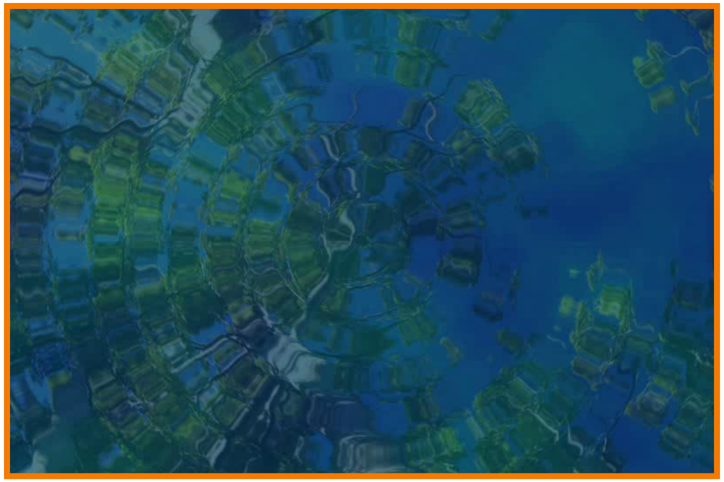
Wei Xiang

More COS426 Examples

Sid Kapur



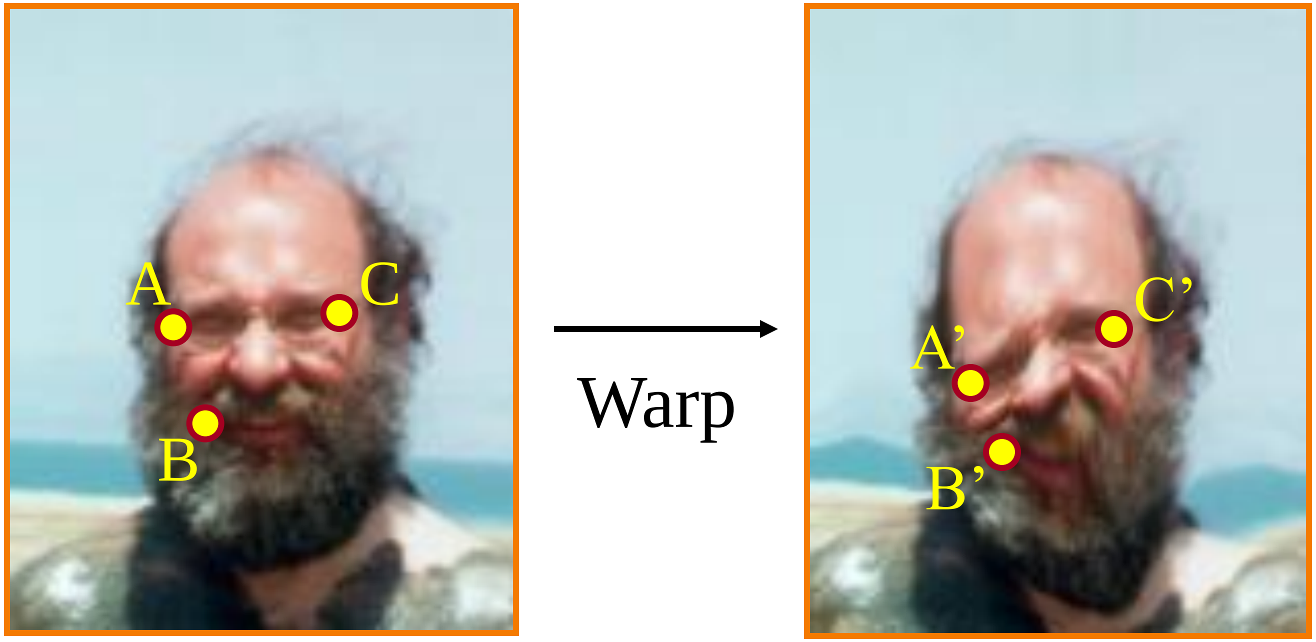
Eirik Bakke



Michael Oranato

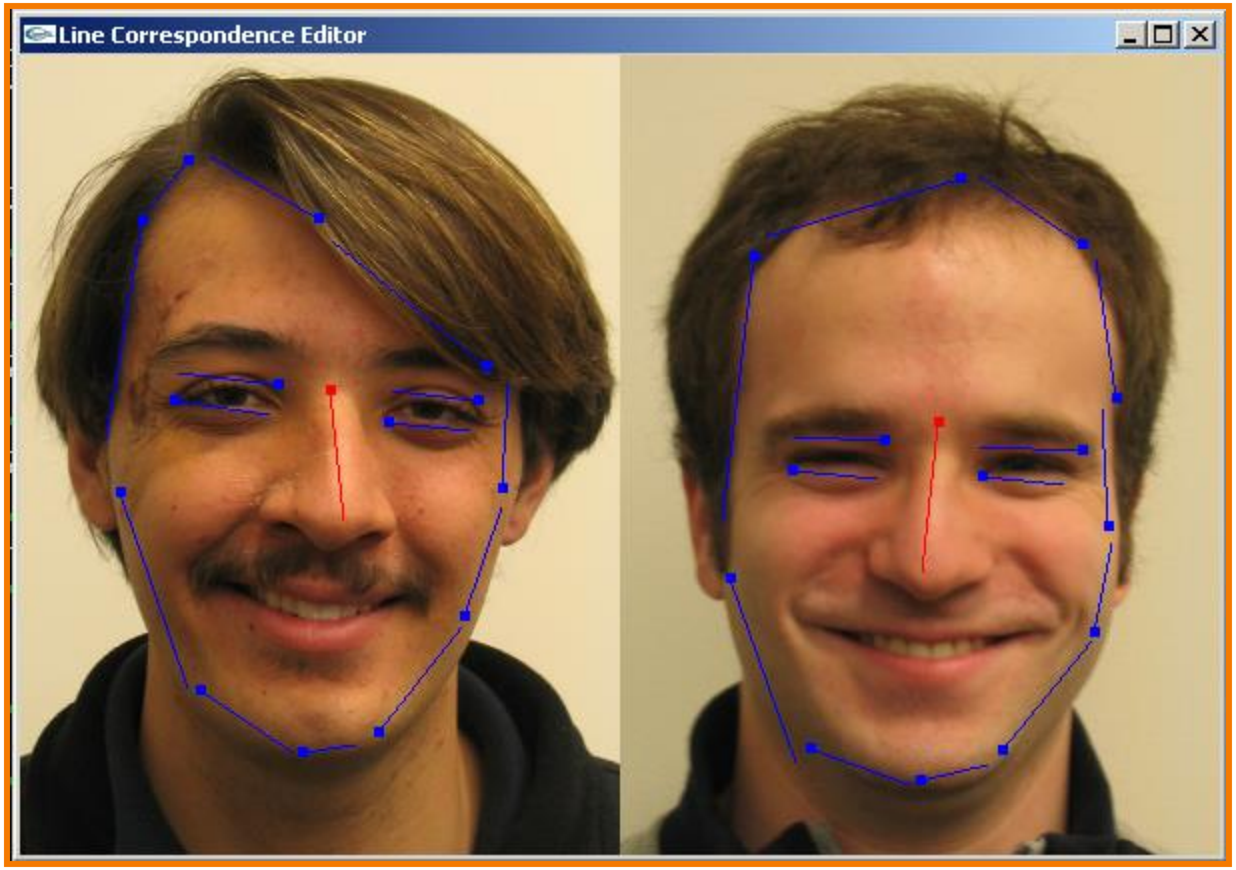
Point Correspondence Mappings

- Mappings implied by correspondences:
  - A ↔ A’
  - B ↔ B’
  - C ↔ C’



Line Correspondence Mappings

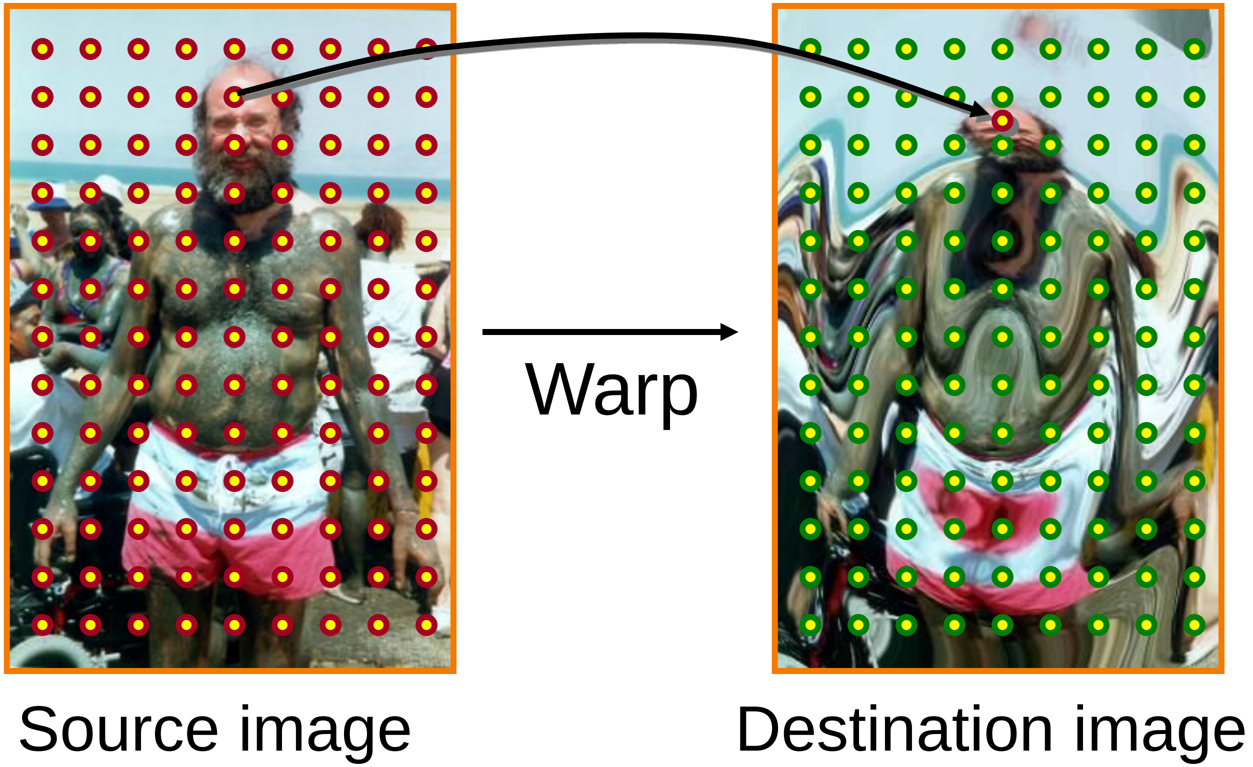
- Beier & Neeley use pairs of lines to specify warps



Beier & Neeley
SIGGRAPH 92

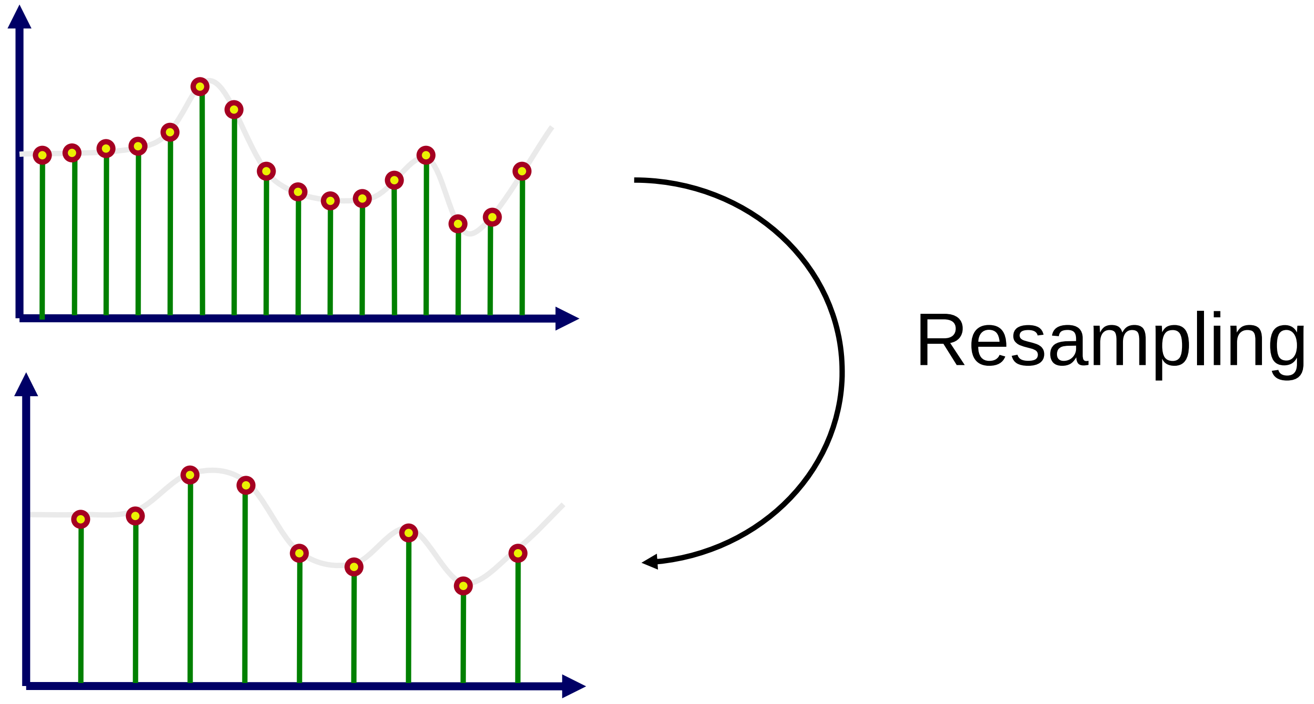
Image Transformation

- Issues:
  1) Specifying where every pixel goes (mapping)
  2) Computing colors at destination pixels (resampling)



Resampling

Simple example: scaling resolution = resampling



Resampling

Example: scaling resolution = resampling

Original

Scaled

Resampling

- Naïve resampling can cause visual artifa

Original

Scaled

What is the Problem?

Aliasing

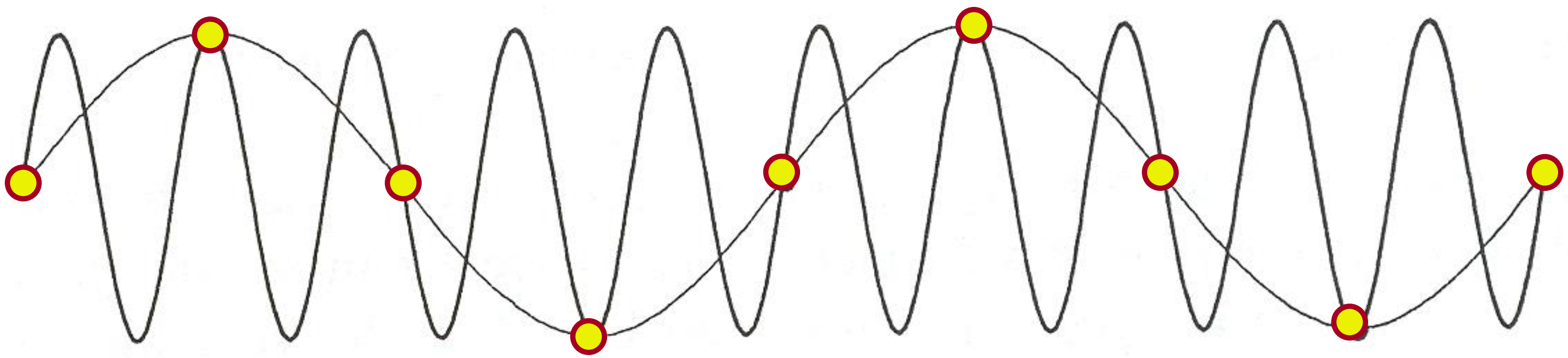


Figure 14.17 FvDFH

Aliasing

Artifacts due to under-sampling

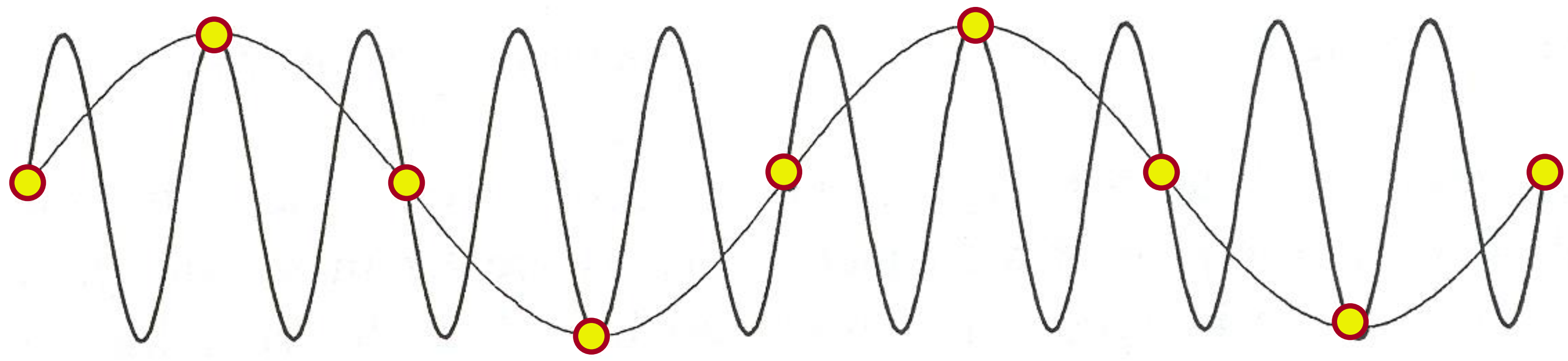
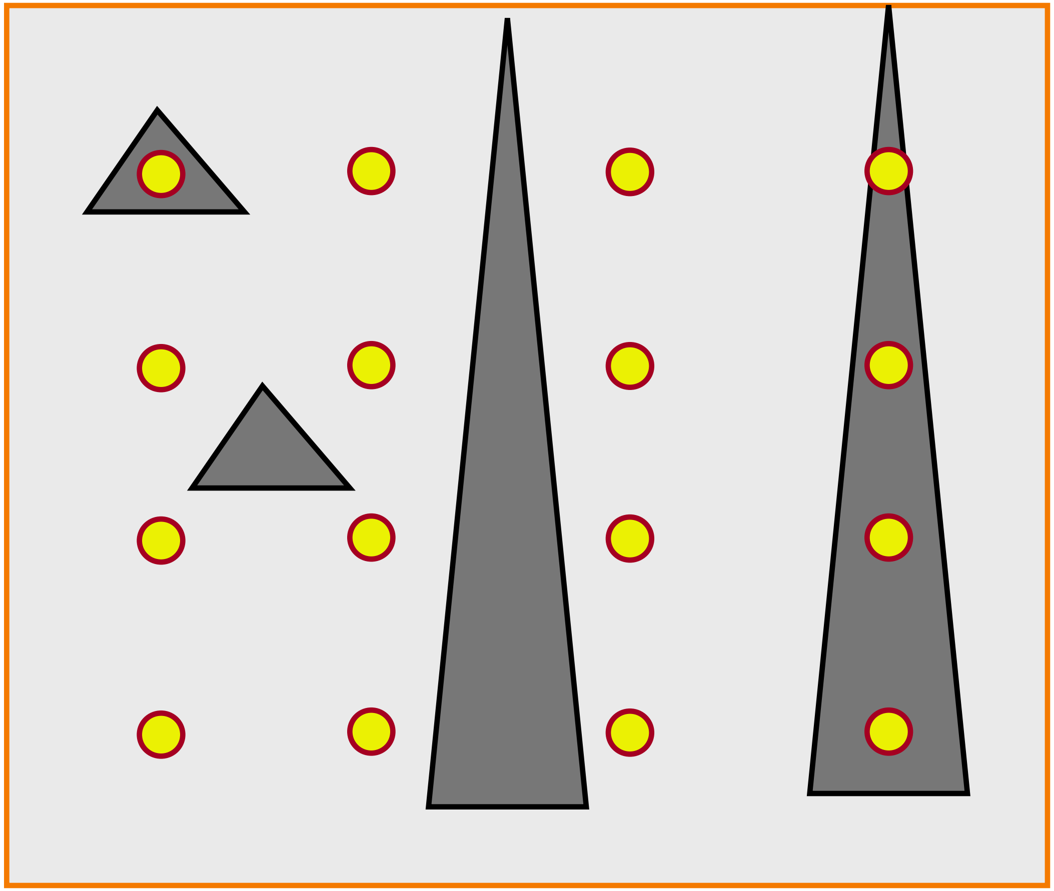


Figure 14.17 FvDFH

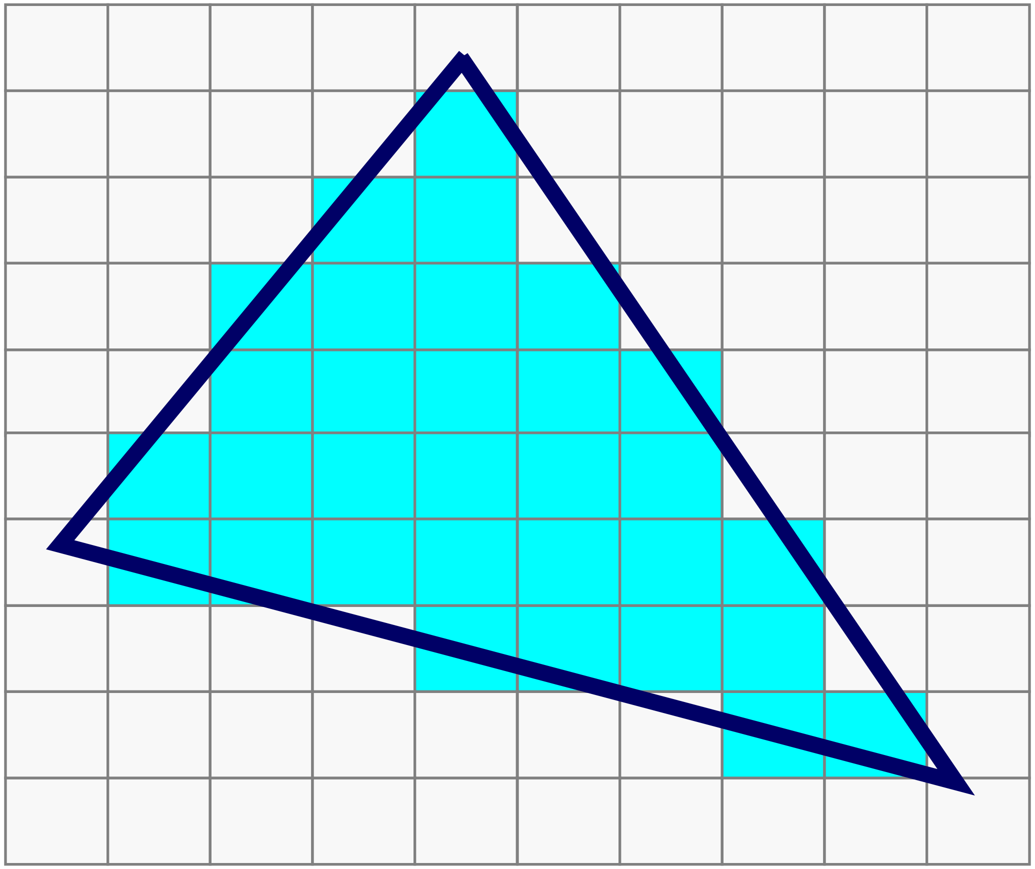
Spatial Aliasing

Artifacts due to under-sampling in x,y



Spatial Aliasing

Artifacts due to under-sampling in x,y

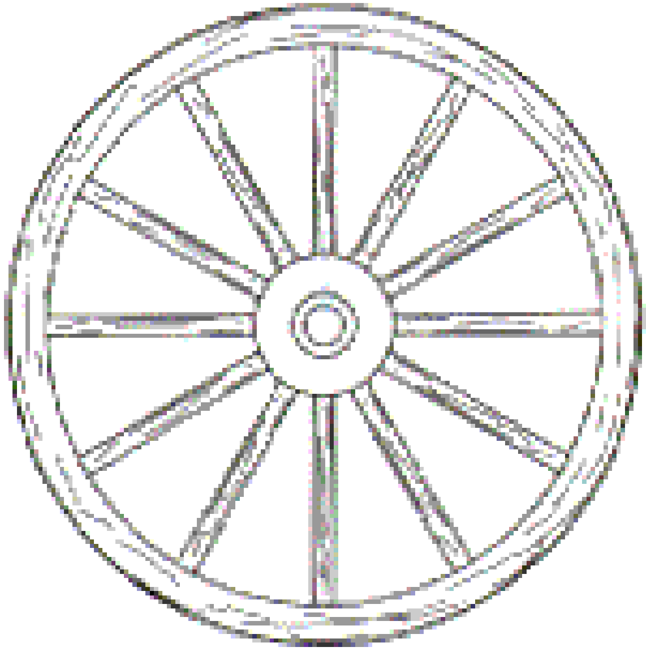


"Jaggies"

Temporal Aliasing

Artifacts due to under-sampling in time

- Strobing
- Flickering



Temporal Aliasing

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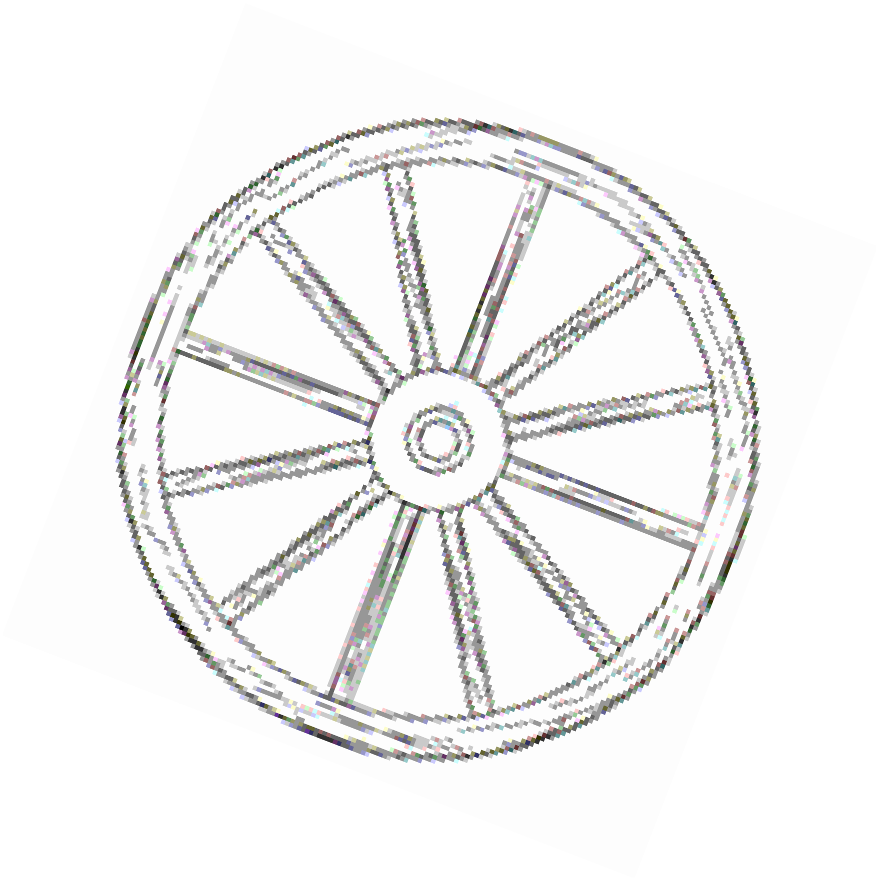

Temporal Aliasing

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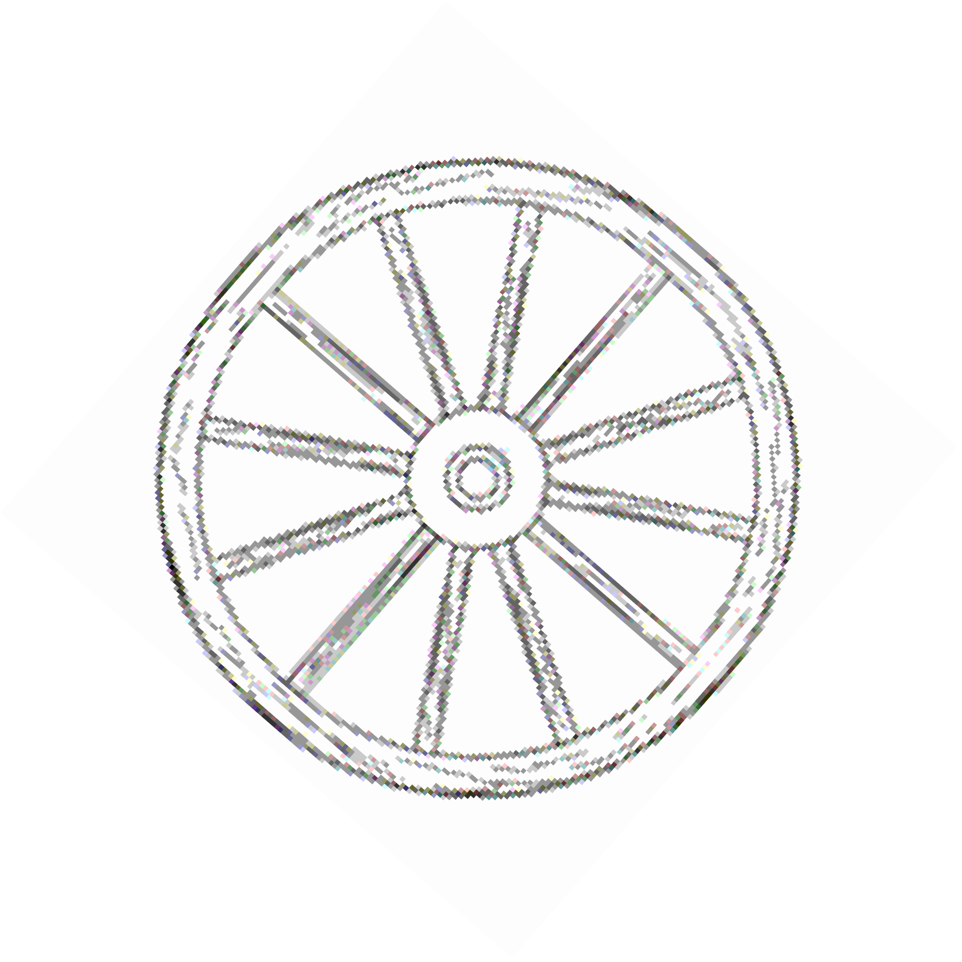

Temporal Aliasing

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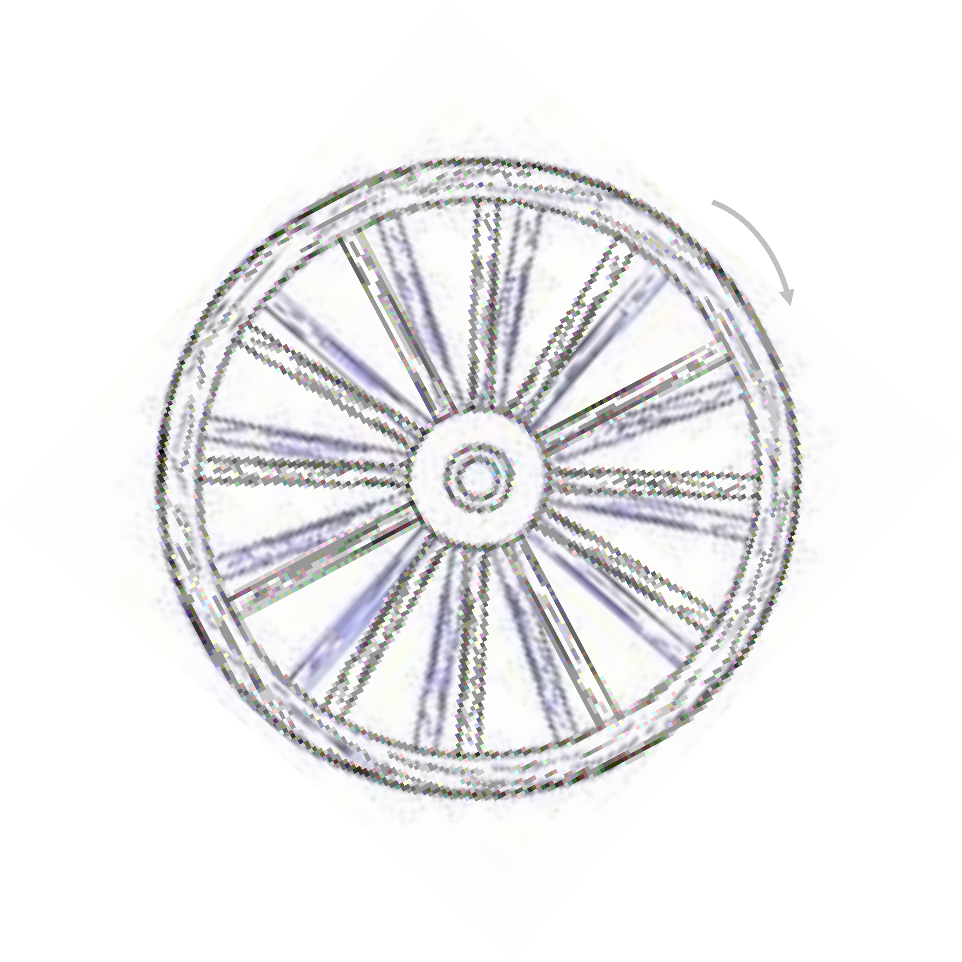
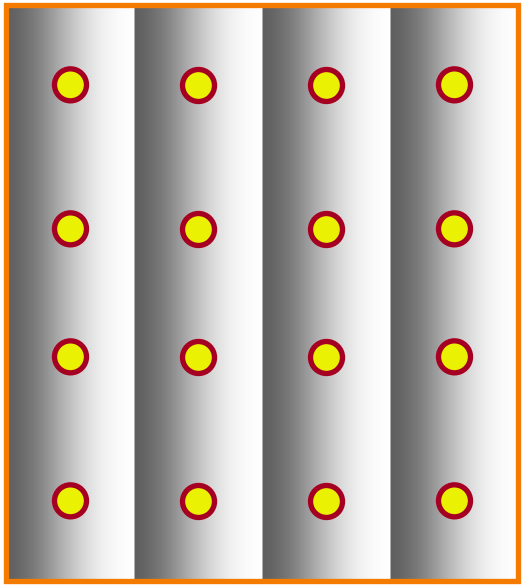
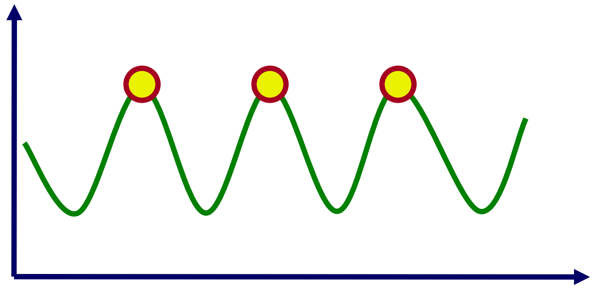


Aliasing

When we under-sample an image,
we can create visual artifacts where
high frequencies masquerade as low ones

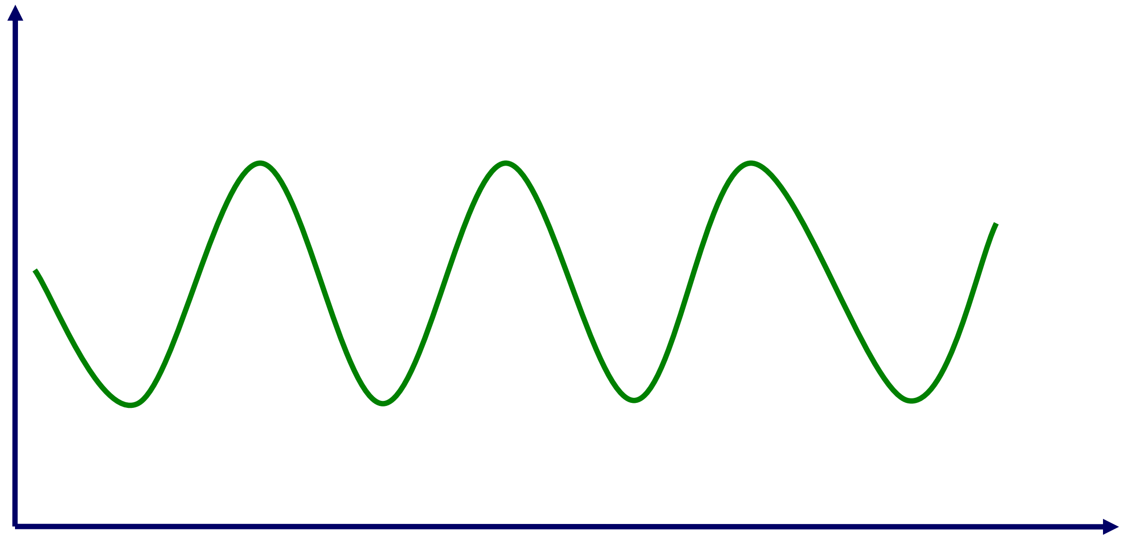




Sampling Theory

How many samples are enough to avoid aliasing?

- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?



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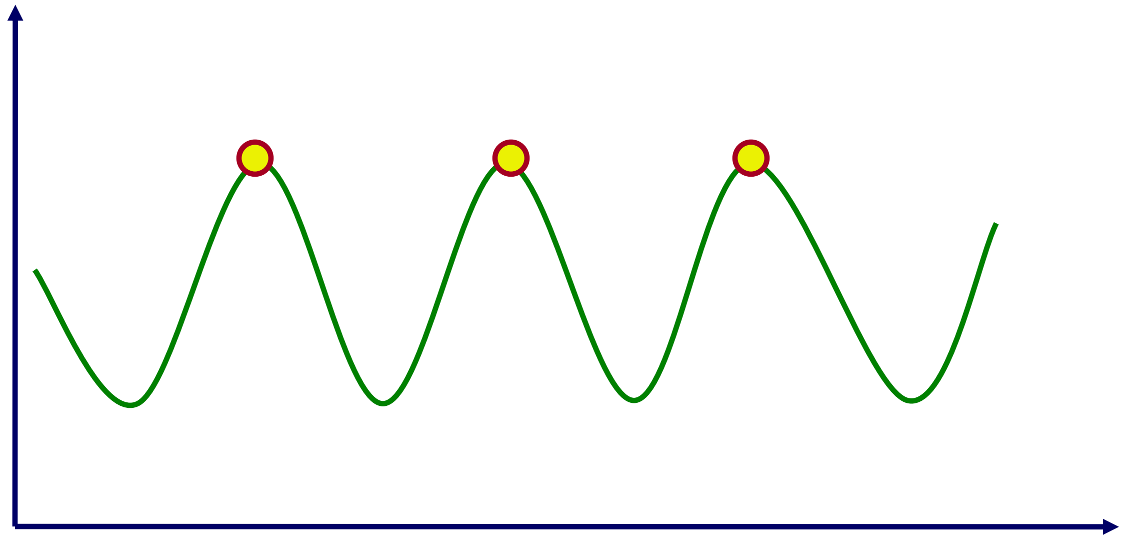

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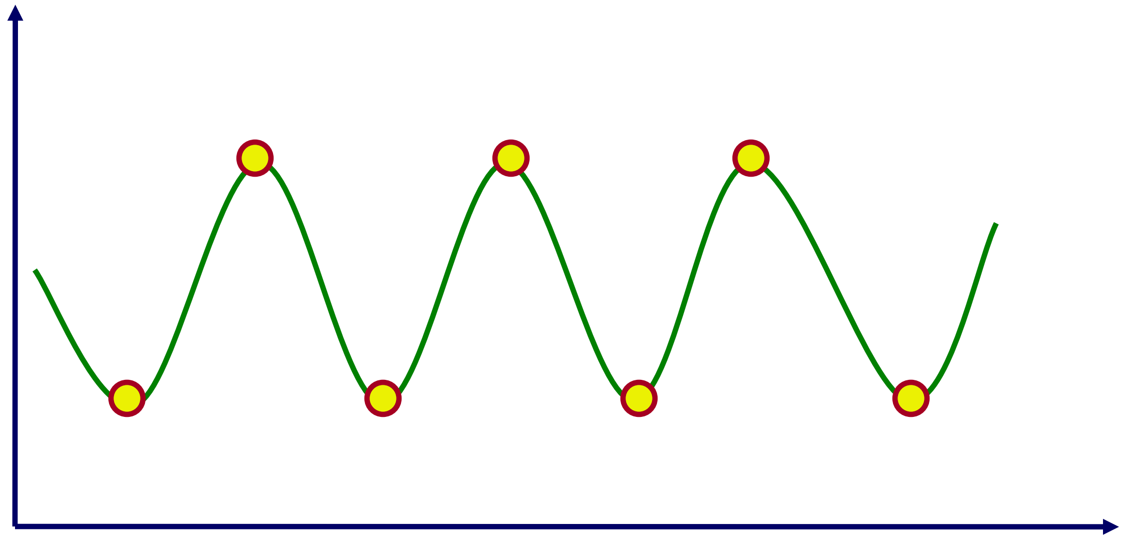

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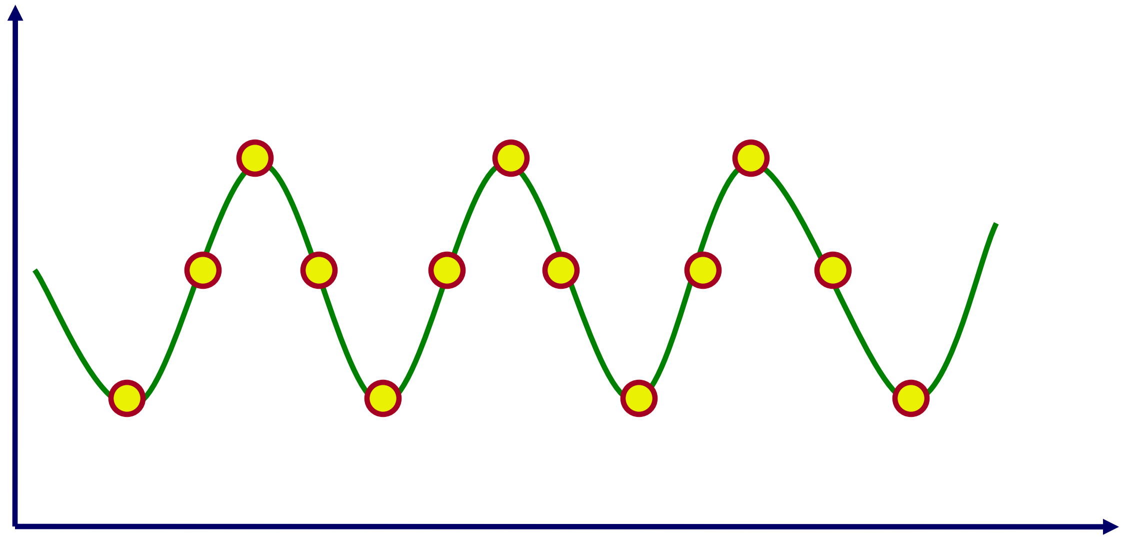

Sampling Theory

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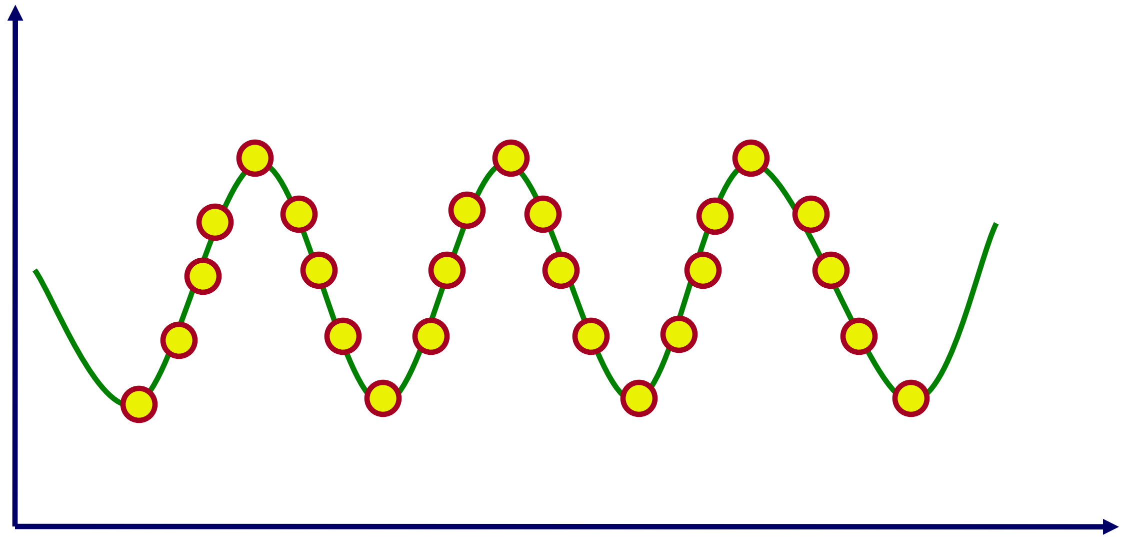
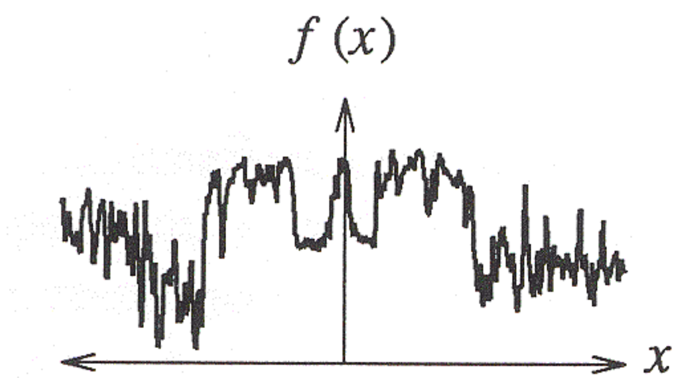
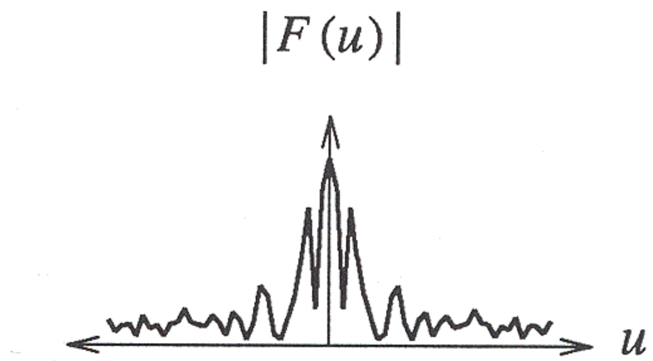


Spectral Analysis

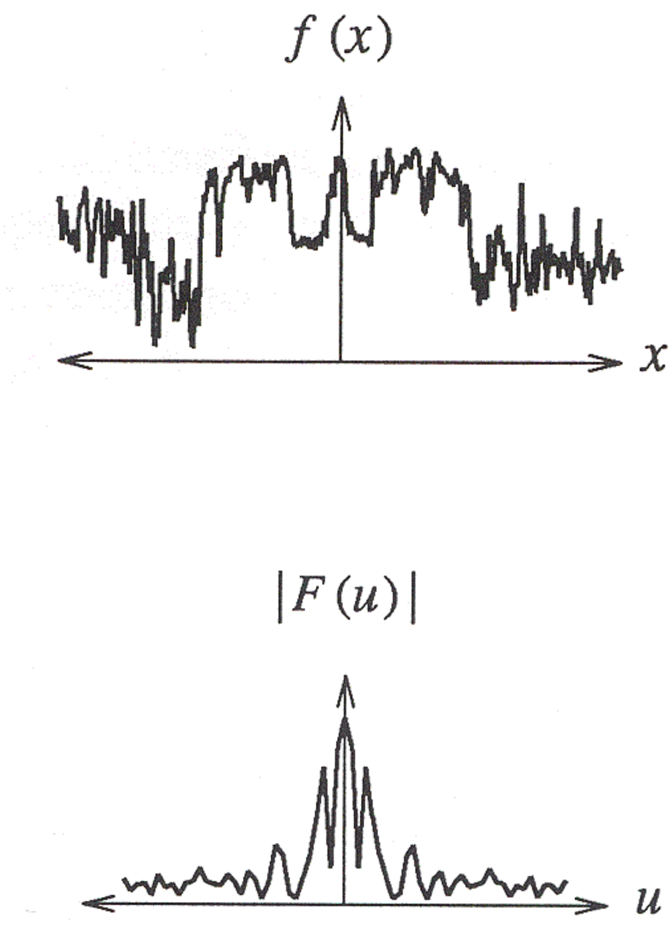
- Spatial domain:
  - Function: f(x)
  - Filtering: convolution
- Frequency domain:
  - Function: F(u)
  - Filtering: multiplication





Any signal can be written as a sum of periodic functions.

Fourier Transform



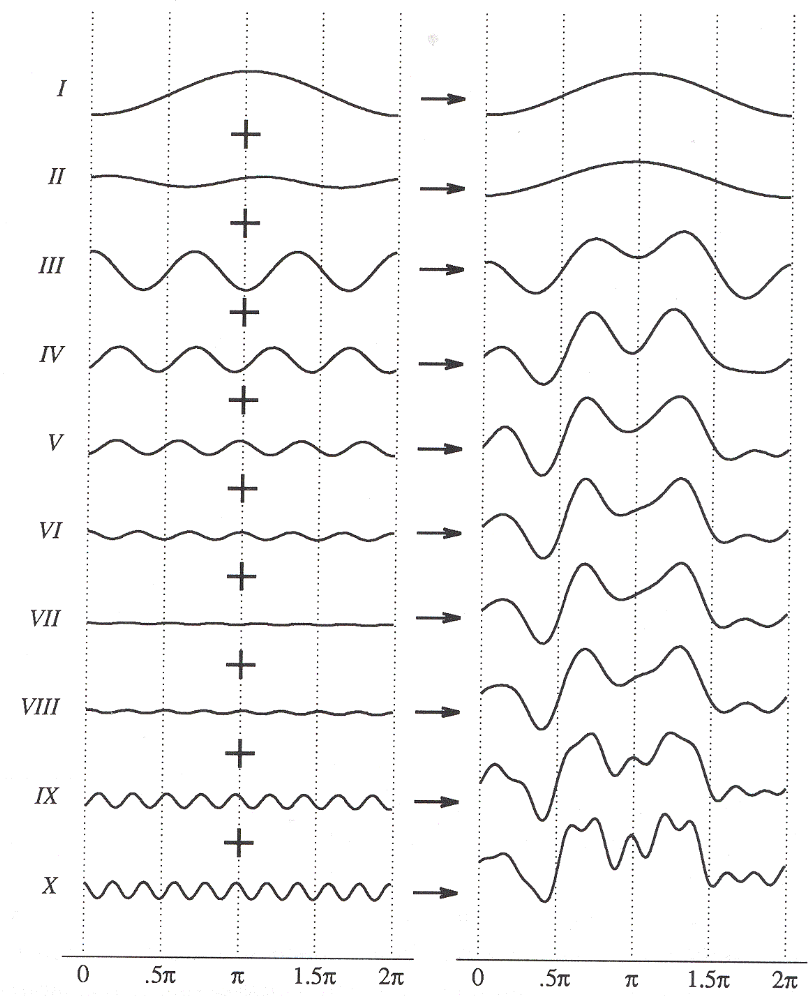


Figure 2.6 Wolberg

Fourier Transform

- Fourier transform:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xu} dx$$

- Inverse Fourier transform:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+i2\pi ux} du$$

Sampling Theorem

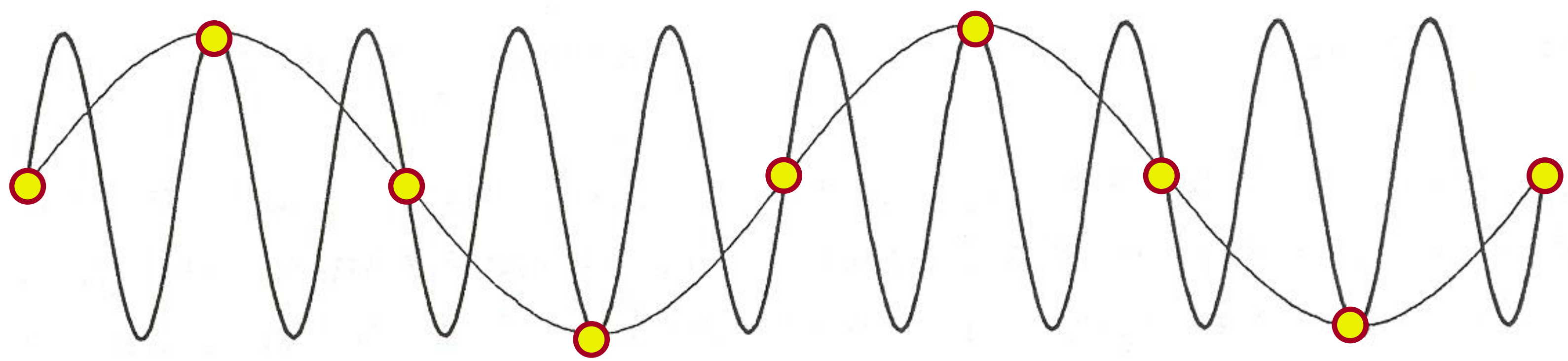
- A signal can be reconstructed from its samples, iff the original signal has no content >= 1/2 the sampling frequency - Shannon
- The minimum sampling rate for bandlimited function is called the "Nyquist rate"

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.

Sampling Theorem

- A signal can be reconstructed from its samples, iff the original signal has no content >= 1/2 the sampling frequency - Shannon

Aliasing will occur if the signal is under-sampled



Under-sampling

Figure 14.17 FvDFH

Sampling and Reconstruction

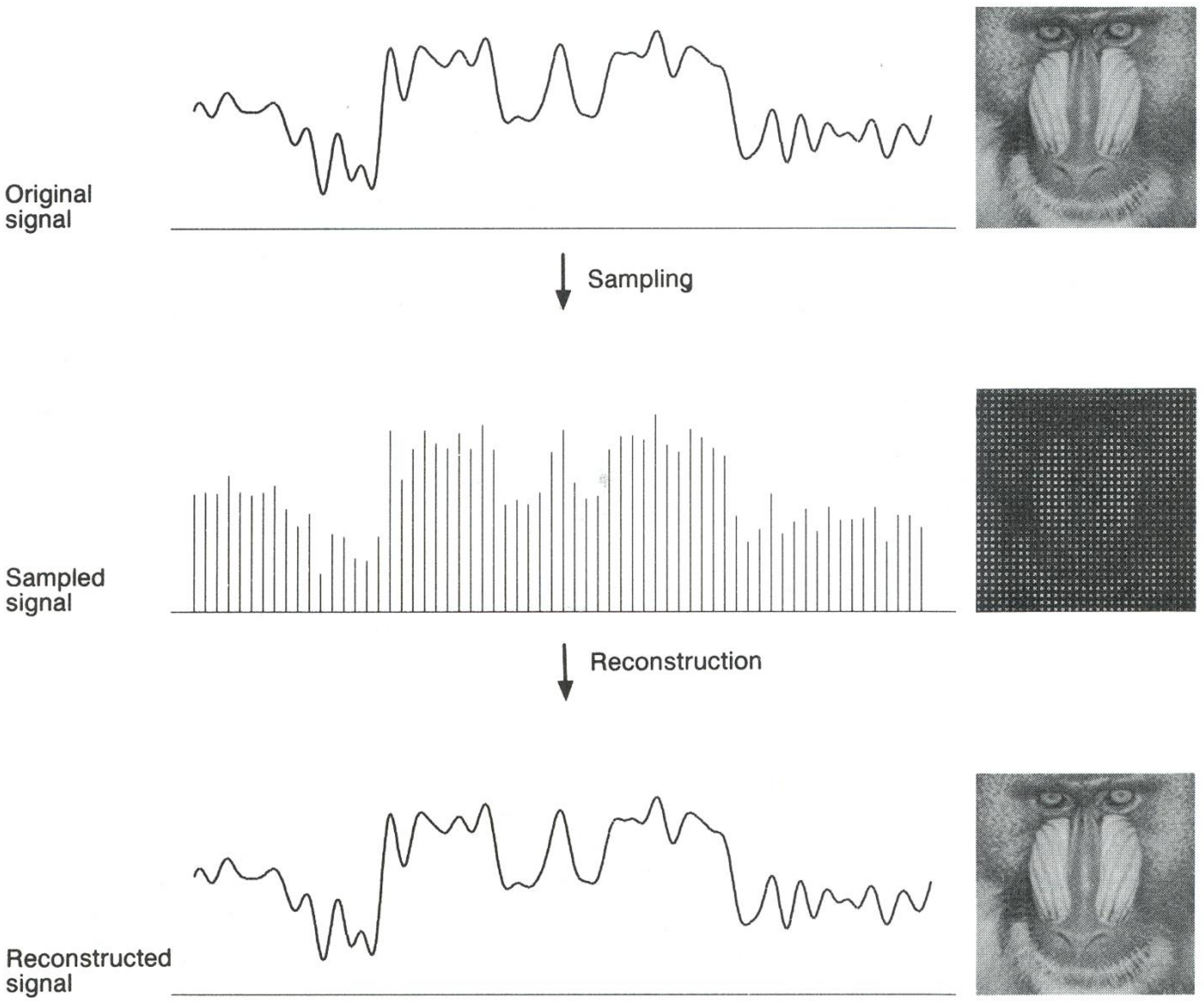
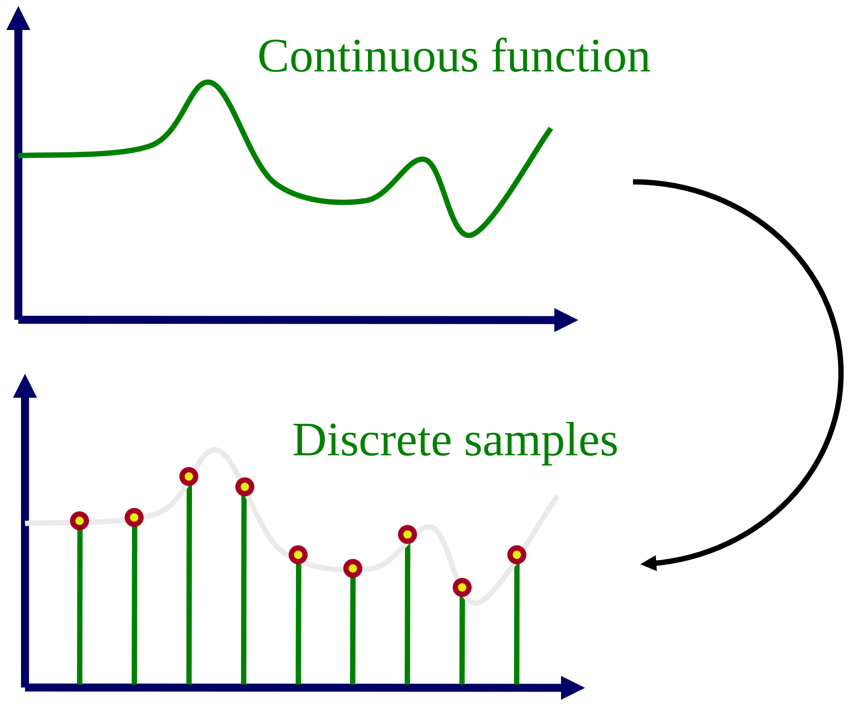


Figure 19.9 FvDFH

Sampling and Reconstruction



Sampling

Sampling and Reconstruction

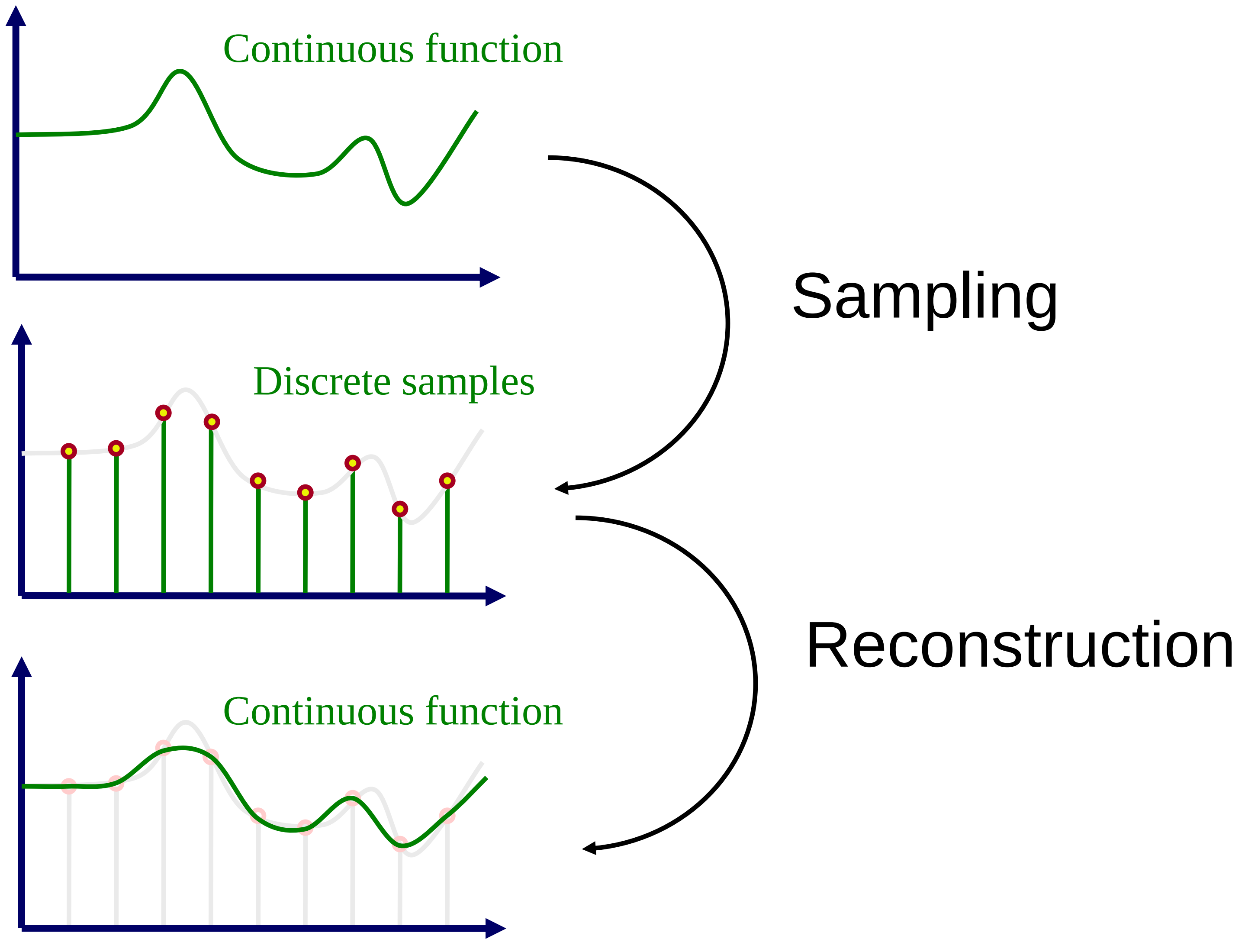
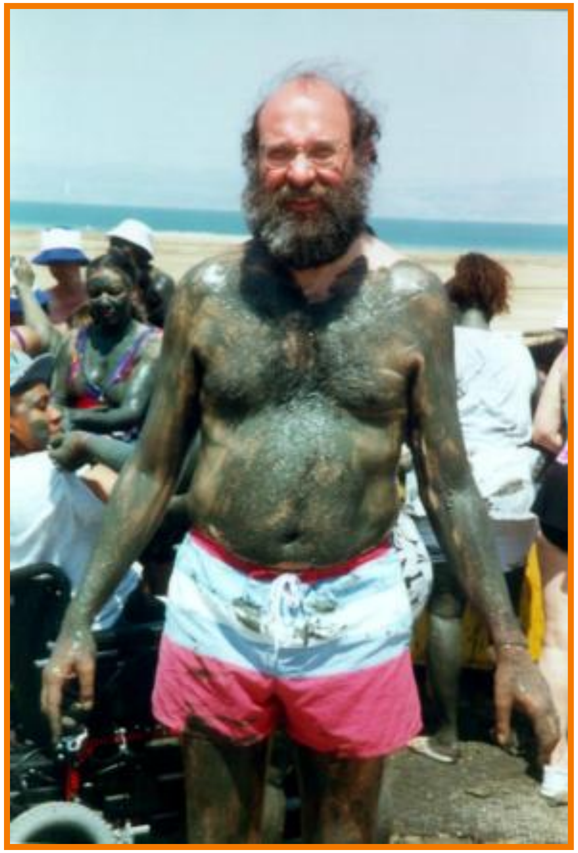


Image Processing

OK … but how does that affect image processing?



Source image

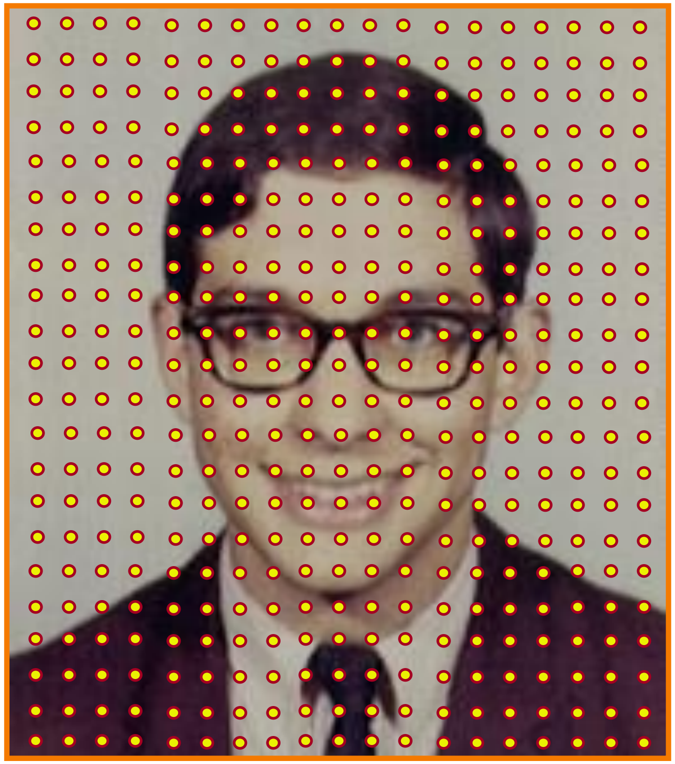
Warp

Destination image

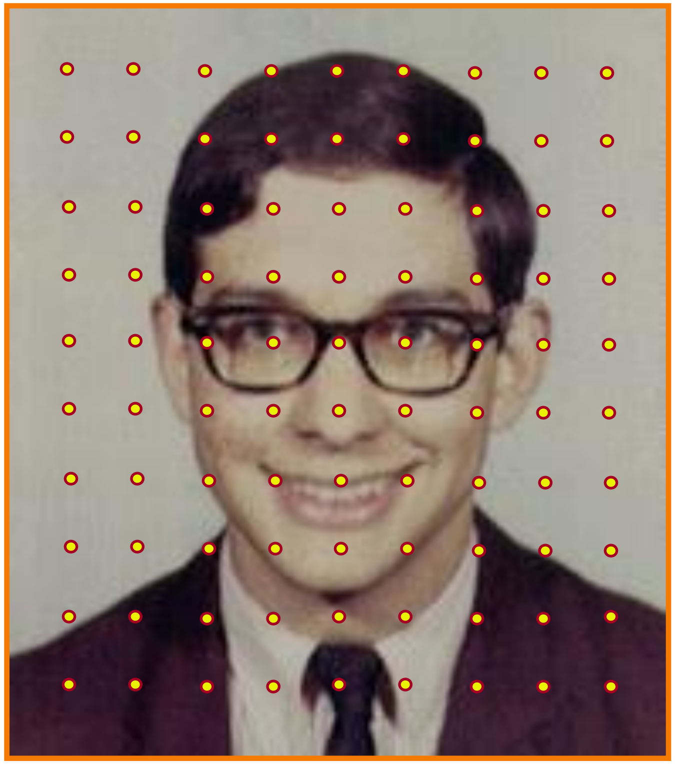
Image Processing

Image processing often requires resampling

- Must band-limit before resampling to avoid aliasing



Original image



1/4 resolution

Ideal Image Processing

↓ Real world

Sample

↓ Discrete samples (pixels)

Reconstruct

↓ Reconstructed function

Transform

↓ Transformed function

Filter

↓ Bandlimited function

Sample

↓ Discrete samples (pixels)

Reconstruct

↓ Display

Ideal Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

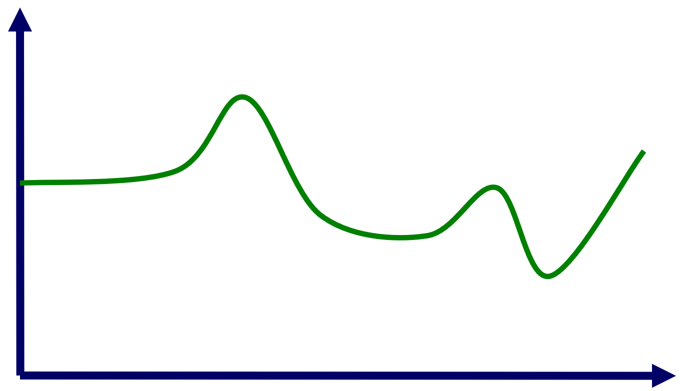
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Continuous Function

Ideal Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

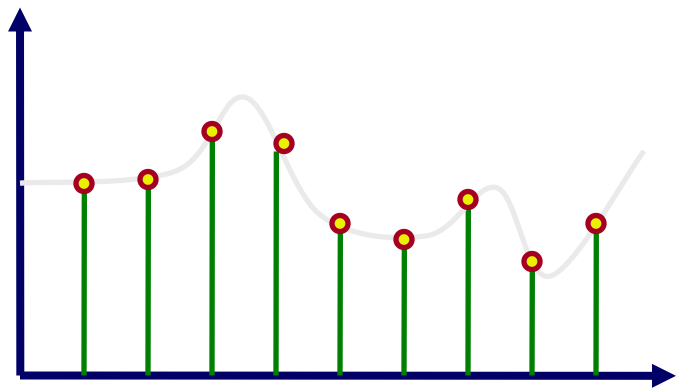
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Discrete Samples

Ideal Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

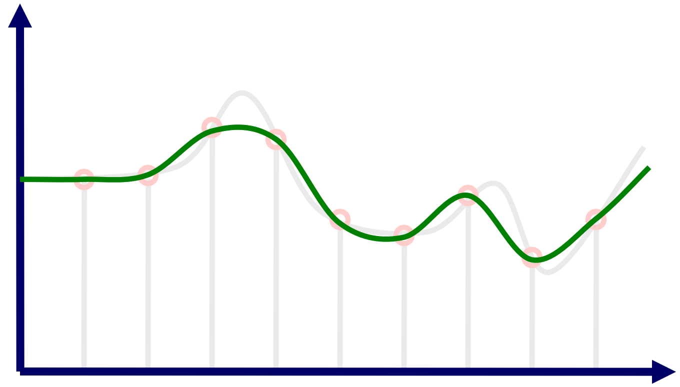
Bandlimited function

Sample

Discrete samples (pixels)

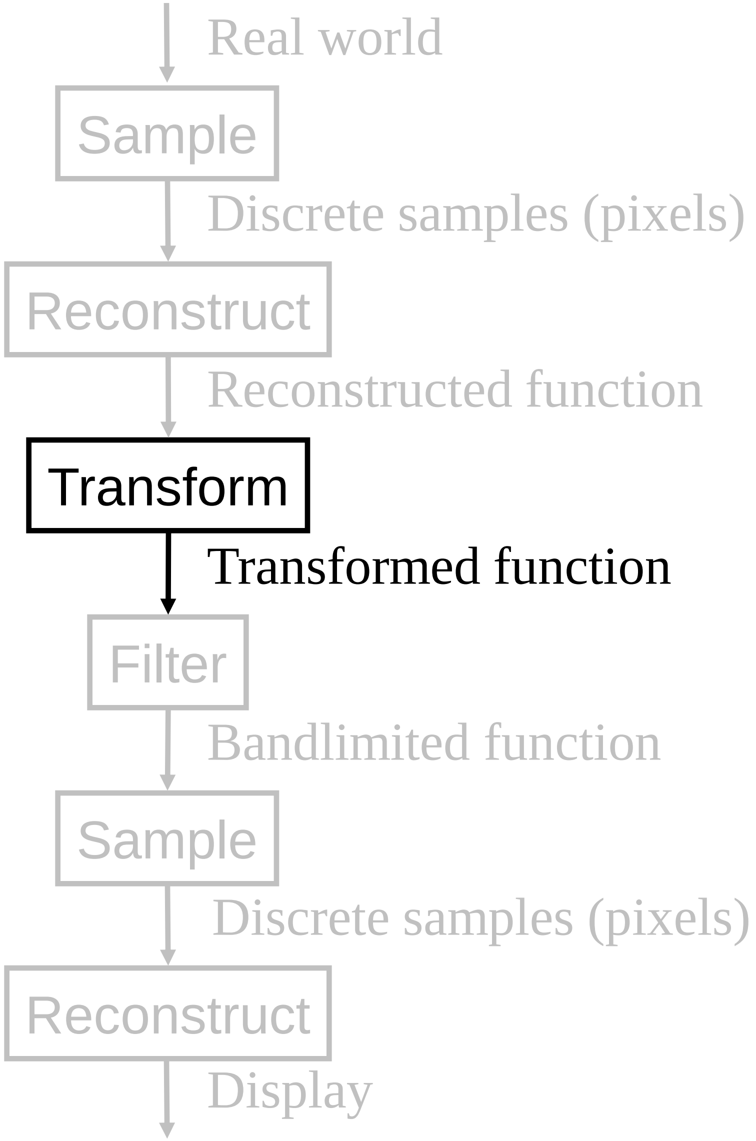
Reconstruct

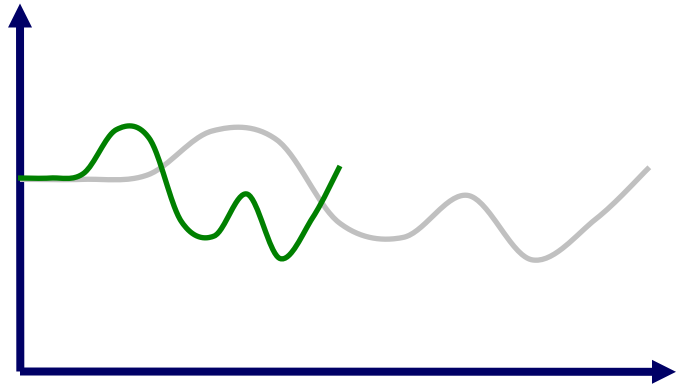
Display



Reconstructed Function

Ideal Image Processing





Transformed Function

Ideal Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

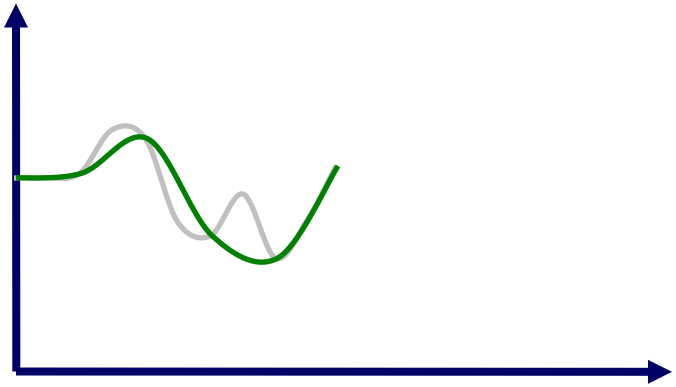
Bandlimited function

Sample

Discrete samples (pixels)

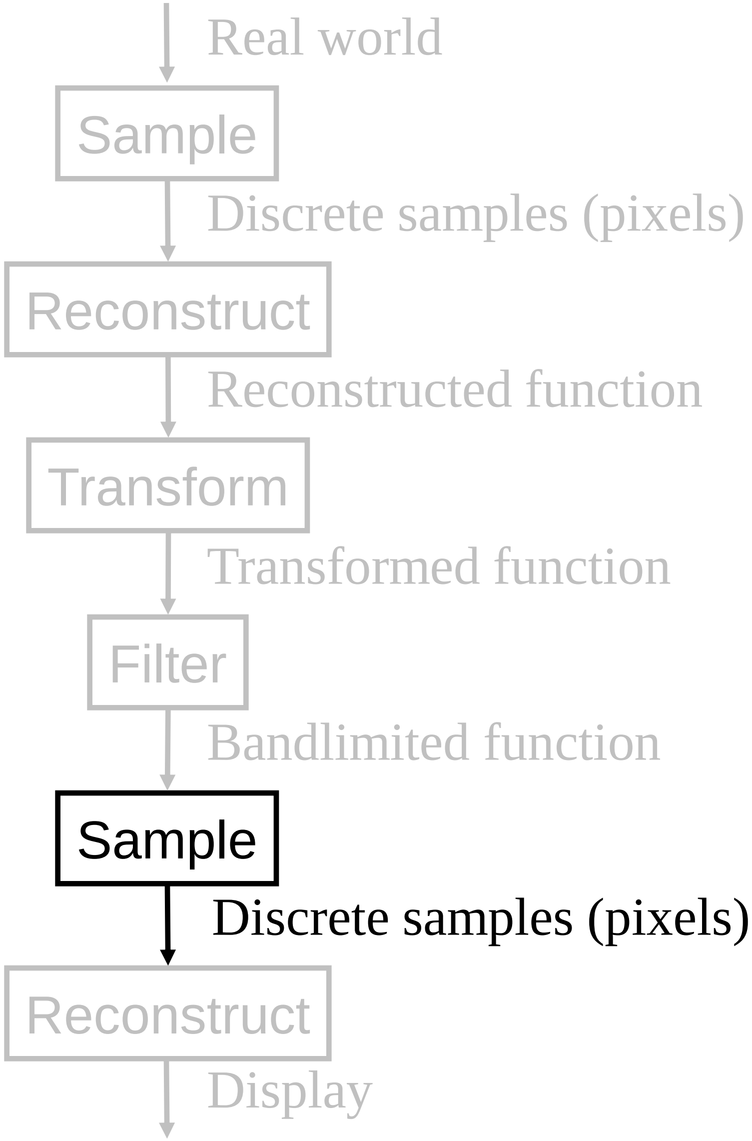
Reconstruct

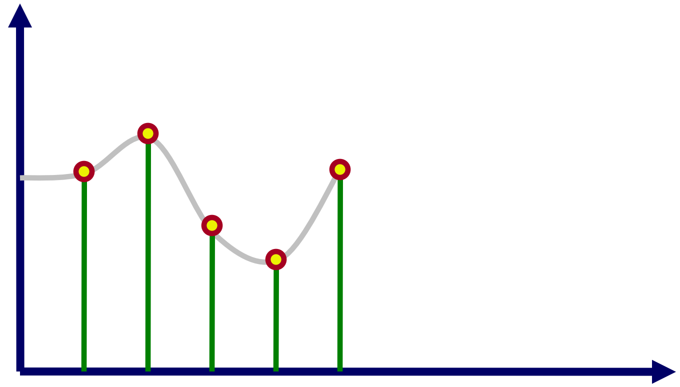
Display



Bandlimited Function

Ideal Image Processing





Discrete samples

Ideal Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

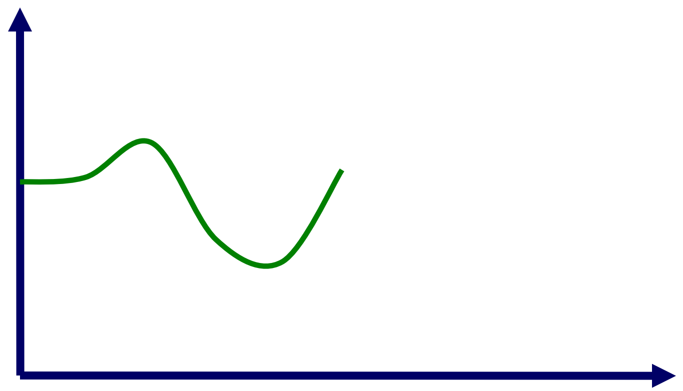
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

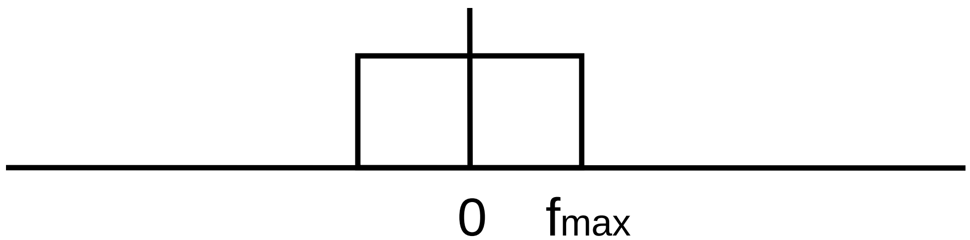
Display



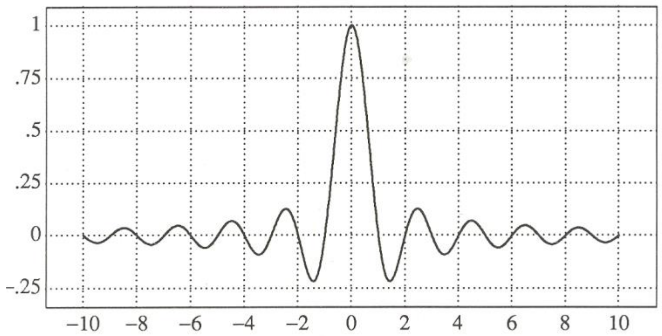
Display

Ideal Bandlimiting Filter

- Frequency domain



- Spatial domain

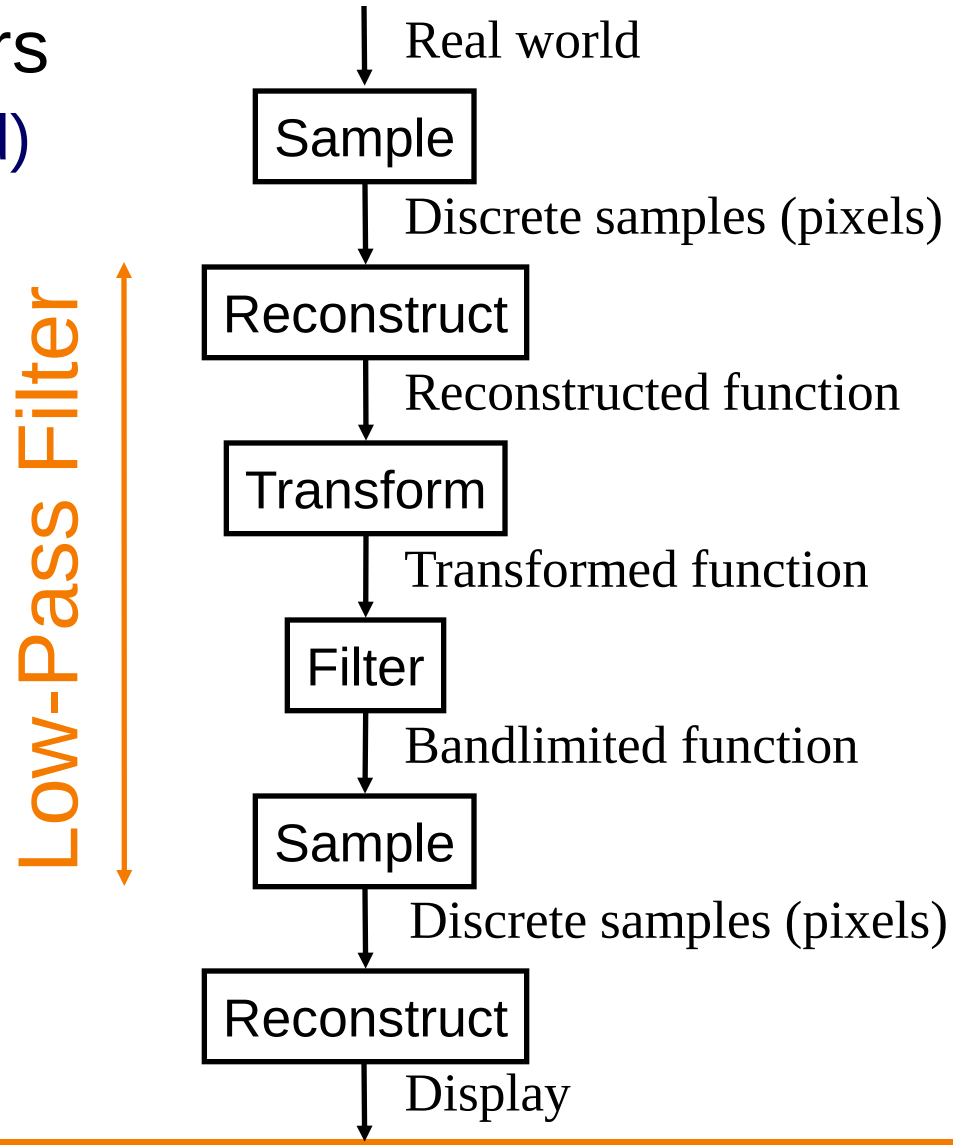


$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

Practical Image Processing

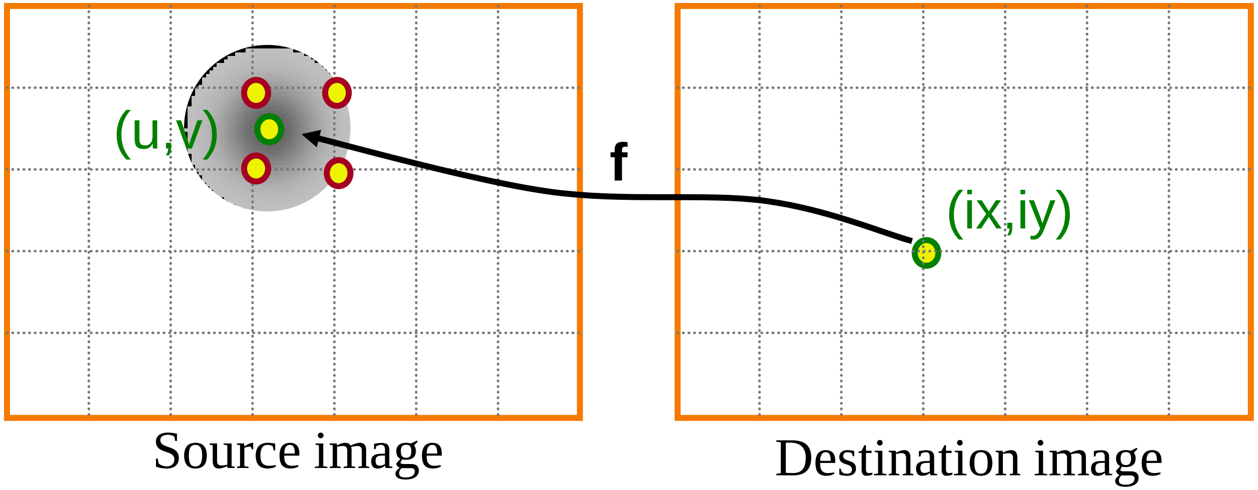
- Finite low-pass filters
  - Point sampling (bad)
  - Box filter
  - Triangle filter
  - Gaussian filter



Practical Image Processing

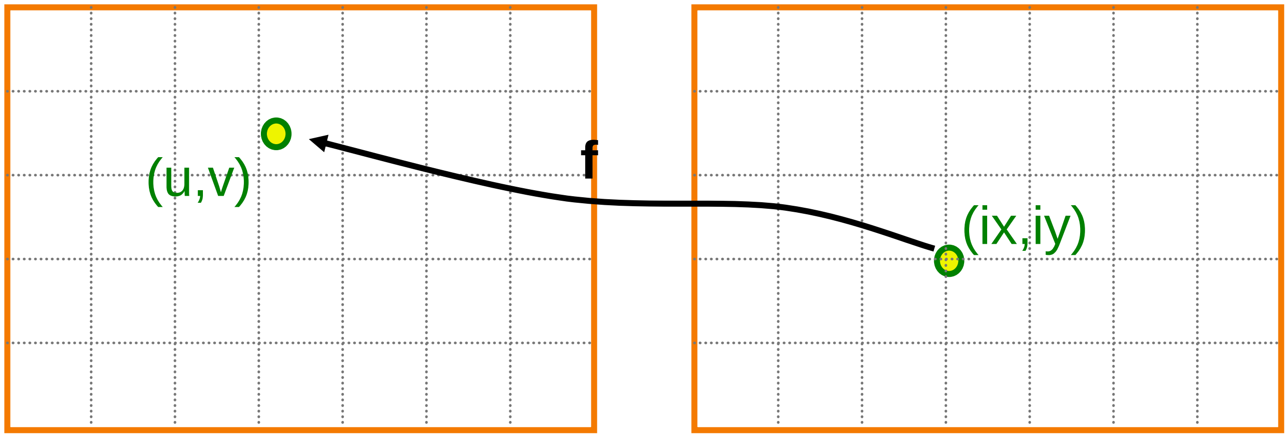
- Reverse mapping:

```
Warp(src, dst) {
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float w ≈ 1 / scale(ix, iy);
      float u = fx^-1(ix,iy);
      float v = fy^-1(ix,iy);
      dst(ix,iy) = Resample(src,u,v,k,w);
    }
  }
}
```



Resampling

- Compute value of 2D function at arbitrary location from given set of samples



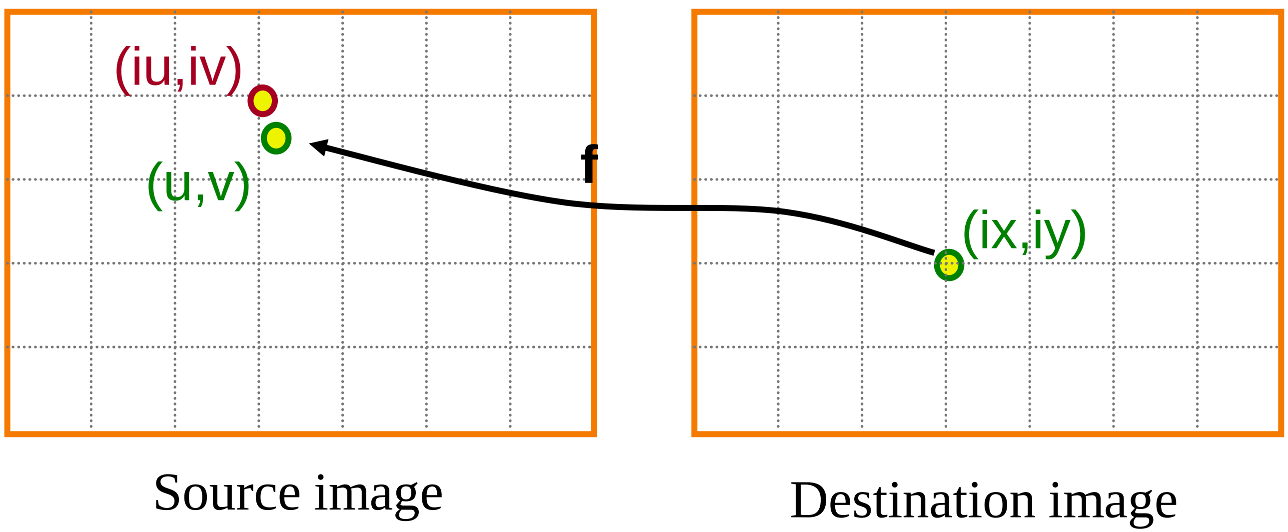
Source image

Destination image

Point Sampling

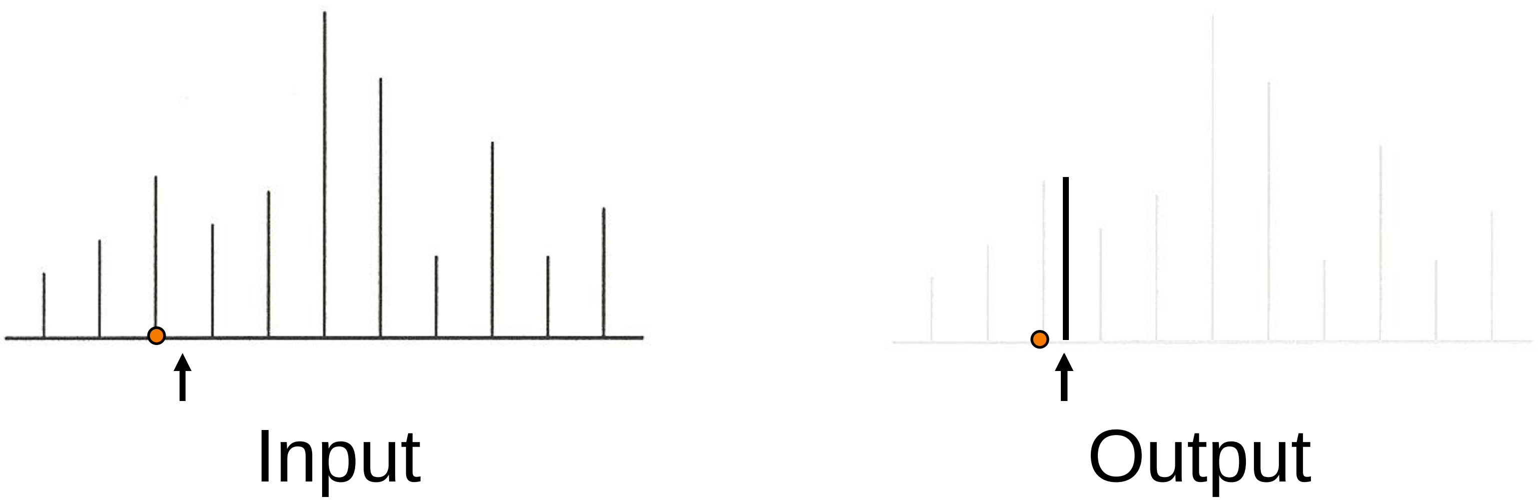
- Possible (poor) resampling implementation:

```
float Resample(src, u, v, k, w) {
   int iu = round(u);
   int iv = round(v);
   return src(iu,iv);
}
```



Point Sampling

- Use nearest sample



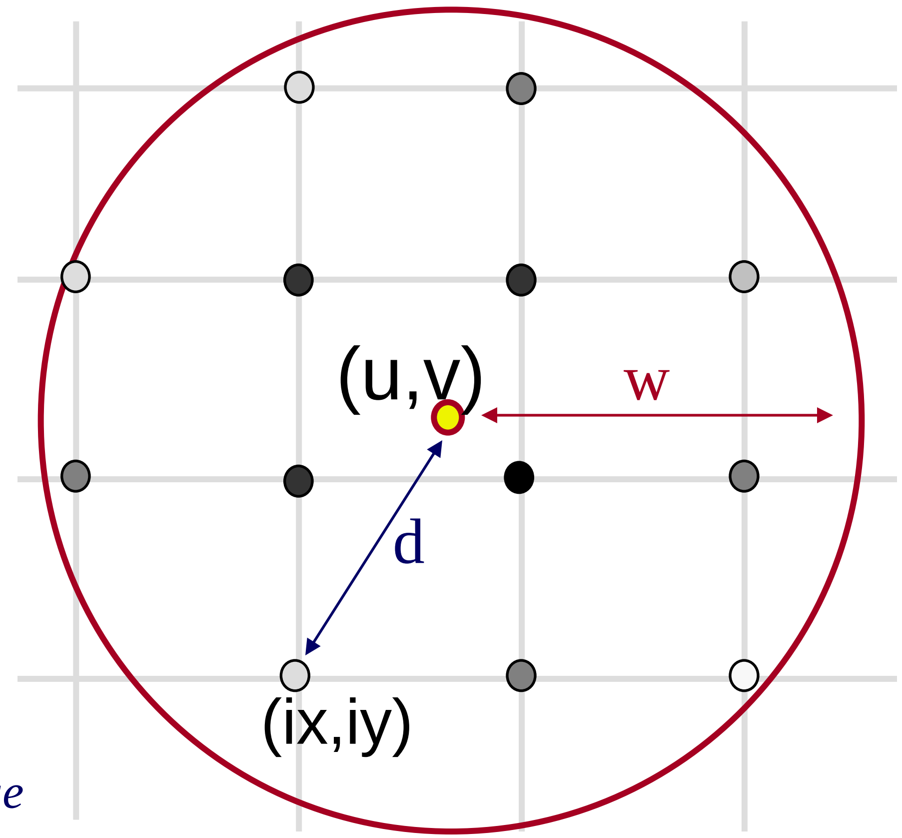
Point Sampling

Point Sampled: Aliasing!

Correctly Bandlimited

Resampling with Low-Pass Filter

- Output is weighted average of input samples, where weights are normalized values of filter (k)

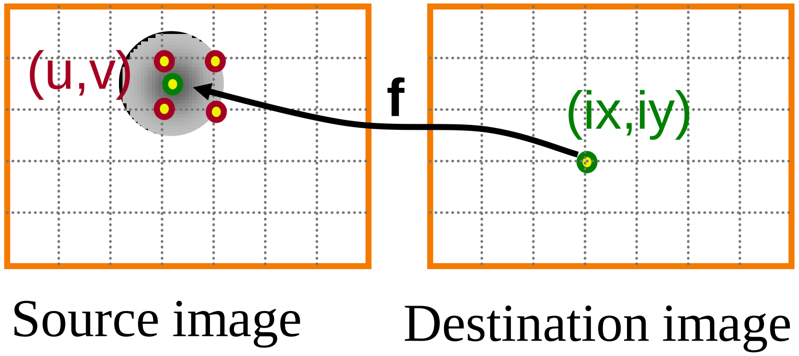


k(ix,iy) represented by gray value

Resampling with Low-Pass Filter

- Possible implementation:

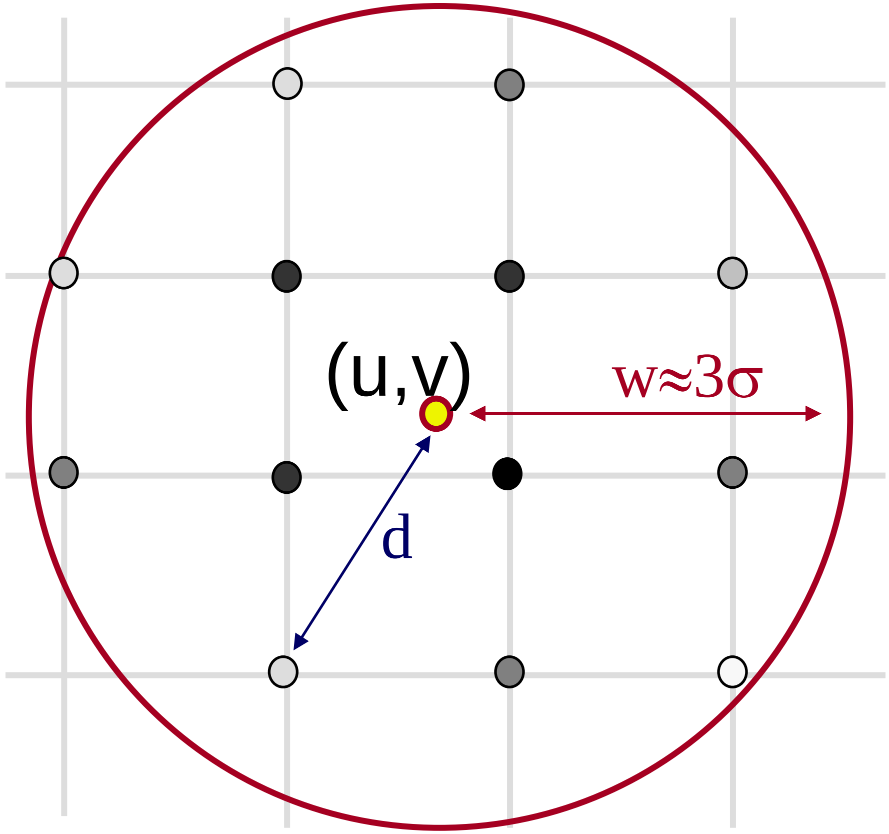
```
float Resample(src, u, v, k, w)
{
  float dst = 0;
  float ksum = 0;
  int ulo = u - w; etc.
  for (int iu = ulo; iu < uhi; iu++) {
    for (int iv = vlo; iv < vhi; iv++) {
      dst += k(u,v,iu,iv,w) * src(u,v)
      ksum += k(u,v,iu,iv,w);
    }
  }
  return dst / ksum;
}
```

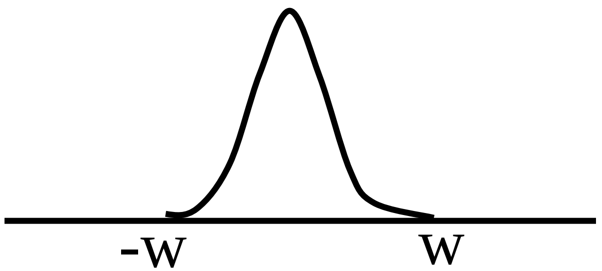


Resampling with Gaussian Filter

- Kernel is Gaussian function

$$G(d,\sigma) = e^{-d^2/(2\sigma^2)}$$



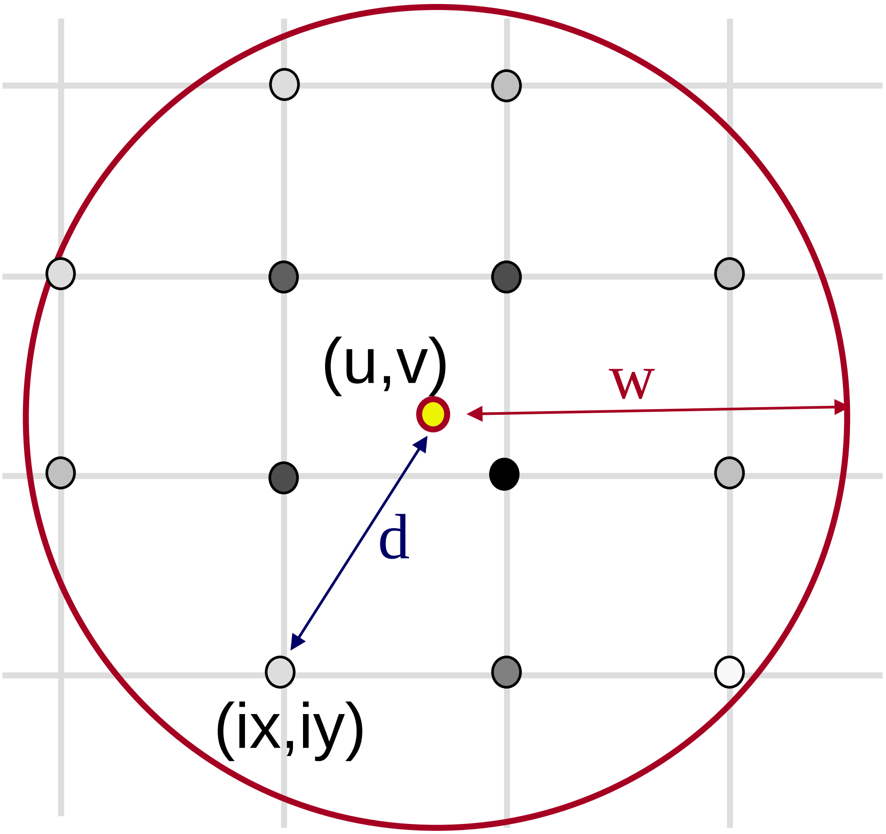


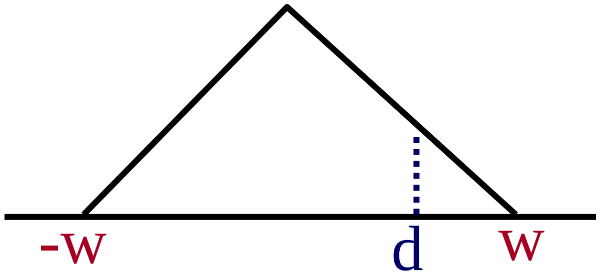
Gaussian Function

- Drops off quickly, but never gets to exactly 0
- In practice: compute out to w ~ 2.5σ or 3σ

Resampling with Triangle Filter

- For isotropic Triangle filter, k(ix,iy) is function of d and w





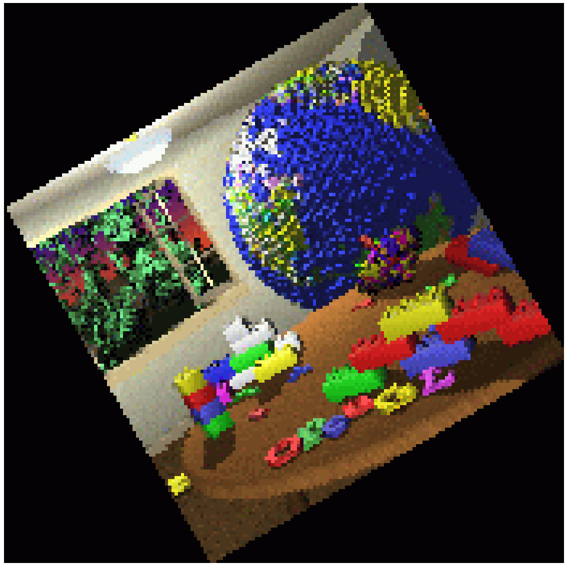
Triangle filter

$$k(i,j)=\max(1 - d/w, 0)$$

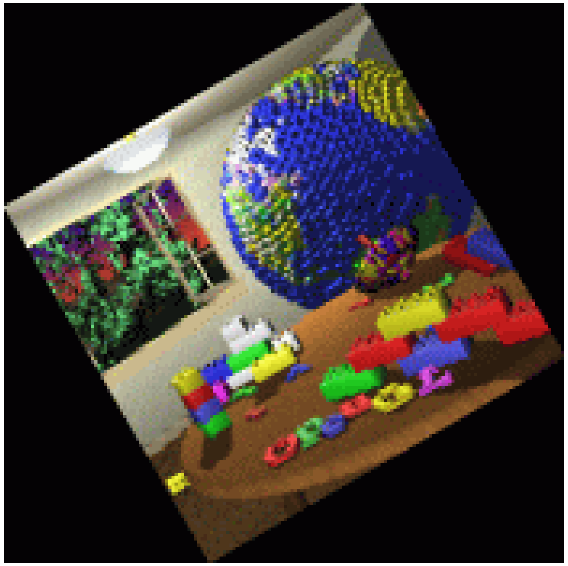
Filter Width = 2

Sampling Method Comparison

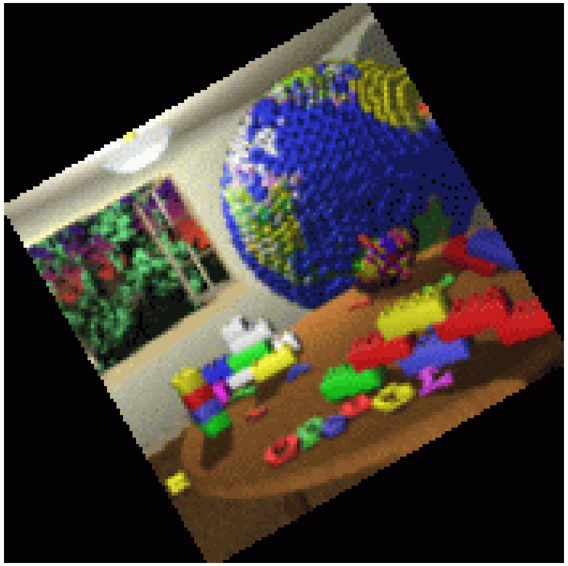
- Trade-offs
  - Aliasing versus blurring
  - Computation speed



Point



Triangle

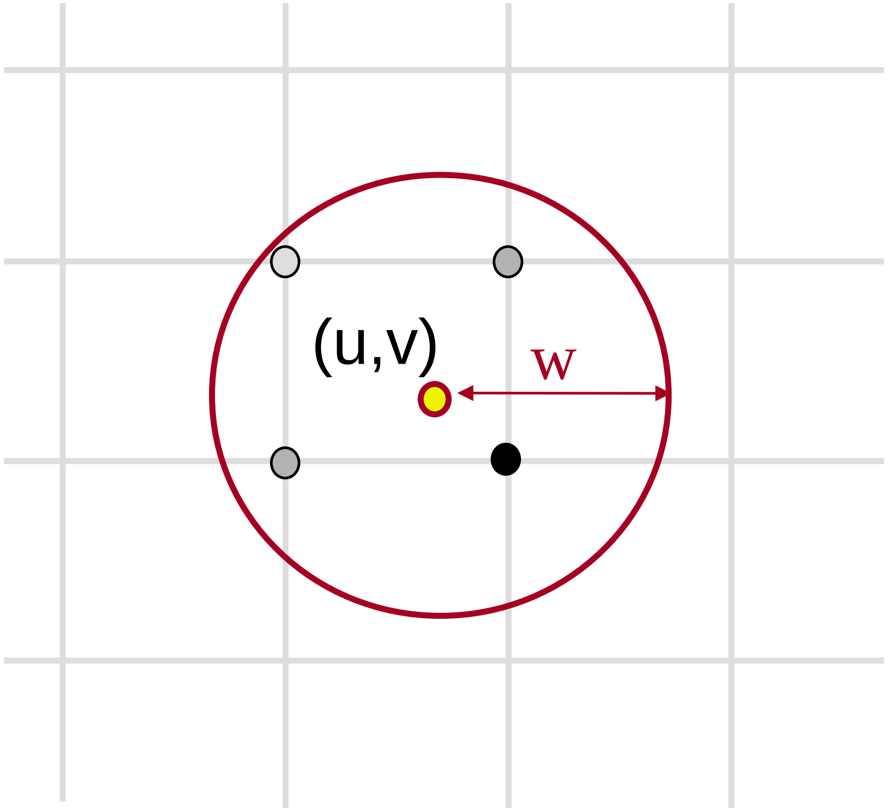


Gaussian

Resampling Details

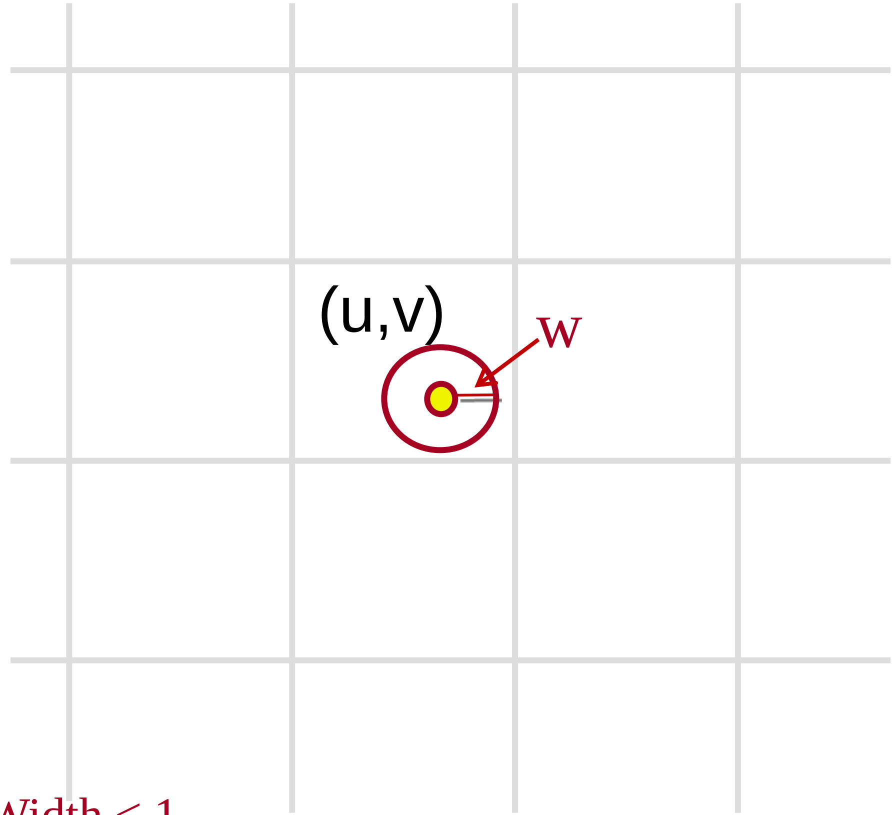
- Filter width chosen based on scale factor of map

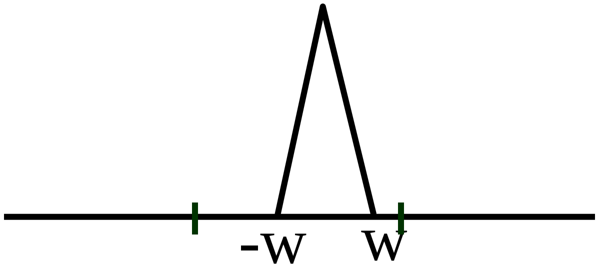
Filter must be wide enough to avoid aliasing



Resampling Details

- What if width (w) is smaller than sample spacing?



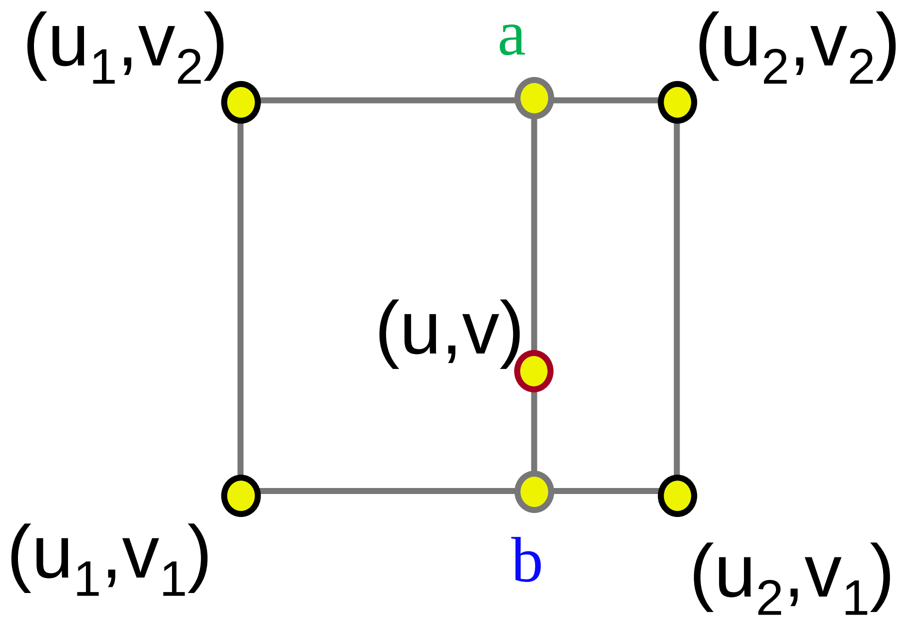


Triangle filter

Filter Width < 1

Resampling Details

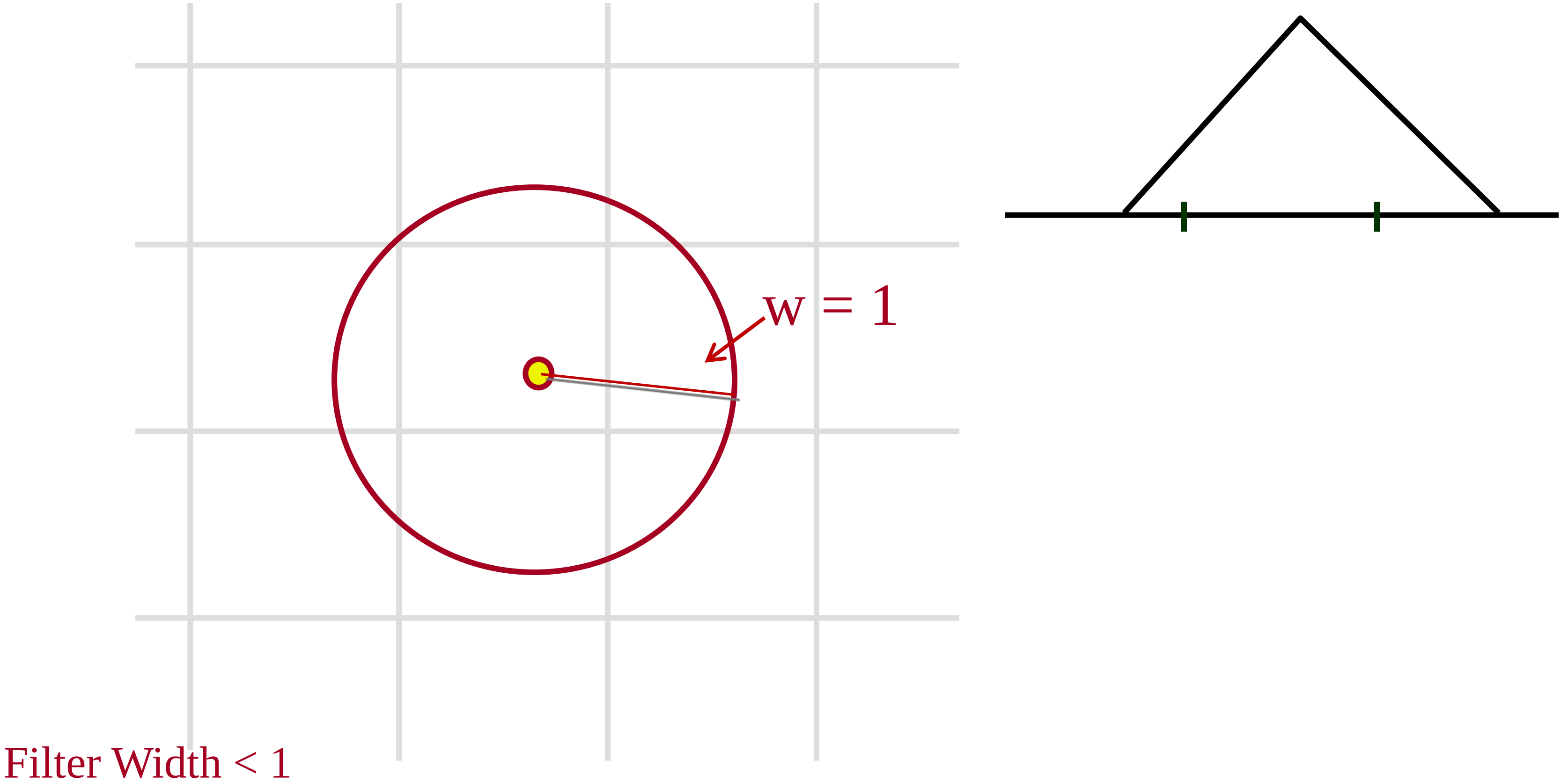
- Alternative 1: Bilinear interpolation of closest pixels
  - a = linear interpolation of src(u_1,v_2) and src(u_2,v_2)
  - b = linear interpolation of src(u_1,v_1) and src(u_2,v_1)
  - dst(x,y) = linear interpolation of "a" and "b"



Filter Width < 1

Resampling Details

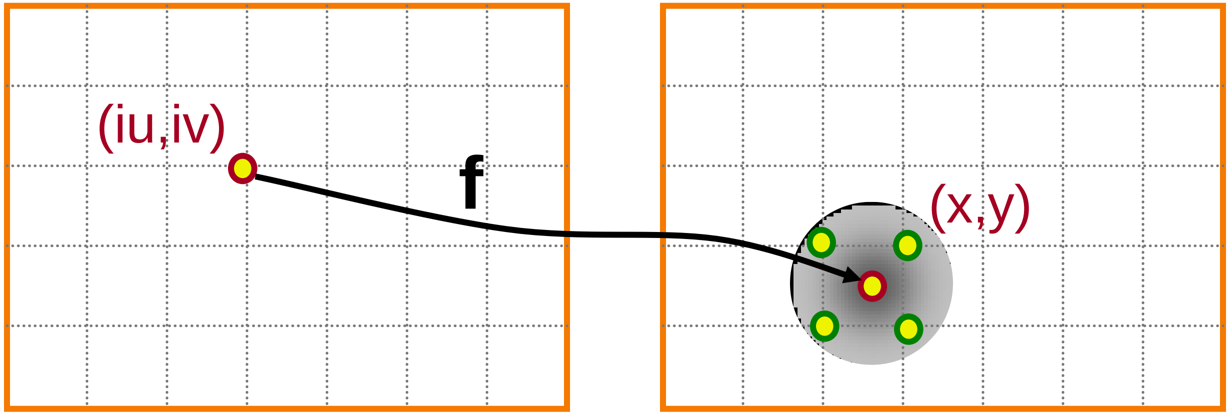
- Alternative 2: force width to be at least 1



Alternative Algorithm

- Forward mapping:

```
Warp(src, dst) {
  for (int iu = 0; iu < umax; iu++) {
    for (int iv = 0; iv < vmax; iv++) {
      float x = fx(iu,iv);
      float y = fy(iu,iv);
      float w ≈ 1 / scale(x, y);
      Splat(src(iu,iv),x,y,k,w);
    }
  }
}
```



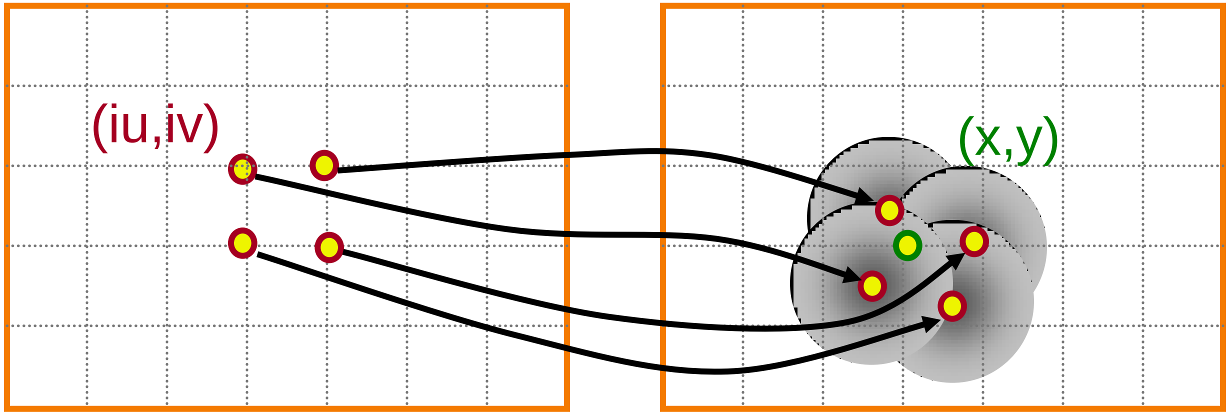
Source image

Destination image

Alternative Algorithm

- Forward mapping:

```
Warp(src, dst) {
  for (int iu = 0; iu < umax; iu++) {
    for (int iv = 0; iv < vmax; iv++) {
      float x = fx(iu,iv);
      float y = fy(iu,iv);
      float w ≈ 1 / scale(x, y);
      Splat(src(iu,iv),x,y,k,w);
    }
  }
}
```



Source image

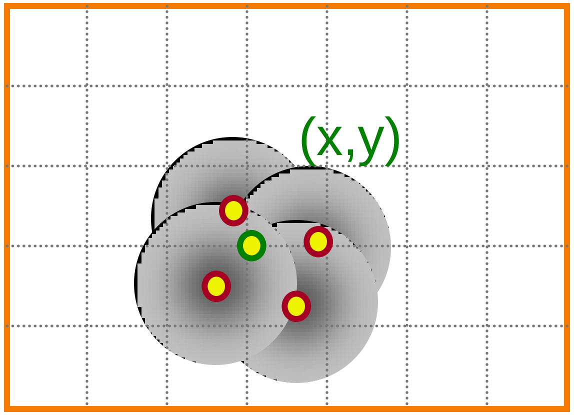
Destination image

Alternative Algorithm

- Forward mapping:

```
for (int iu = 0; iu < umax; iu++) {
  for (int iv = 0; iv < vmax; iv++) {
    float x = fx(iu,iv);
    float y = fy(iu,iv);
    float w ≈ 1 / scale(x, y);
    for (int ix = xlo; ix <= xhi; ix++) {
      for (int iy = ylo; iy <= yhi; iy++) {
        dst(ix,iy) += k(x,y,ix,iy,w) * src(iu,iv);
      }
    }
  }
}
```

Problem?

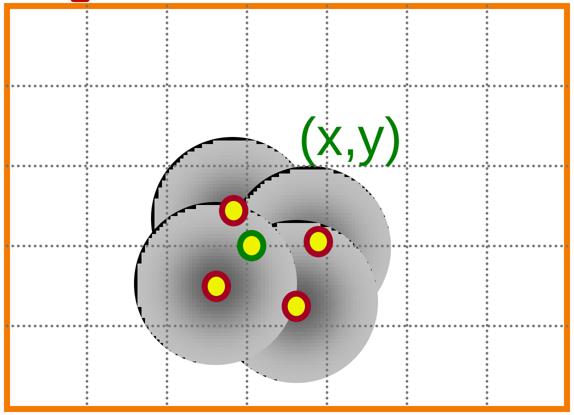


Destination image

Alternative Algorithm

- Forward mapping:

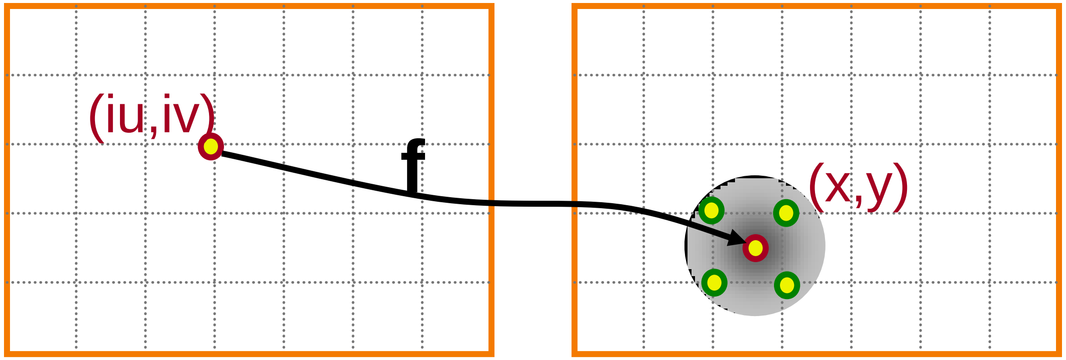
```
for (int iu = 0; iu < umax; iu++) {
  for (int iv = 0; iv < vmax; iv++) {
    float x = fx(iu,iv);
    float y = fy(iu,iv);
    float w ≈ 1 / scale(x, y);
    for (int ix = xlo; ix <= xhi; ix++) {
      for (int iy = ylo; iy <= yhi; iy++) {
        dst(ix,iy) += k(x,y,ix,iy,w) * src(iu,iv);
        ksum(ix,iy) += k(x,y,ix,iy,w);
      }
    }
  }
}
for (ix = 0; ix < xmax; ix++)
  for (iy = 0; iy < ymax; iy++)
    dst(ix,iy) /= ksum(ix,iy)
```



Destination image

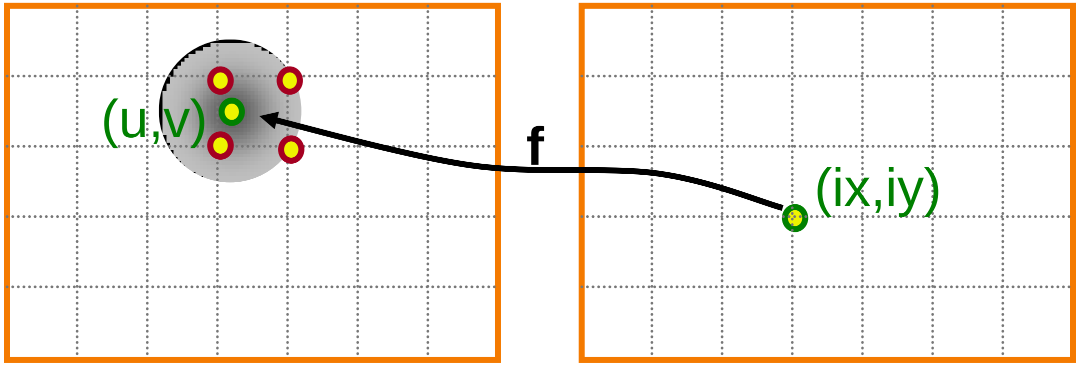
Forward vs. Reverse Mapping?

- Forward mapping



Source image　　　Destination image

- Reverse mapping



Source image　　　Destination image

Forward vs. Reverse Mapping

- Tradeoffs:
  - Forward mapping:
    - Requires separate buffer to store weights
  - Reverse mapping:
    - Requires inverse of mapping function, random access to original image

Reverse mapping is usually preferable

Putting it All Together

- Possible implementation of image blur:

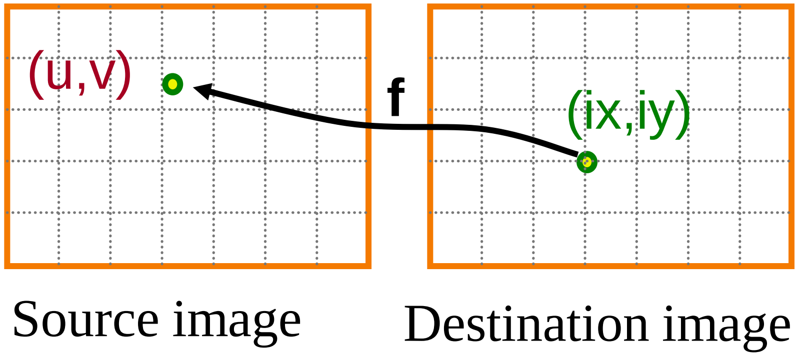
```
Blur(src, dst, sigma) {
  w ≈ 3*sigma;
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float u = ix;
      float v = iy;
      dst(ix,iy) = Resample(src,u,v,k,w);
    }
  }
}
```

Increasing sigma

Putting it All Together

- Possible implementation of image scale:

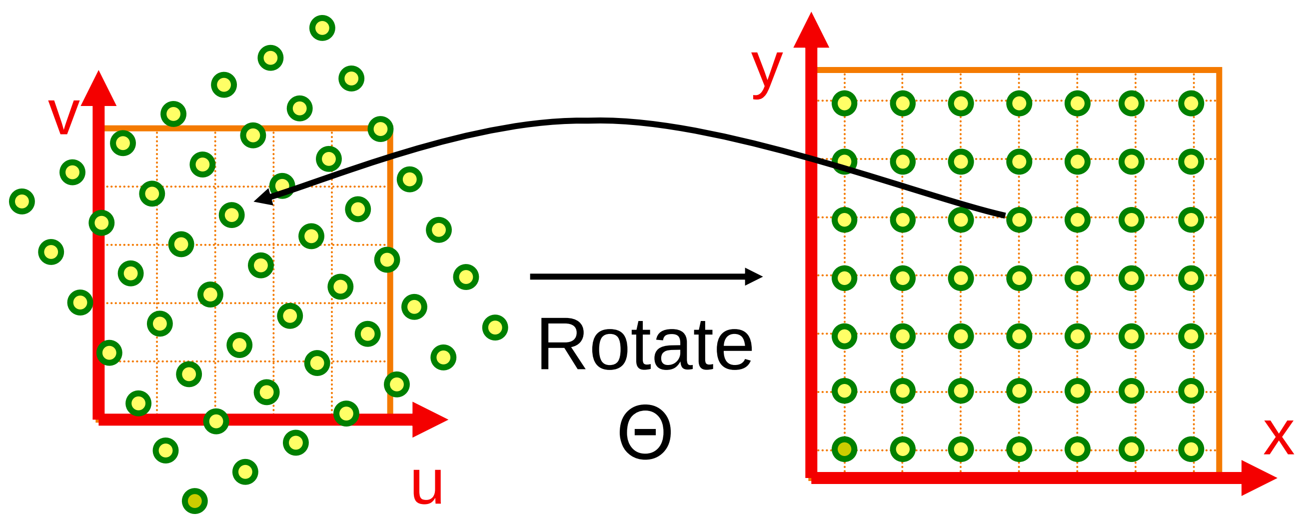
```
Scale(src, dst, sx, sy) {
   w ≈ max(1/sx,1/sy);
   for (int ix = 0; ix < xmax; ix++) {
      for (int iy = 0; iy < ymax; iy++) {
         float u = ix / sx;
         float v = iy / sy;
         dst(ix,iy) = Resample(src,u,v,k,w);
      }
   }
}
```



Putting it All Together

- Possible implementation of image rotation:

```
Rotate(src, dst, Θ) {
  w ≈ 1
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float u = ix*cos(-Θ) – iy*sin(-Θ);
      float v = ix*sin(-Θ) + iy*cos(-Θ);
      dst(ix,iy) = Resample(src,u,v,k,w);
    }
  }
}
```



Summary

- Mapping
  - Parametric
  - Correspondences
- Sampling, reconstruction, resampling
  - Frequency analysis of signal content
  - Filter to avoid aliasing
  - Reduce visual artifacts due to aliasing
    - » Blurring is better than aliasing
- Image processing
  - Forward vs. reverse mapping