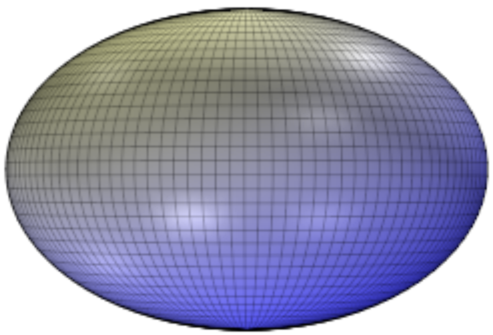
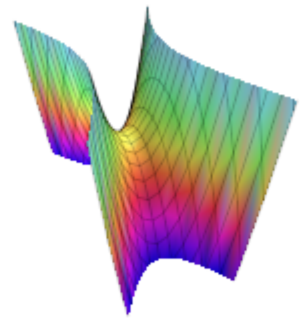
COS 426
Computer Graphics
Princeton University

Ray Tracing

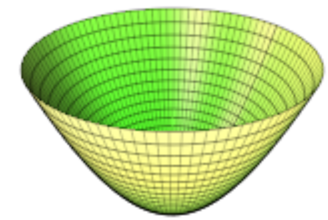
- **Ray/primitive intersection**
- Acceleration



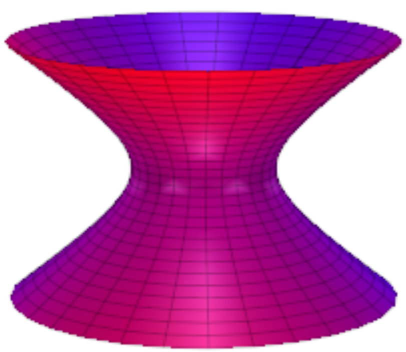
Ellipsoid



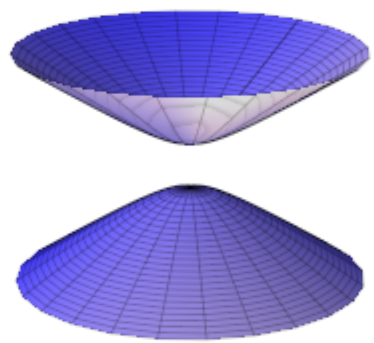
Hyperbolic paraboloid



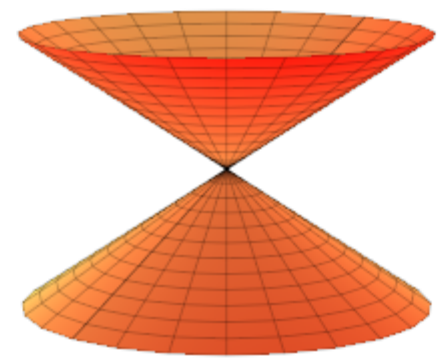
Elliptic paraboloid



Hyperboloid of one sheet

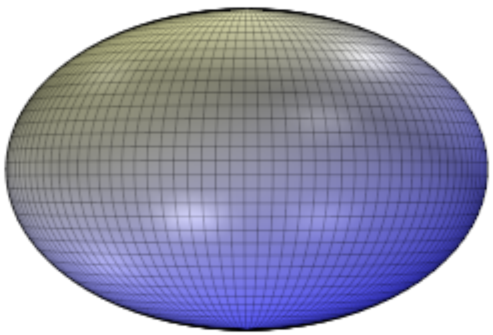


Hyperboloid of two sheets

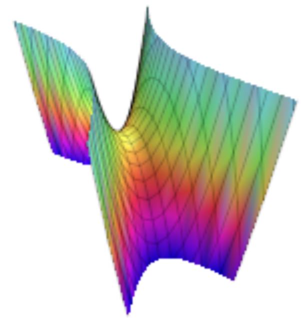


Cone

Quadrics



Ellipsoid



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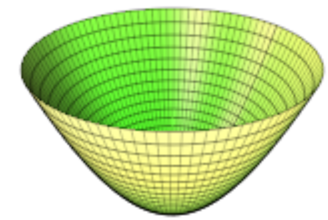

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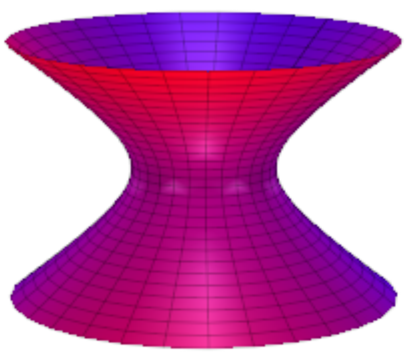

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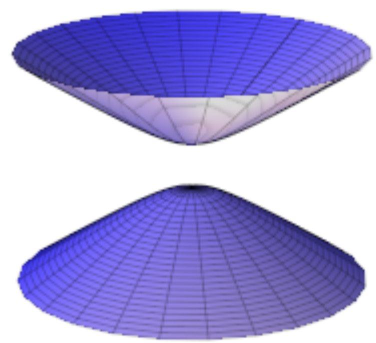
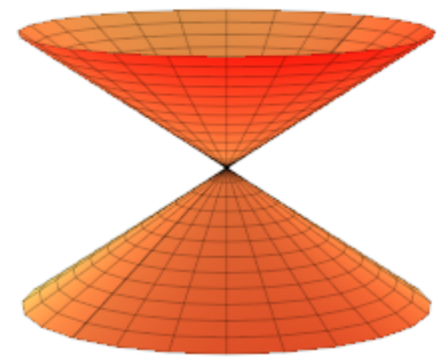
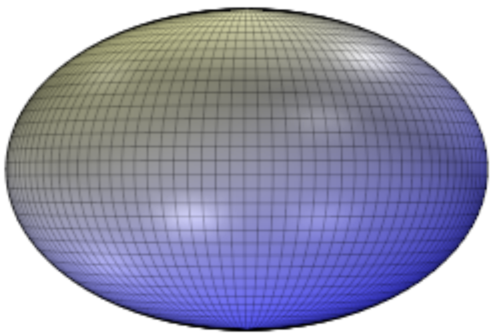


Hyperboloid of two sheets

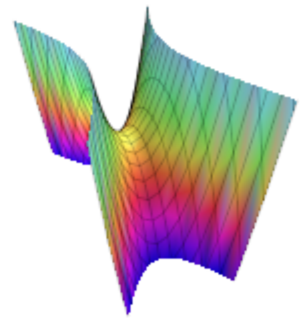


Cone

Quadrics



Ellipsoid



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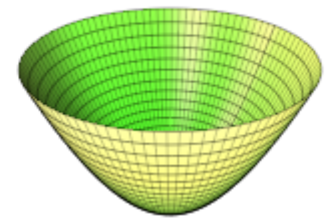
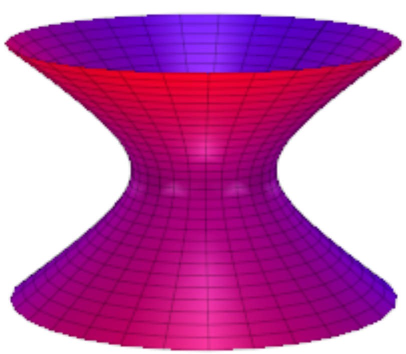


Elliptic paraboloid



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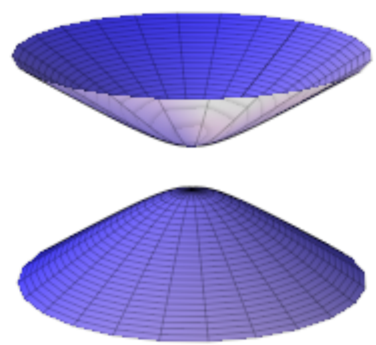

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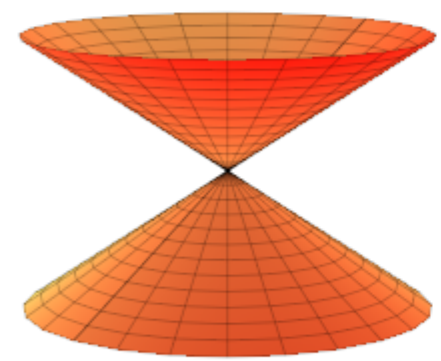

Cone

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

Quadrics

- Ray/primitive intersection:
  - Write down all equations
  - Solve for intersection

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- Ray:
  - ???

Quadrics

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  - Write down all equations
  - Solve for intersection
- Quadric:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

- Ray:

$$p = p_0 + t \cdot v$$

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v \quad \left\{ \begin{array}{l} x = x_0 + t \cdot v_x \\ y = y_0 + t \cdot v_y \\ z = z_0 + t \cdot v_z \end{array} \right.$$

Quadrics

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$$\cdots$$

$$K \cdot t^2 + L \cdot t + M = 0$$

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v \quad \begin{cases} x = x_0 + t \cdot v_x \\ y = y_0 + t \cdot v_y \\ z = z_0 + t \cdot v_z \end{cases}$$

$$K \cdot t^2 + L \cdot t + M = 0$$

A positive real solution exists

Two complex solutions

Two real negative solution

Quadrics

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$$K \cdot t^2 + L \cdot t + M = 0$$

A positive real solution exists

Pick smallest positive value to find intersection

Two complex solutions

Does not intersect

Two real negative solution

Does not intersect

Quadrics
Simpler Derivation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

Quadrics
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$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

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Quadrics
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$$p = p_0 + t \cdot v$$

$$p\underset{3 \times 3}{Q}p^T + \underset{1 \times 3}{P}p^T + \underset{1 \times 1}{R} = 0$$

Quadrics
Simpler Derivation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

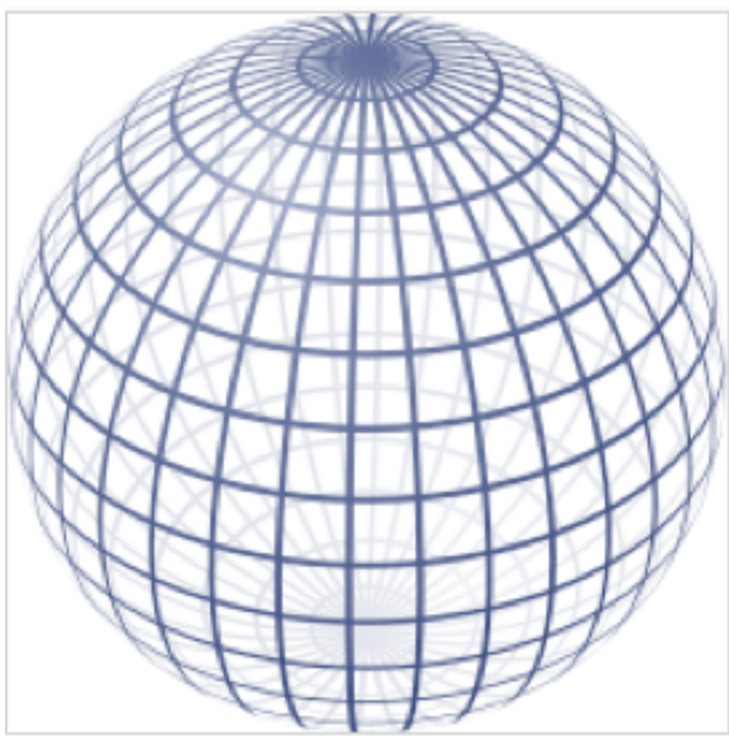
$$\boxed{p\underset{3\times 3}{\underline{Q}}p^T + \underset{1\times 3}{\underline{P}}p^T + \underset{1\times 1}{\underline{R}} = 0}$$

$$(p_0 + tv)Q(p_0 + tv)^T + P(p_0 + tv)^T + R = 0$$

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$



Quadrics

- If you use general quadric for sphere

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  - What do you need to define?

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$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Quadrics

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  – What do you need to define?

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$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array} \right]$$

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$

  – What do you need to define?

  – Q, P, R

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = -r^2$$

$$P = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array} \right]$$

Ray Tracing

- Ray/primitive intersection
- **Acceleration**

Acceleration

- **Bounding Volume**
- Generate Structure (e.g. octree)
- Traverse Structure

Bounding Volume

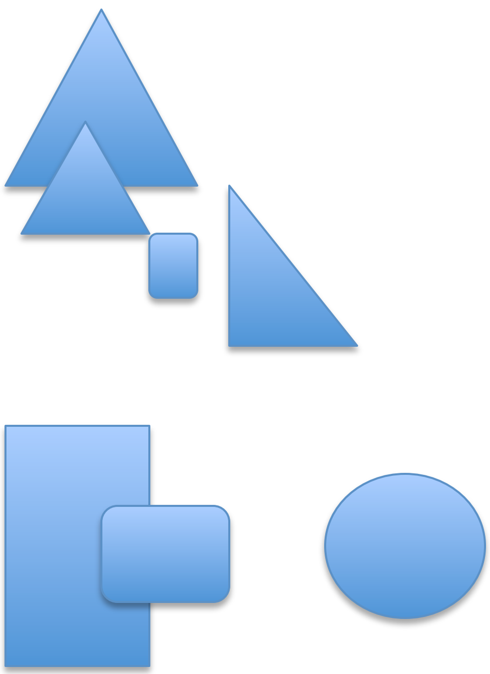
- Bounding sphere
- Axis-aligned bounding box (AABB)
- Oriented bounding box (OBB)
- Bounding volume hierarchy

Acceleration

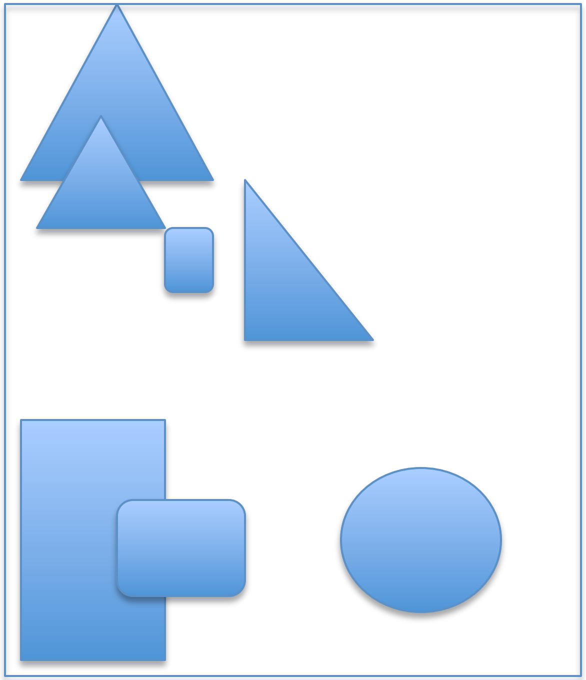
- **Generate Structure (e.g. octree)**

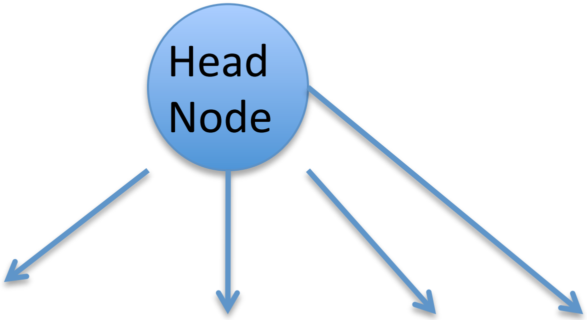
- Traverse Structure

Acceleration

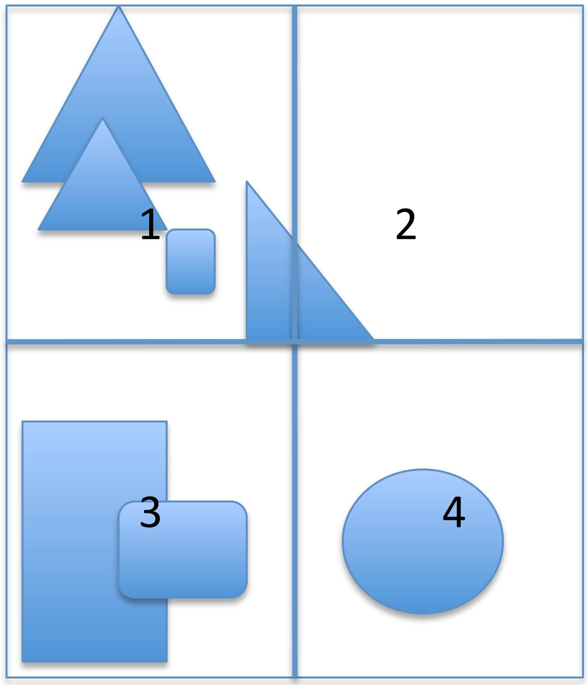


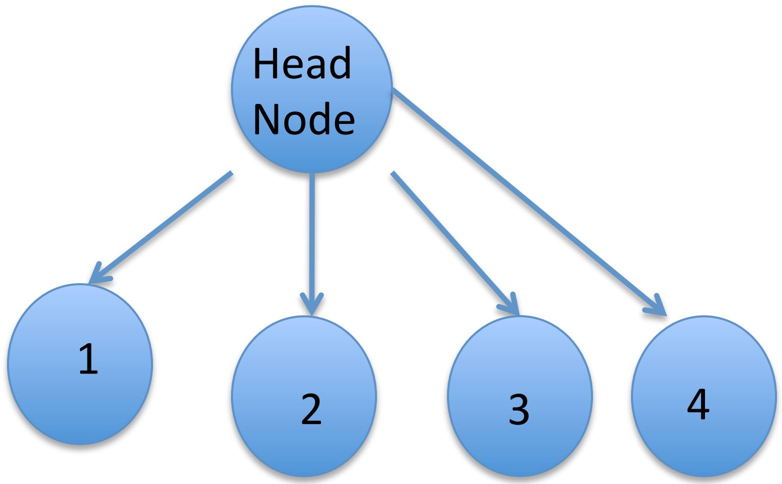
Acceleration





Acceleration





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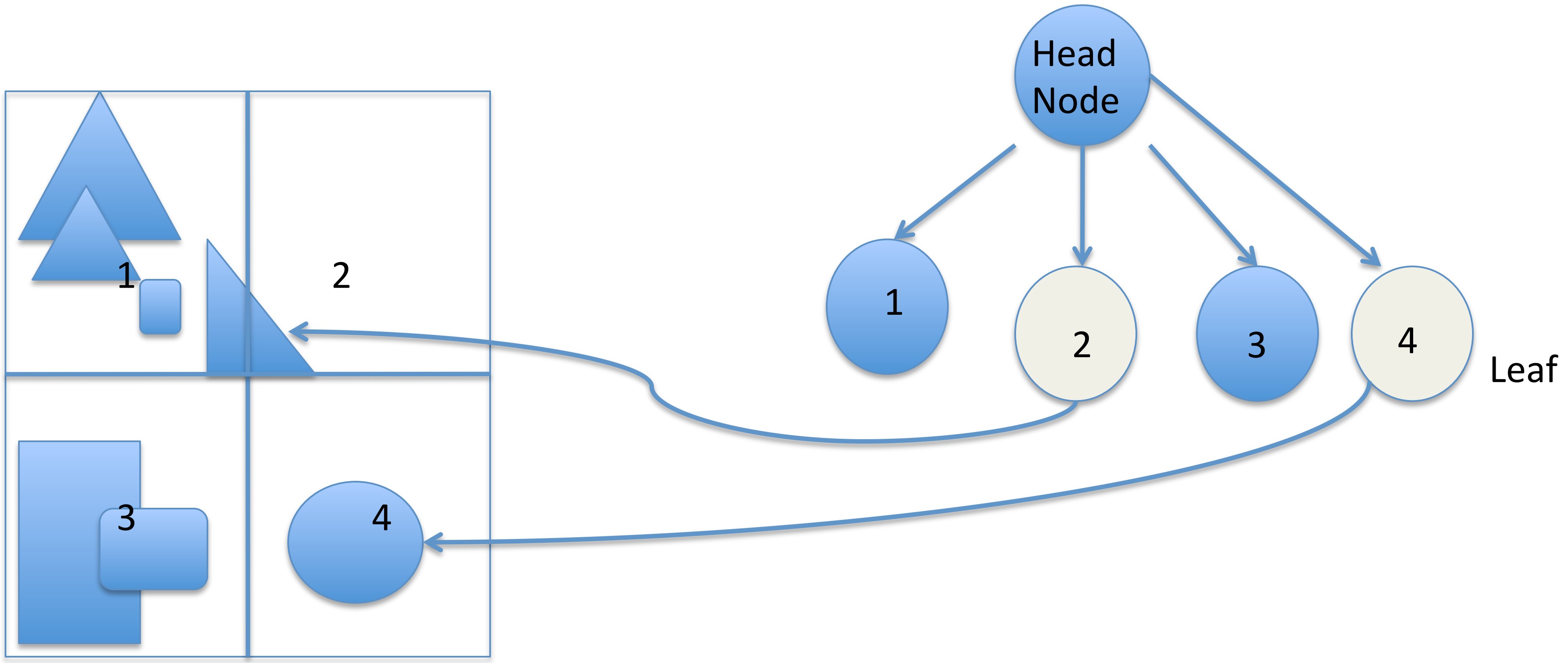

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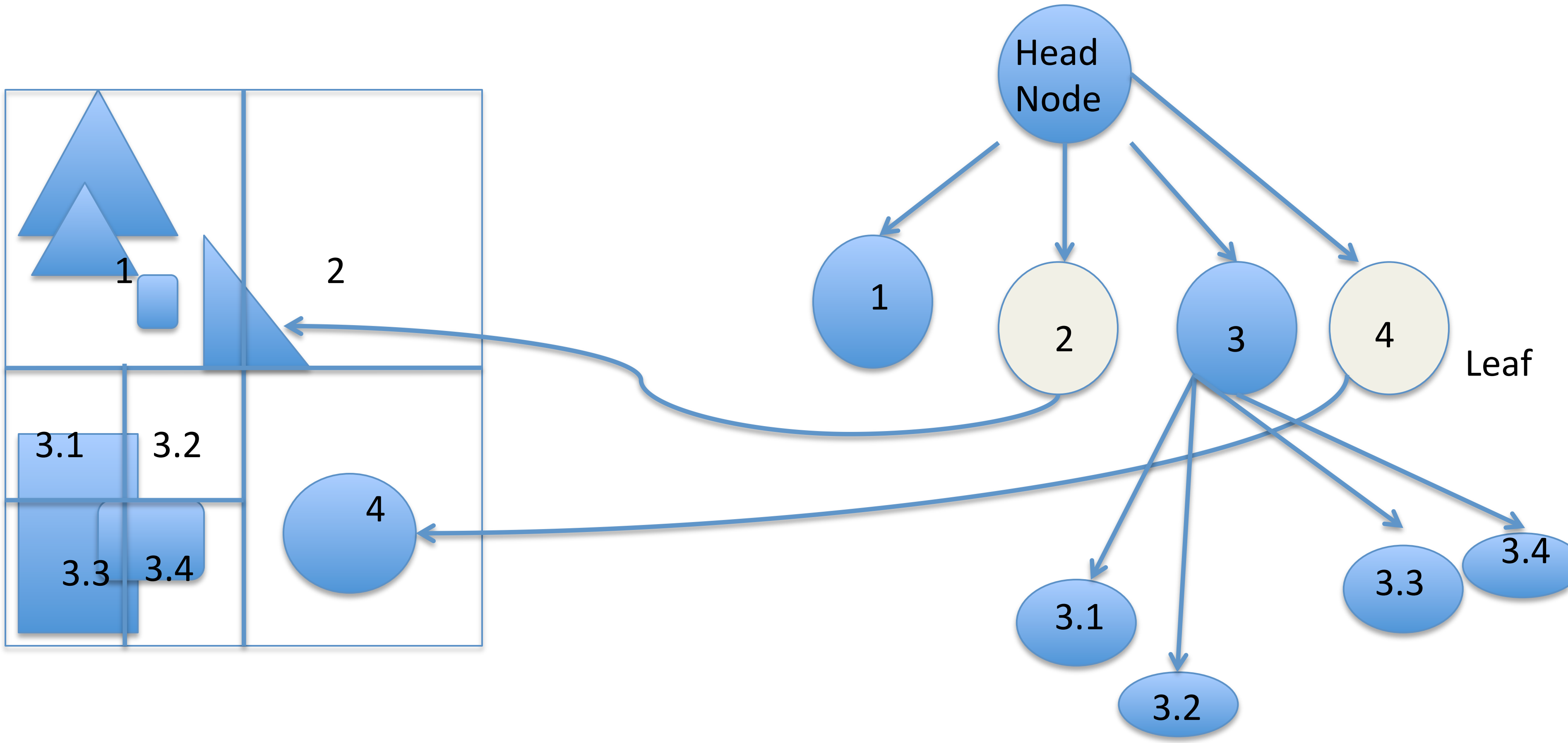
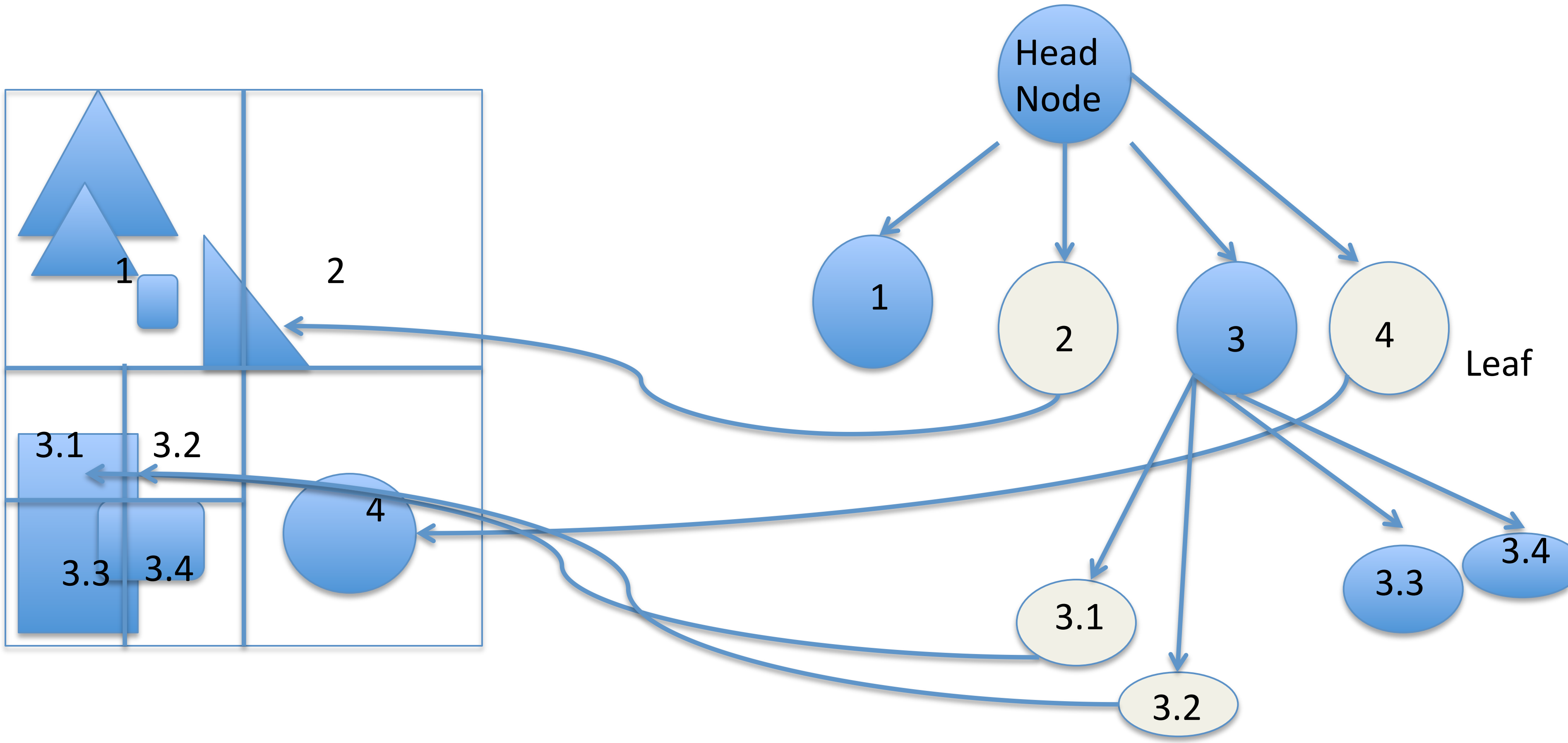
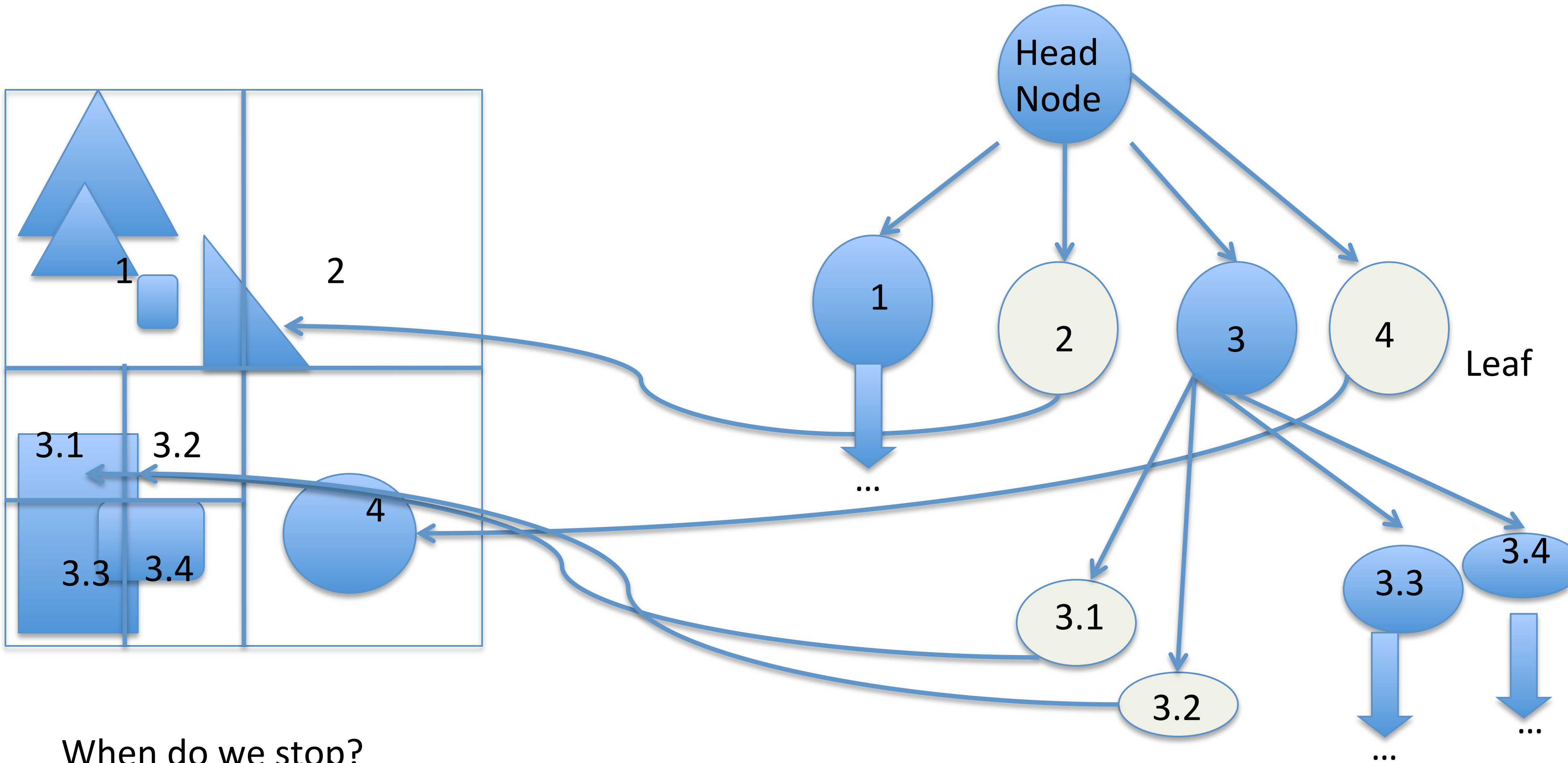


Acceleration



Acceleration



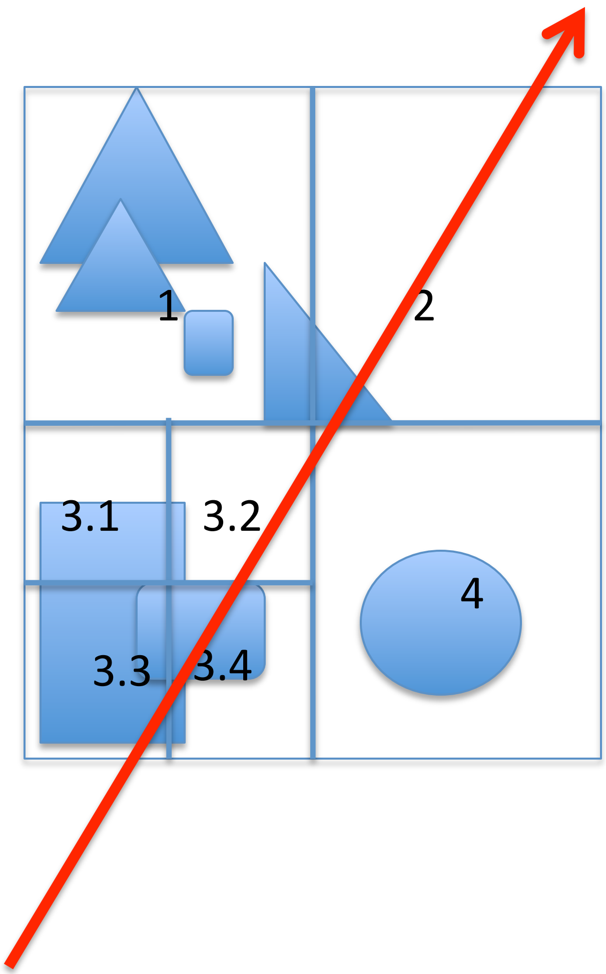
When do we stop?

Acceleration

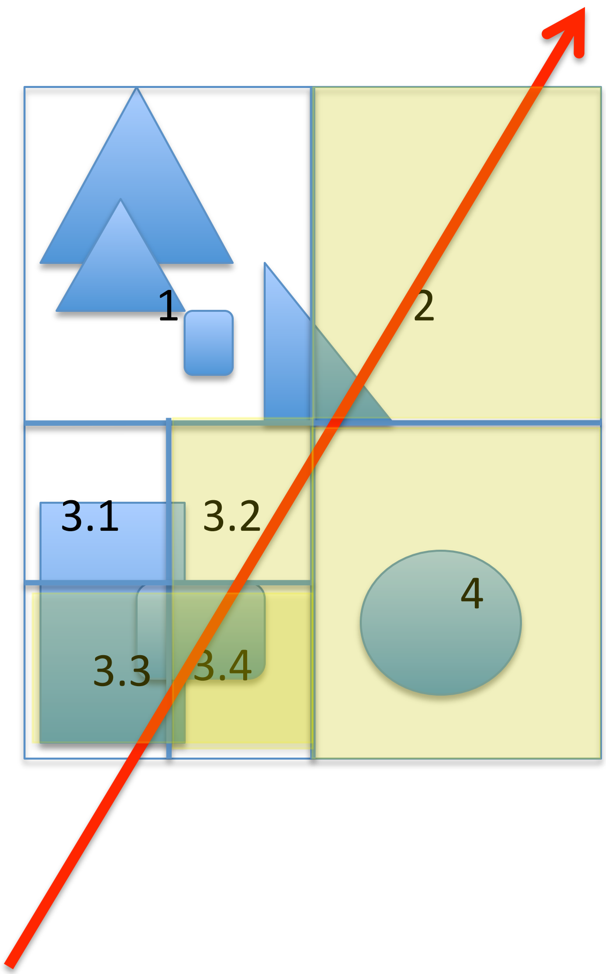
- Generate Structure (e.g. octree)

- **Traverse Structure**

Acceleration



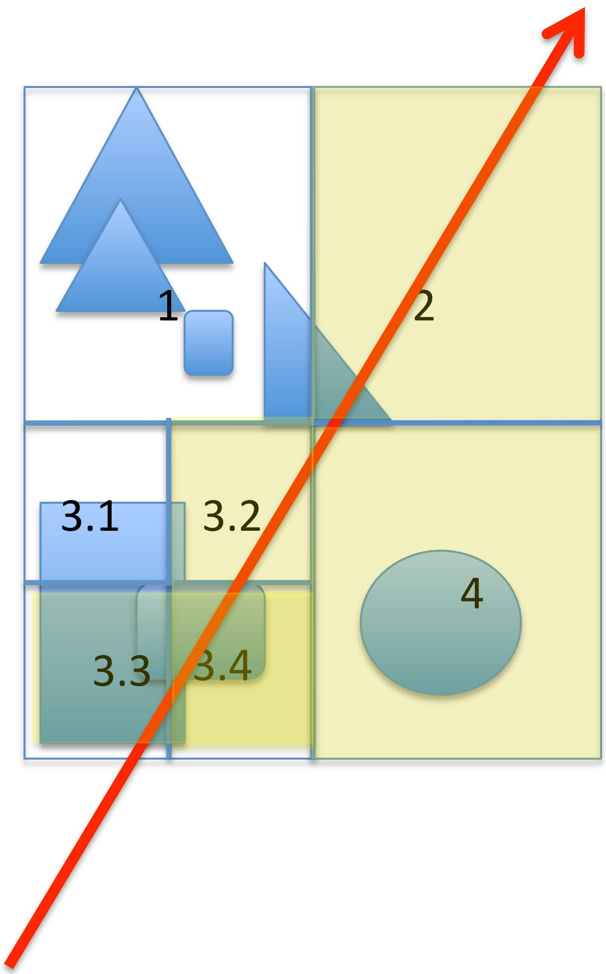
Acceleration



ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3,
can it intersect something (with a smaller t)
in a later node?

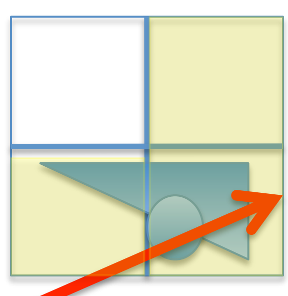
Acceleration



ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3,
can it intersect something (with a smaller t)
in a later node?

Think about objects on boundaries.



Example: should intersect the circle,
but the triangle is visited first

The End

Questions?