COS 426

Computer Graphics

Princeton University
Ray Tracing

• Ray/primitive intersection

• Acceleration
Ellipsoid

Hyperbolic paraboloid

Elliptic paraboloid

Hyperboloid of one sheet

Hyperboloid of two sheets

Cone
Quadrics

Ellipsoid

Hyperbolic paraboloid

Elliptic paraboloid

Hyperboloid of one sheet

Hyperboloid of two sheets

Cone
Quadrics

Ellipsoid

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Hyperboloid of two sheets

Cone

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]
Quadrics

• Ray/primitive intersection:
  – Write down all equations
  – Solve for intersection
Quadrics

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  – Write down all equations
  – Solve for intersection

• Quadric:

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

• Ray:
  – ???
Quadrics

• Ray/primitive intersection:
  – Write down all equations
  – Solve for intersection

• Quadric:

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

• Ray:

\[ p = p_0 + t \cdot v \]
Quadrics

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

\[ p = p_0 + t \cdot v \]

\[ \begin{align*}
  x &= x_0 + t \cdot v_x \\
  y &= y_0 + t \cdot v_y \\
  z &= z_0 + t \cdot v_z
\end{align*} \]
Quadrics

\[ A x^2 + B y^2 + C z^2 + D x y + E x z + F y z + G x + H y + I z + J = 0 \]

\[ p = p_0 + t \cdot v \]
\[ x = x_0 + t \cdot v_x \]
\[ y = y_0 + t \cdot v_y \]
\[ z = z_0 + t \cdot v_z \]

\[ K \cdot t^2 + L \cdot t + M = 0 \]
Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

$$\begin{align*}
x &= x_0 + t \cdot v_x \\
y &= y_0 + t \cdot v_y \\
z &= z_0 + t \cdot v_z
\end{align*}$$

$$K \cdot t^2 + L \cdot t + M = 0$$

- A positive real solution exists
- Two complex solutions
- Two real negative solution
Quadrics

\( A x^2 + B y^2 + C z^2 + D x y + E x z + F y z + G x + H y + I z + J = 0 \)

\[ p = p_0 + t \cdot v \]

\[
\begin{align*}
    x &= x_0 + t \cdot v_x \\
    y &= y_0 + t \cdot v_y \\
    z &= z_0 + t \cdot v_z 
\end{align*}
\]

\[ K \cdot t^2 + L \cdot t + M = 0 \]

- A positive real solution exists
- Two complex solutions
- Two real negative solution

Pick smallest positive value to find intersection

Does not intersect

Does not intersect
Quadrics
Simpler Derivation

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

\[ p = p_0 + t \cdot v \]
Quadrics
Simpler Derivation

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

\[ p = p_0 + t \cdot v \]

\[ pQp^T + Pp^T + R = 0 \]
Quadrics
Simpler Derivation

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

\[ p = p_0 + t \cdot v \]

\[ pQp^T + Pp^T + R = 0 \]

\( 3 \times 3 \) \( 1 \times 3 \) \( 1 \times 1 \)
Quadrics
Simpler Derivation

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]
\[ p = p_0 + t \cdot v \]

\[
\begin{array}{c}
pQp^T + Pp^T + R = 0 \\
3 \times 3 & 1 \times 3 & 1 \times 1 \\
\end{array}
\]

\[
(p_0 + tv)Q(p_0 + tv)^T + P(p_0 + tv)^T + R = 0
\]
Quadrics

- If you use general quadric for sphere
  \[ pQp^T + Pp^T + R = 0 \]
Quadrics

• If you use general quadric for sphere

\[ pQp^T + Pp^T + R = 0 \]

– What do you need to define?
Quadrics

• If you use general quadric for sphere

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– What do you need to define?
– Q, P, R
Quadrics

• If you use general quadric for sphere

\[ pQp^T + Pp^T + R = 0 \]

– What do you need to define?
– \( Q, P, R \)

\[
Q = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
Quadrics

• If you use general quadric for sphere

\[ pQp^T + PP^T + R = 0 \]

– What do you need to define?
– Q, P, R

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Q = \begin{bmatrix}
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\end{bmatrix}
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\[
P = \begin{bmatrix}
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\end{bmatrix}
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Quadrics

• If you use general quadric for sphere

\[ pQp^T + Pp^T + R = 0 \]

– What do you need to define?

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Q = \begin{bmatrix}
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0 & 0 & 1 \\
\end{bmatrix}
\]

\[
P = \begin{bmatrix}
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
R = -r^2
\]
Ray Tracing

- Ray/primitive intersection
- Acceleration
Acceleration

• **Bounding Volume**
• Generate Structure (e.g. octree)
• Traverse Structure
Bounding Volume

- Bounding sphere
- Axis-aligned bounding box (AABB)
- Oriented bounding box (OBB)
- Bounding volume hierarchy
Acceleration

• Generate Structure (e.g. octree)

• Traverse Structure
Acceleration
Acceleration

Head Node
Acceleration
Acceleration

Head Node

1

2

3

4

Leaf

3.1

3.2

3.3

3.4

3.1

3.2
When do we stop?
Acceleration

• Generate Structure (e.g. octree)

• Traverse Structure
Acceleration
ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3, can it intersect something (with a smaller $t$) in a later node?
Acceleration

ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3, can it intersect something (with a smaller t) in a later node?

Think about objects on boundaries.

Example: should intersect the circle, but the triangle is visited first
The End

Questions?