

NAME:

Login name:

**Computer Science 426 Midterm
7PM-9PM**

This test is 8 questions, of equal weight. Do all of your work on these pages (use the back for scratch space), giving the answer in the space provided. This is a closed-book exam -- you may use one-page of notes with writing on both sides during the exam. **Put your name on every page, and write out and sign the Honor Code pledge before turning in the test.**

“I pledge my honor that I have not violated the Honor Code during this examination.”

Question	Score
1	
2	
3	
4	
5	
6	
7	
8	
Total:	

NAME:

Q1: Image Processing

(a) What is a bandlimited signal (one sentence)?

(b) What signals can be reconstructed without loss for a given sampling rate (one sentence)?

(c) Give three examples of aliasing in computer graphics (three phrases).

(d) Assume you work for Boeing and your job is to analyze the output of your colleagues' wind tunnel simulation in which air velocity $V(x,y,z)$ has been measured at every vertex of a uniform, rectilinear 3D grid. Write the equation for how you would use trilinear interpolation to estimate the air velocity V at an arbitrary point (x,y,z) within the grid cell defined by (x_1,x_2) , (y_1,y_2) , and (z_1, z_2) . Support your answer with a labeled drawing.

NAME:

Q2: Modeling Transformations

For each of the following cases, write a 4x4 matrix which applies the specified transformation to any 3D point $(x,y,z,1)$. Assume that points to be transformed are represented as a column vectors and they are left-multiplied by the matrix. If it is impossible to define a matrix for the given transformation, say so and explain why.

(a) Transform $(x, y, z, 1)$ to $(2x-3, -2y, 3z + 4, 1)$:

(b) Transform $(x, y, z, 1)$ to $(x/z, y/z, 1, z)$:

(c) Transform $(x, y, z, 1)$ to $(x, y, 1, z)$:

(d) Scale $(x, y, z, 1)$ by a factor S around an arbitrary “center of scale” point $C = (cx, cy, cz)$
(you can leave your answer as a product of matrices):

(e) Rotate $(x, y, z, 1)$ by Θ degrees around the line specified by $x=y$ and $z=0$
(you can leave your answer as a product of matrices):

NAME:

Q3: General Transformations

(a) What is the defining property of a linear transformation (one equation)?

(b) Circle the entries of the 4x4 matrix that must be set to specify transformations of the following three classes (assume points are represented by column vectors and matrices are premultiplied):

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

Linear

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

Affine

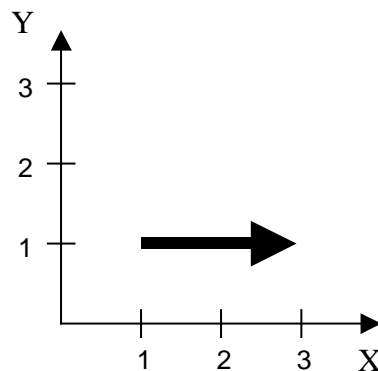
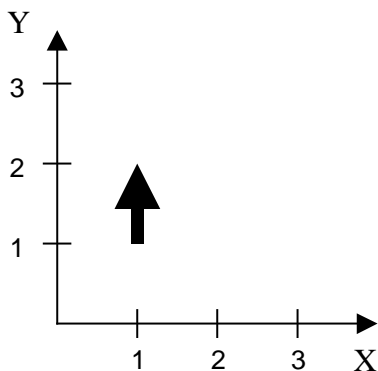
$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

Projective

(c) Fill out the table below by writing "TRUE" in each box for which the properties listed in its row applies to the transformation class listed in its column:

	Linear	Affine	Projective
Origin maps to origin			
Lines generally map to lines			
Parallel lines remain parallel			
Lengths are preserved			
Length ratios are preserved			

(d) Specify a sequence of transformations that transforms the arrow drawn on the left to the arrow drawn on the right. Assume 2D points are represented by column vectors. Write translations as $T(dx, dy)$, scales as $S(sx, sy)$, and counterclockwise rotations as $R(\Theta)$.



(e) Is the above transformation linear, affine, or projective (one word)?

NAME:

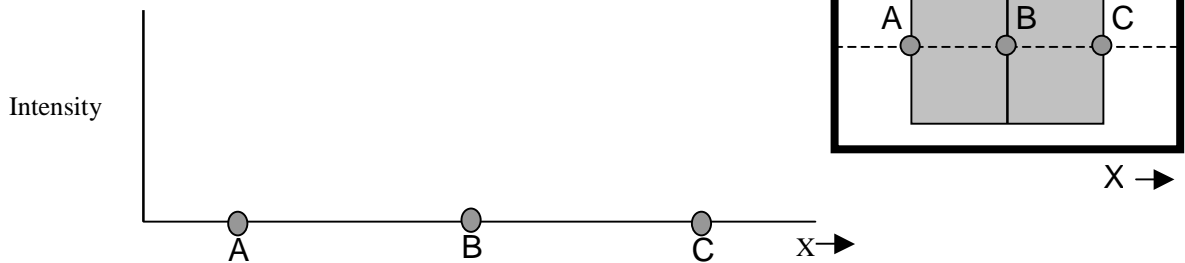
Q4: Polygon Shading

Consider a gray-level image of the 3D cylinder shown below. The cylinder is being viewed with an orthographic projection perpendicular to its vertical axis (shown as a solid line), and it is being lit "head on" by a single directional light source whose direction vector matches the viewer's. Assuming the surface of the cylinder is completely diffuse, draw the gray-level values rendered along a horizontal scan line (e.g., the dashed line) for each of the following rendering techniques.

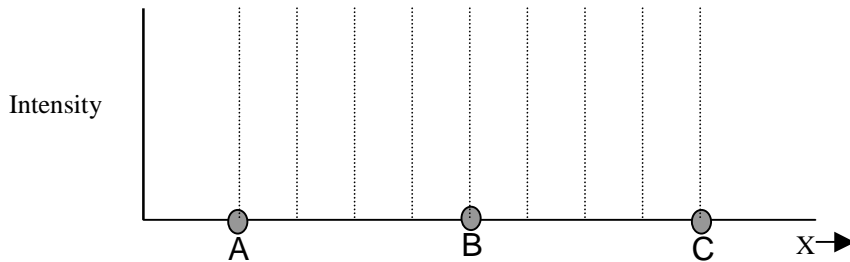
(a) Write an expression for the intensity across the scan line as a function of x :

Intensity(x) =

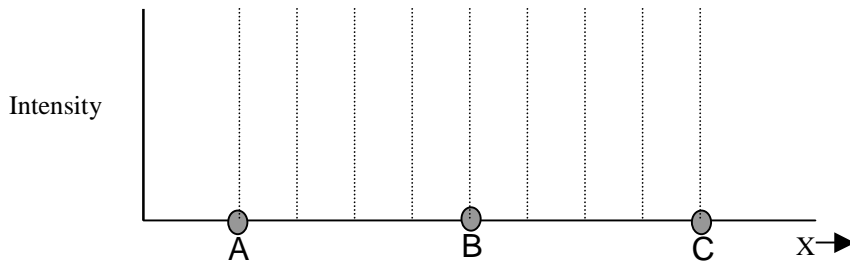
(b) Ray casting:



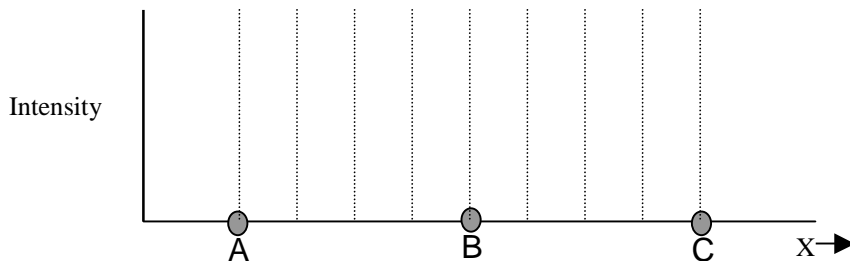
(c) Flat shading of a prism approximating the cylinder with 8 front-facing rectangles:



(d) Gouraud shading of a prism approximating the cylinder with 8 front-facing rectangles:



(e) Phong shading of a prism approximating the cylinder with 8 front-facing rectangles:



NAME:

Q4: Polygon Shading (continued)

(e) Circle each of the following problems that generally appears when images of curved surfaces are approximated by rendering triangle meshes with Phong shading:

Piecewise-linear silhouette curves

Failure to capture illumination highlights except at vertices

C^0 discontinuities along most interior edges of the mesh

Distortion of interior pixels for perspective projections.

(f) Briefly describe the following two visible surface algorithms. Support your answer with a picture. For each algorithm, specify at least one way in which it utilizes screen space coherence.

Scan-line algorithm (a few sentences and a picture):

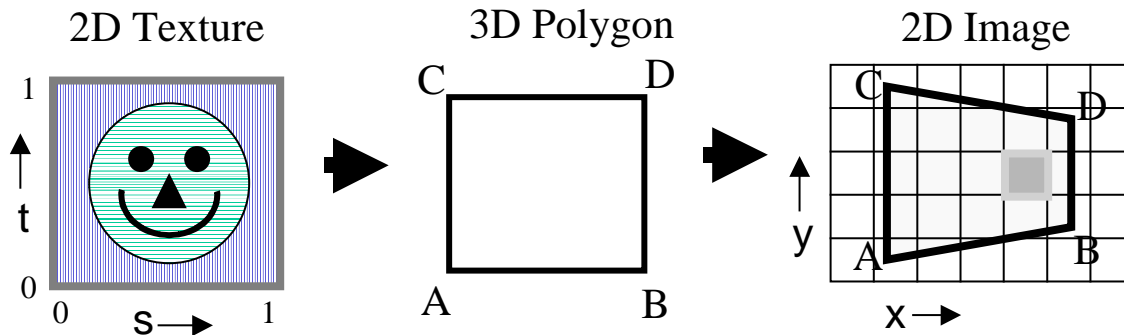
Warnock's area subdivision algorithm (a few sentences and a picture):

NAME:

Q5: Textures

The figure below shows three coordinate systems used for texture mapping.
The thick trapezoid indicates the projection of the 3D polygon in the 2D image.
The texture coordinates (s,t) for the 3D polygon vertices are:

$$A=(\frac{1}{2}, \frac{1}{2}), B=(1, \frac{1}{2}), C=(\frac{1}{2}, 1), \text{ and } D=(1, 1)$$



- (a) Draw a contour in the 2D texture indicating the region mapped onto the polygon.
Draw another contour in the 2D texture around the region mapped onto the shaded pixel.
Fill in the blank areas of the 3D polygon and 2D image with the texture mapped onto them.
- (b) What type of viewing transformation is used to render the 3D polygon into the 2D image?
Perspective or parallel (one word)? How do you know (one sentence)?
- (c) What is a mip map (one sentence)? Draw an example for the smiley face texture shown above.
- (d) Describing a situation in which mip mapping is useful and explain why it helps (two sentences).

NAME:

Q6: Global Illumination

(a) Circle the term used to describe the light carried by a single ray (e.g., as in your ray tracer of assignment #2).

Flux

Radiant intensity

Radiance

Radiosity

(b) If L represents light leaving a light source, E represents light arriving at an eye point, D represents a diffuse reflection at a surface, and S represents a pure specular reflection at a surface, circle ALL the ray paths among the following that are modeled a classical ray tracer (e.g., as in your ray tracer of assignment #2).

LSSE

LDDSE

LSSDE

LDSSE

LSDSE

LDSDE

(c) Briefly describe distributed ray tracing.

(d) List at least three examples of rendering effects for which distributed ray tracing is used.

NAME:

Q6: Global Illumination (continued)

Recall that the radiosity equation: $B_i = E_i + \rho_i \sum_j B_j F_{ij}$

(e) Describe the meaning of the form factor F_{ij} between two polygons i and j (one sentence).

(f) Circle each of the following properties of form factors that are TRUE for scenes with polygonal surfaces:

$F_{ij} > 0$, for all i and j

$F_{ij} = F_{ji}$, for all i and j

$F_{ii} = 0$, for all i

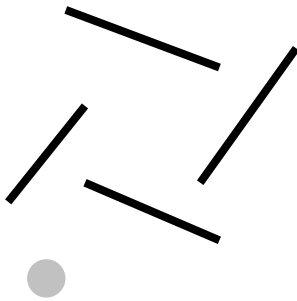
$\sum_j F_{ij} \leq 1$, for all i

NAME:

Q7: BSP Trees

- (a) Write the implicit representation for a 3D plane (write one equation)?
- (b) Provide a geometric interpretation for each 3D plane equation coefficient.
Hint: draw a 2D picture.

- (c) Draw a 2D BSP with a split along every line segment (black lines) for the scene shown below. Label each split plane with a lower-case letter and each cell (leaf node) with an upper-case letter. Draw the corresponding tree structure on the right with matching labels for both interior nodes and leaf nodes.



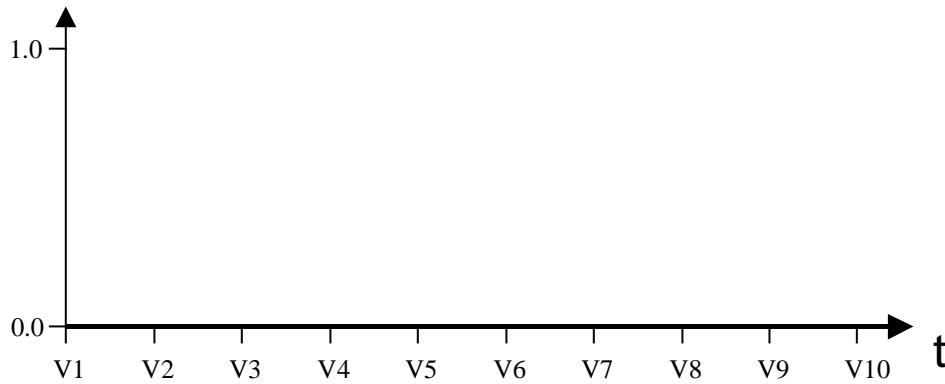
- (d) Write the order that cells of your BSP are traversed in back-to-front BSP visibility order for the viewpoint shown as a gray dot (a list of capital letters).

- (e) What is the worst-case complexity of the number of cells in a BSP formed by splitting on all surfaces of a scene with N polygons (e.g., $O(N)$, $O(N \log N)$, etc.)?

NAME:

Q8: Bsplines

- (a) Consider a piecewise-cubic uniform Bspline curve defined by 10 control points. Draw the cubic BSpline blending functions used to weight each control point.



- (b) What property of these blending functions ensures that the Bspline curve lies within the convex hull of the control points (write one sentence)?
- (c) What property of these blending functions indicates that the Bspline curve does not generally interpolate its control points (write one sentence)?
- (d) What property of these blending functions indicates that the Bspline curve provides local control (write one sentence)?

NAME:

Q8: Bsplines (continued)

- (e) Does every isoparametric curve (e.g., along a constant u value) on the surface of a bicubic B-spline tensor product surface patch define a cubic B-spline curve? If so, write a formula for the curve's control points Q_i in terms of the surface's control points $V_{i,j}$. If not, explain why not. Support your answer with a labeled picture.