Topics

• Morphing

[Beier 1992]

• Bilateral Filtering

[Paris 2008]
Morphing

- Beier and Neely, 1992:
  - Align facial features
Morphing

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  – Align facial features

Why align features?
Morphing

• Beier and Neely, 1992:
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Why align features?
Morphing: Align Features

- Associate primitives: e.g. lines
Morphing: Align Features

- Associate primitives: e.g. lines

- Move primitives so that they align (at some intermediate location) -> warp accordingly
Morphing: Align Features

• A simple case: 1 image, 1 primitive:
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

Where this pixel should come from?
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

1. Find local coordinates: $u,v$

\[ X = u \cdot PQ + v \cdot PQ^\perp \]
Morphing: Align Features

- A simple case: 1 image, 1 primitive:

1. Find local coordinates: $u, v$
   \[ X = P + u \cdot PQ + v \cdot PQ^\perp \]

2. Location in original image: $X' = P' + u \cdot P'Q' + v \cdot P'Q'^\perp$
Morphing: Align Features

• Multiple lines?

\[ X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \]
Morphing: Align Features

- Multiple lines?

\[ X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 = 0 \neq 0 \]

Line 1 did not move
Morphing: Align Features

• Multiple lines?

- Find $X' = X + w_1 \cdot d_1 + w_2 \cdot d_2$

- $d_2 \neq 0$

- $\approx 0.6 \approx 0.4 \neq 0$

Line 1 is longer and closer
Morphing

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Topics

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Bilateral Filtering

Taken from
SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/

Input

Gaussian Blur

Bilateral Filtering
Bilateral Filtering

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Input

Gaussian Blur

Bilateral Filtering

Edge-preserving
Bilateral Filtering

• How?
Bilateral Filtering

• How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]
Bilateral Filtering

• How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x Later
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x

Later

Go over every pixel $\xi$ in image
Bilateral Filtering

• How?

\[ h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x

Later

Go over every pixel \( \xi \) in image

Value at a pixel
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)
Later
Go over every pixel \( \xi \) in image
Value at a pixel
Is pixel close to \( x \)?
Bilateral Filtering

- How?

\[ h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x

Go over every pixel \( \xi \) in image

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

- How?

\[ h(x) = \frac{1}{2\pi \sigma^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) \, d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\( x \)

\( \xi \)

0

???
Bilateral Filtering

• How?

\[ h(x) = \frac{k}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi} \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?

\[ \approx 1 \]

???
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) \, d\xi \]

Filtered value at pixel x

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \text{Later} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

- How?

\[ h(x) = \frac{1}{da1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \) ?

≈1

≈0

EDGE!

Value at a pixel

Is pixel close to \( x \)?

≈1
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{da} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \) ?

Is pixel close to \( x \) ?

\( x \)

\( \xi \)
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Later

Value at a pixel

Is pixel similar to \( x \)?

\( \approx 0 \)

\( \approx 0 \)

\( \approx 1 \)

Is pixel close to \( x \)?

\( \approx 0 \)

FAR!
Bilateral Filtering

- **How?**

\[ h(x) = \frac{1}{d_x} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

• How?

$$h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x
Later
Go over every pixel $\xi$ in image
Value at a pixel
Is pixel close to $x$?
Is pixel similar to $x$?
$\approx 1$
$\approx 0$
FAR!
EDGE!

$\approx 0$
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{k_d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?

\[ c(\xi - x) = e^{-\frac{1}{2} \left( \frac{\| \xi - x \|}{\sigma_d} \right)^2} \]

\[ s(\xi - x) = e^{-\frac{1}{2} \left( \frac{\| f(\xi) - f(x) \|}{\sigma_r} \right)^2} \]
Bilateral Filtering

- How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) \, d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

Normalization:

\[ k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x) s(\xi - x) \, d\xi \]
Bilateral Filtering

- In Practice?
  - Not going to infinity, but using a window
  - Using ‘for’ loops instead of integral
  - Similar for
    \[
    k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x)s(\xi - x)d\xi
    \]
Questions?

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