Introduction to Theoretical Computer Science

Introduction to Theoretical CS

Fundamental questions:

Q. What can a computer do?

Q. What can a computer do with limited resources?

General approach.

- Don't talk about specific machines or problems.
- Consider minimal abstract machines.
- Consider general classes of problems.

Why Learn Theory?

In theory …

- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice …

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

Why Learn Theory?

In theory …

- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice …

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

> *" In theory there is no difference between theory and practice. In practice there is."* – *Yogi Berra*

Regular Expressions

Pattern matching problem. Is a given string in a specified set of strings?

Ex. [genomics]

- Fragile X syndrome is a common cause of mental retardation.
- Human genome contains triplet repeats of CGG or AGG, bracketed by GCG at the beginning and CTG at the end.
- Number of repeats is variable, and correlated with syndrome.

Specified set of strings: "all strings of G, C, T, A having some occurrence of GCG followed by any number of CGG or AGG triplets, followed by CTG"

Q: "Is this string in the set?"

`GCGGCGTGTGTGCGAGAGAGTGGGTTTAAAGCTGGCGCGGAGGCGGCTGGCGCGGAGGCTG`

A: Yes `GCG|CGG|AGG|CGG|CTG`

First step:

Regular expression. A formal notation for specifying a set of strings.

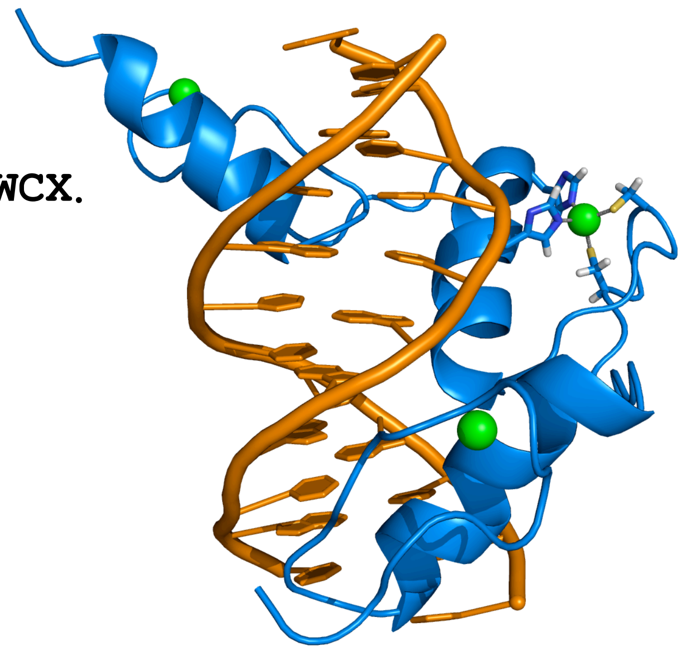
Pattern Matching Application

PROSITE. Huge database of protein families and domains.

Q. How to describe a protein motif?

Ex. [signature of the C_2H_2-type zinc finger domain]

1. C
2. Between 2 and 4 amino acids.
3. C
4. 3 more amino acids.
5. One of the following amino acids: **LIVMFYWCX**.
6. 8 more amino acids.
7. H
8. Between 3 and 5 more amino acids.
9. H



A. Use a regular expression.

CAASCGGPYACGGWAGYHAGWH

Pattern Matching Applications

Test if a string matches some pattern.

- Process natural language.
- Scan for virus signatures.
- Access information in digital libraries.
- Search-and-replace in a word processors.
- Filter text (spam, NetNanny, ads, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).
- Search for markers in human genome using PROSITE patterns.

Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read in data stored in TOY input file format.
- Automatically create Java documentation from Javadoc comments.

Regular Expressions: Basic Operations

Regular expression. Notation to specify a set of strings.

"in specified set" "not in specified set"

operation	regular expression	matches	does not match
concatenation	`aabaab`	`aabaab`	*every other string*
wildcard	`.u.u.u.`	`cumulus` `jugulum`	`succubus` `tumultuous`
union	`aa | baab`	`aa` `baab`	*every other string*
closure	`ab*a`	`aa` `abbba`	`ab` `ababa`
parentheses	`a(a|b)aab`	`aaaab` `abaab`	*every other string*
	`(ab)*a`	`a` `ababababa`	`aa` `abbba`

Regular Expressions: Examples

Regular expression. Notation is surprisingly expressive.

regular expression	matches	does not match
`.*spb.*` *contains the trigraph* **spb**	raspberry crispbread	subspace subspecies
`a* | (a*ba*ba*ba*)*` *multiple of three* **b***'s*	bbb aaa bbbaababbaa	b bb baabbbaa
`.*0....` *fifth to last digit is* **0**	1000234 98701234	111111111 403982772
`gcg(cgg|agg)*ctg` *fragile X syndrome indicator*	gcgctg gcgcggctg gcgcggaggctg	gcgcgg cggcggcggctg gcgcaggctg

Generalized Regular Expressions

Regular expressions are a standard programmer's tool.

- Built in to Java, Perl, Unix, Python, ….
- Additional operations typically added for convenience.
  - Ex 1: `[a-e]+` is shorthand for `(a|b|c|d|e)(a|b|c|d|e)*`.
  - Ex 2: `\s` is shorthand for "any whitespace character" (space, tab, …).

operation	regular expression	matches	does not match
one or more	`a(bc)+de`	abcde abcbcde	ade bcde
character class	`[A-Za-z][a-z]*`	lowercase Capitalized	camelCase 4illegal
exactly k	`[0-9]{5}-[0-9]{4}`	08540-1321 19072-5541	111111111 166-54-1111
negation	`[^aeiou]{6}`	rhythm	decade

Regular Expression Challenge 1

Q. Consider the RE

`a*bb(ab|ba)*`

Which of the following strings match (is in the set described)?

a. **abb**

b. **abba**

c. **aaba**

d. **bbbaab**

e. **cbb**

f. **bbababbab**

Regular Expression Challenge 2

Q. Give an RE that describes the following set of strings:

- characters are **A**, **C**, **T** or **G**
- starts with **ATG**
- length is a multiple of 3
- ends with **TAG**, **TAA**, or **TTG**

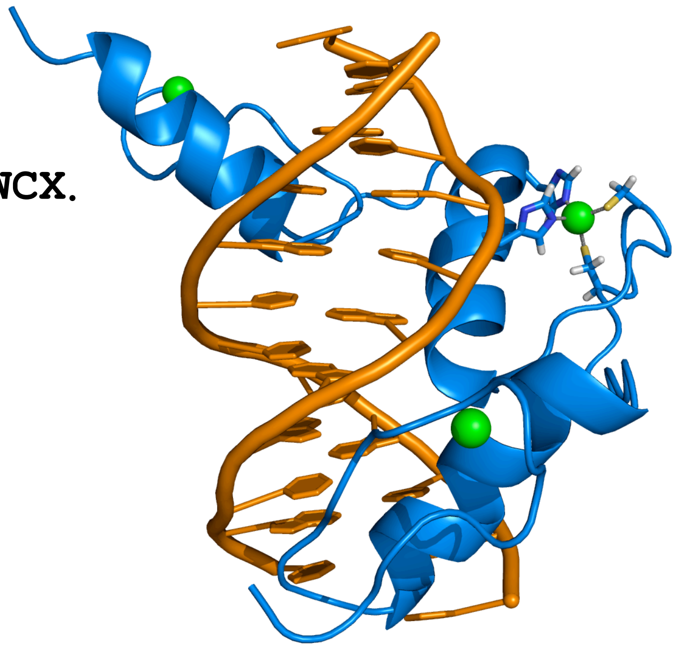
Describing a Pattern

PROSITE. Huge database of protein families and domains.

Q. How to describe a protein motif?

Ex. [signature of the C_2H_2-type zinc finger domain]

1. **C**
2. Between 2 and 4 amino acids.
3. **C**
4. 3 more amino acids.
5. One of the following amino acids: **LIVMFYWCX**.
6. 8 more amino acids.
7. **H**
8. Between 3 and 5 more amino acids.
9. **H**



A. **C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H**

CAASCGGPYACGGWAGYHAGWH

REs in Java

`public class String` *(Java's String library)*

`boolean matches(String re)`	*does this string match the given regular expression?*
`String replaceAll(String re, String str)`	*replace all occurrences of regular expression with the replacement string*
`int indexOf(String r, int from)`	*return the index of the first occurrence of the string r after the index from*
`String[] split(String re)`	*split the string around matches of the given regular expression*

```
String re = "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H ";
String input = "CAASCGGPYACGGAAGYHAGAH";
boolean test = input.matches(re);
```

is the input string in the set described by the RE?

REs in Java

Validity checking. Is `input` in the set described by the `re`?

```
public class Validate
{
   public static void main(String[] args) {
      String re    = args[0];
      String input = args[1];
      StdOut.println(input.matches(re));
   }
}
```

powerful string library method

C_2H_2 type zinc finger domain

```
% java Validate "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H" CAASCGGPYACGGAAGYHAGAH
true
```

legal Java identifier

```
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true
```

valid email address (simplified)

```
% java Validate "[a-z]+@([a-z]+\.)+(edu|com)" doug@cs.princeton.edu
true
```

need quotes to "escape" the shell

REs in Java

`public class String` *(Java's String library)*

`boolean matches(String re)`	*does this string match the given regular expression?*
`String replaceAll(String re, String str)`	*replace all occurrences of regular expression with the replacement string*
`int indexOf(String r, int from)`	*return the index of the first occurrence of the string r after the index from*
`String[] split(String re)`	*split the string around matches of the given regular expression*

```
String s = StdIn.readAll();
s = s.replaceAll("\\s+", " ");
```

RE that matches any sequence of whitespace characters (at least 1).

Extra \ distinguishes from the string \s+

replace each sequence of at least one whitespace character with a single space

REs in Java

`public class String` *(Java's String library)*

`boolean matches(String re)`	*does this string match the given regular expression?*
`String replaceAll(String re, String str)`	*replace all occurrences of regular expression with the replacement string*
`int indexOf(String r, int from)`	*return the index of the first occurrence of the string r after the index from*
`String[] split(String re)`	*split the string around matches of the given regular expression*

```
String s = StdIn.readAll();
String[] words = s.split("\\s+");
```

create an array of the words in StdIn

DFAs

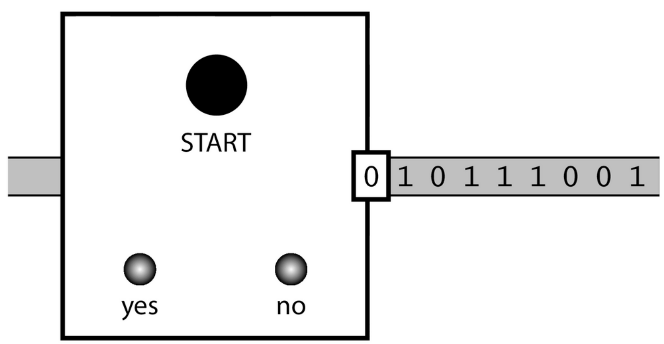
Solving the Pattern Match Problem

Regular expressions are a concise way to describe patterns.

- How would you implement the method **`matches()`** ?
- Hardware: build a deterministic finite state automaton (DFA).
- Software: simulate a DFA.

DFA: simple machine that solves a pattern match problem.

- Different machine for each pattern.
- Accepts or rejects string specified on input tape.
- Focus on **`true`** or **`false`** questions for simplicity.

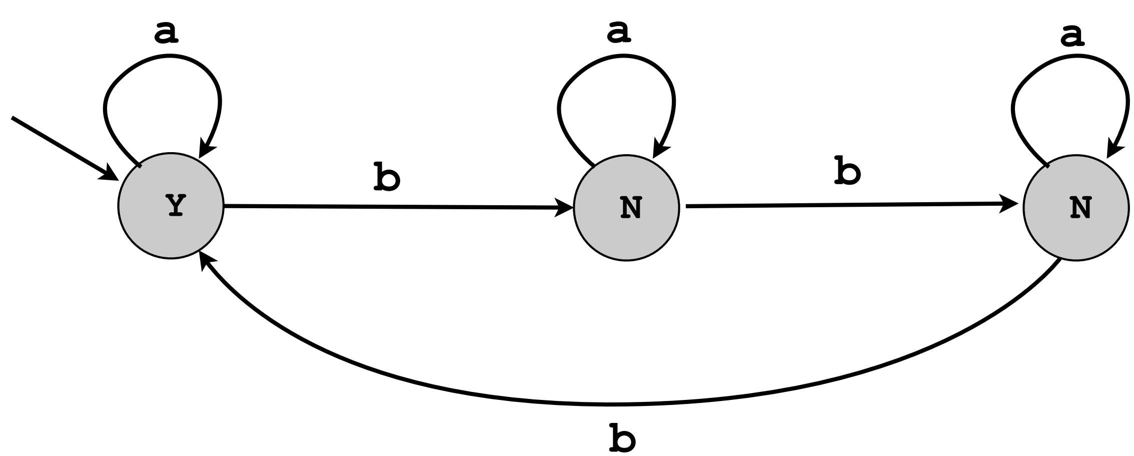


Deterministic Finite State Automaton (DFA)

Simple machine with N states.

- Begin in start state.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- Accept input string if last state is labeled Y.

DFA



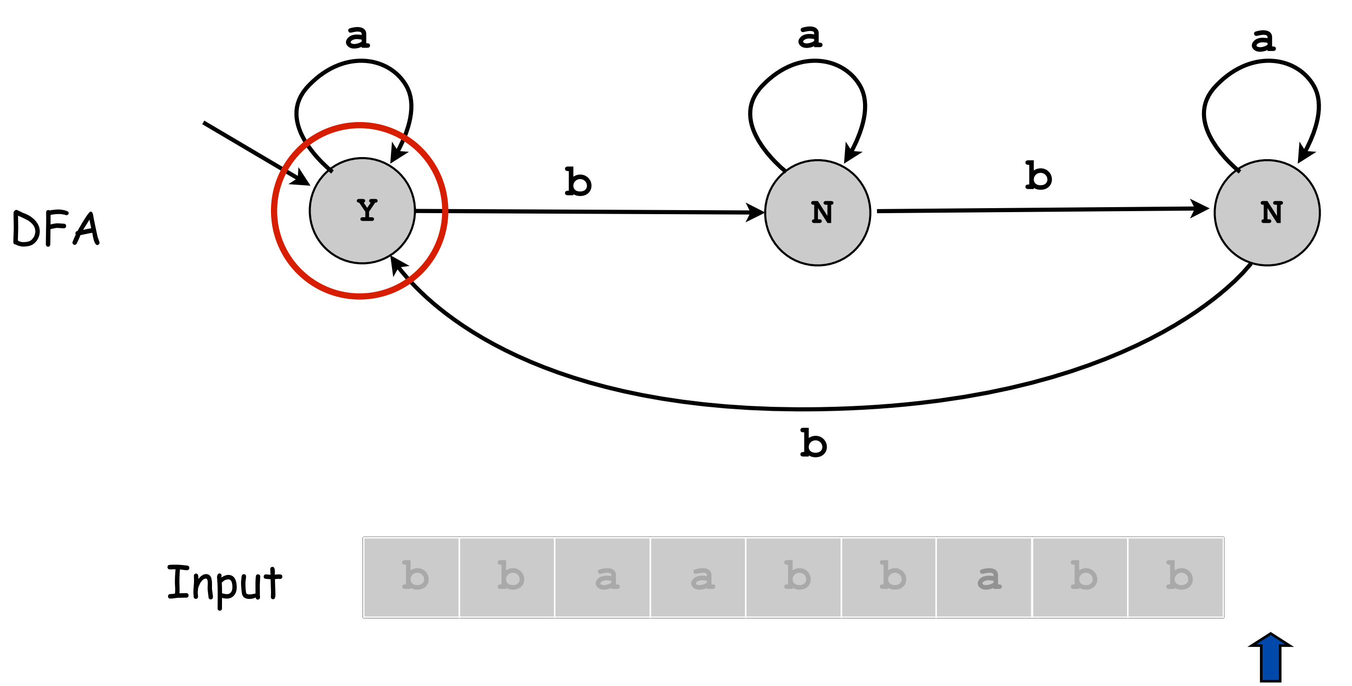
Input

b	b	a	a	b	b	a	b	b

Deterministic Finite State Automaton (DFA)

Simple machine with N states.

- Begin in start state.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- Accept input string if last state is labeled Y.



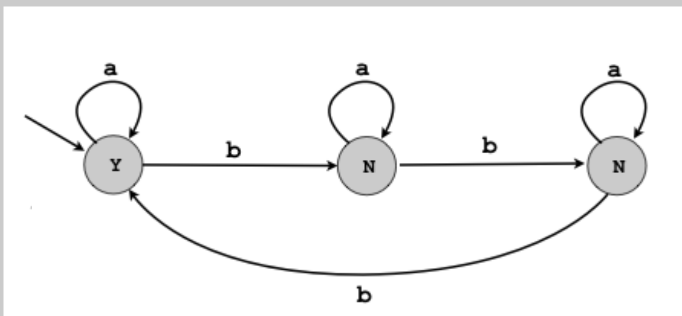
DFA and RE Duality

RE. Concise way to describe a set of strings.

DFA. Machine to recognize whether a given string is in a given set.

Duality.

- For any DFA, there exists a RE that describes the same set of strings.
- For any RE, there exists a DFA that recognizes the same set.



multiple of 3 b's

`a* | (a*ba*ba*ba*)*`

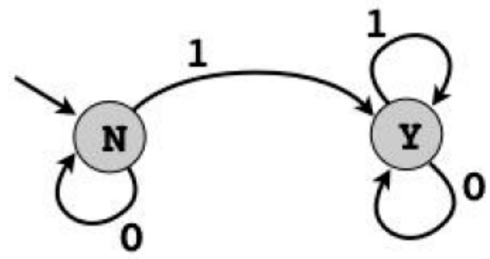
multiple of 3 b's

Practical consequence of duality proof: to match RE

- build DFA
- simulate DFA on input string.

DFA Challenge 1

Q. Consider this DFA:



Which of the following sets of strings does it recognize?

a. Bitstrings with at least one 1

b. Bitstrings with an equal number of occurrences of 01 and 10

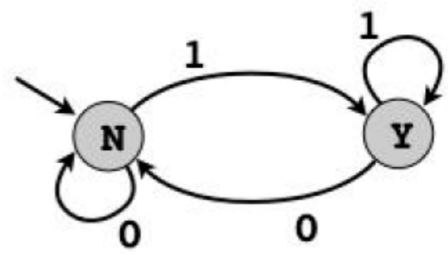
c. Bitstrings with more 1s than 0s

d. Bitstrings with an equal number of occurrences of 0 and 1

e. Bitstrings that end in 1

DFA Challenge 2

Q. Consider this DFA:



Which of the following sets of strings does it recognize?

a. Bitstrings with at least one 1

b. Bitstrings with an equal number of occurrences of 01 and 10

c. Bitstrings with more 1s than 0s

d. Bitstrings with an equal number of occurrences of 0 and 1

e. Bitstrings that end in 1

Implementing a Pattern Matcher

Problem. Given a RE, create program that tests whether given input is in set of strings described.

Step 1. Build the DFA.

- A compiler!
- See COS 226 or COS 320.

It is actually better to use an NFA, an equivalent (but more efficient) representation of a DFA. We ignore that distinction in this lecture.

Step 2. Simulate it with given input.

```
State state = start;
while (!StdIn.isEmpty())
{
   char c = StdIn.readChar();
   state = state.next(c);
}
StdOut.println(state.accept());
```

Direct Application: Harvester

Harvest information from input stream.

- Harvest patterns from DNA.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg
```

- Harvest email addresses from web for spam campaign.

```
% java Harvester "[a-z]+@([a-z]+\.)+(edu|com)" http://www.princeton.edu/~cos126
rs@cs.princeton.edu
dgabai@cs.princeton.edu
doug@cs.princeton.edu
wayne@cs.princeton.edu
```

Direct Application: Harvester

Harvest information from input stream.

- Use `Pattern` data type to compile regular expression to NFA.
- Use `Matcher` data type to simulate NFA.

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester
{
   public static void main(String[] args)
   {
      String re         = args[0];
      In in             = new In(args[1]);
      String input      = in.readAll();
      Pattern pattern = Pattern.compile(re);
      Matcher matcher = pattern.matcher(input);

      while (matcher.find())
         StdOut.println(matcher.group());
   }
}
```

create NFA from RE

create NFA simulator

look for next match

the match most recently found

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg
```

Real-World Application: Parsing a Data File

Java's **`Pattern`** and **`Matcher`** classes

- use REs for pattern matching (previous slide)
- extend REs to facilitate processing string-based data

Ex: parsing an NCBI genome data file.

header info

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus
ORIGIN
      1 tgtatttcat ttgaccgtgc tgtttttcc cggtttttca gtacggtgtt agggagccac
     61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
    121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
    ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

line numbers

spaces

comments

Goal. Extract the data as a single **`actg`** string.

Real-World Application: Parsing a Data File

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class ParseNCBI
{
   public static void main(String[] args)
   {
      String re = "[ ]*[0-9]+([actg ]*).*";
      Pattern pattern = Pattern.compile(re);
      In in = new In(args[0]);
      String data = "";
      while (!in.isEmpty())
      {
         String line = in.readLine();
         Matcher matcher = pattern.matcher(line);
         if (matcher.find())
            data += matcher.group(1).replaceAll(" ", "");
      }
      System.out.println(data);
   }

}
```

identify a "group" in any match

extract the part of match in () [just a, c, t, g and spaces, not line numbers or comments]

remove spaces

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus
ORIGIN
     1 tgtatttcat ttgaccgtgc tgttttttcc cggtttttca gtacggtgtt agggagccac
    61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
   121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
   ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

Limitations of DFA

No DFA can recognize the language of all bit strings with an equal number of 0's and 1's.

- Suppose some N-state DFA can recognize this language.
- Consider following input: $\underbrace{00000000}_{N+1\ 0's}\underbrace{11111111}_{N+1\ 1's}$
- Our DFA must accept this string.
- Some state x is revisited during first N+1 0's since only N states.

0000000011111111
x x

- Machine would accept same string without intervening 0's.

0000011111111
x

- This string doesn't have an equal number of 0's and 1's.

Summary

Programmer.

- Regular expressions are a powerful pattern matching tool.
- Implement regular expressions with finite state machines.

Theoretician.

- Regular expression is a compact description of a set of strings.
- DFA is an abstract machine that solves pattern match problem for regular expressions.
- DFAs and regular expressions have limitations.

Variations

- Yes (accept) and No (reject) states sometimes drawn differently
- Terminology: Deterministic Finite State Automaton (DFA), Finite State Machine (FSM), Finite State Automaton (FSA) are the same
- DFA's can have output, specified on the arcs or in the states
  - These may not have explicit Yes and No states

Fundamental Questions

Q. Are there patterns that cannot be described by any RE/DFA?

A. Yes.

- Bit strings with equal number of 0s and 1s.
- Decimal strings that represent prime numbers.
- DNA strings that are Watson-Crick complemented palindromes.
- and many, many more . . .

Q. Can we extend RE/DFA to describe richer patterns?

A. Yes.

- Context free grammar (e.g., Java).
- Turing machines.

7.4 Turing Machines

Alan Turing (1912-1954)

Turing Machine

Desiderata. Simple model of computation that is "as powerful" as conventional computers.

Intuition. Simulate how humans calculate.

Ex. Addition.

			1	2	3	4	5	6		
		+	3	1	4	1	5	9		

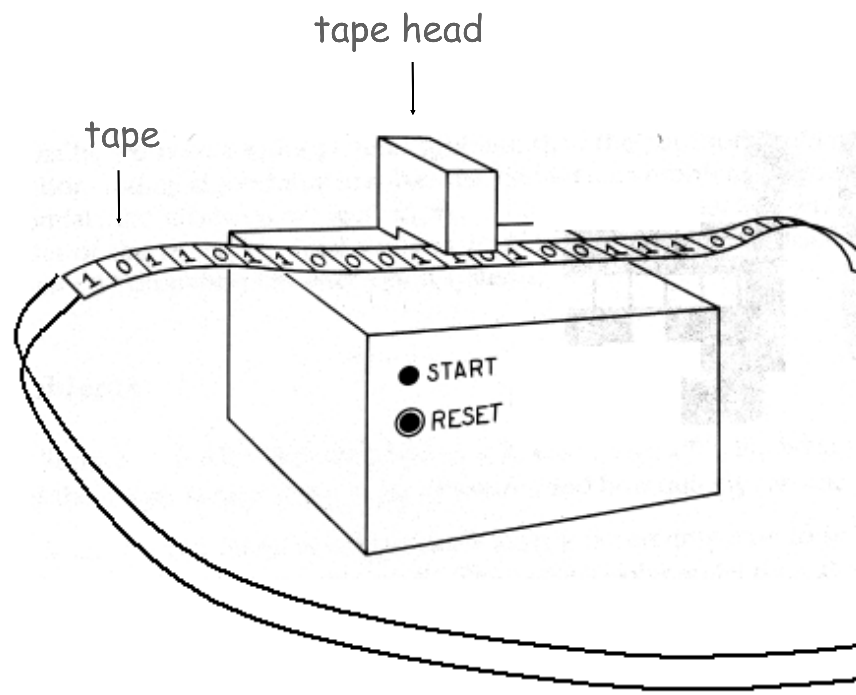
0	0	0	0	1	1	
	1	2	3	4	5	6
+	3	1	4	1	5	9
	4	3	7	6	1	5

Tape.

- Stores input, output, and intermediate results.
- One arbitrarily long strip, divided into cells.
- Finite alphabet of symbols.

Tape head.

- Points to one cell of tape.
- Reads a symbol from active cell.
- Writes a symbol to active cell.
- Moves left or right one cell at a time.



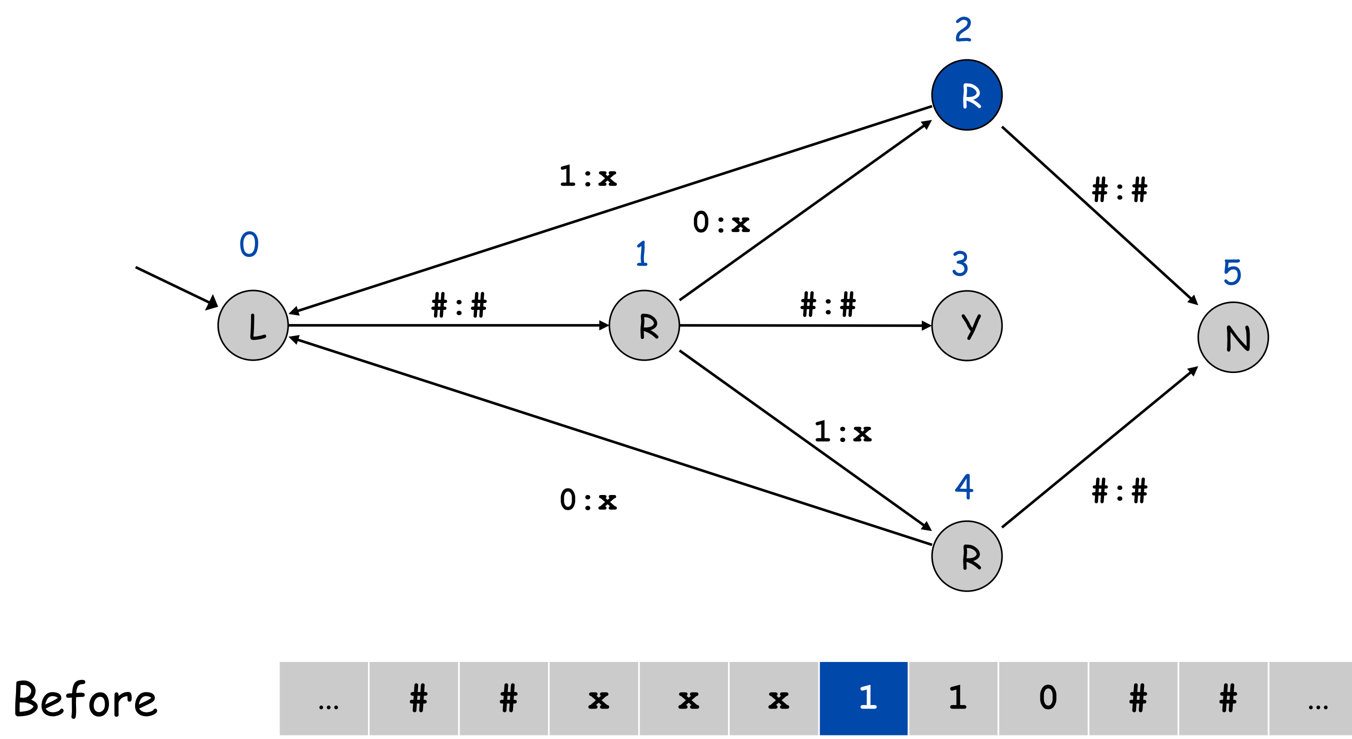
tape head ↓

tape	...	#	1	1	0	0	+	1	0	1	1	#	...

Turing Machine: Execution

States.
- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

State transition diagram.
- Ex. if in state 2 and input symbol is `1` then: overwrite the `1` with `x`, move to state 0, move tape head to left.



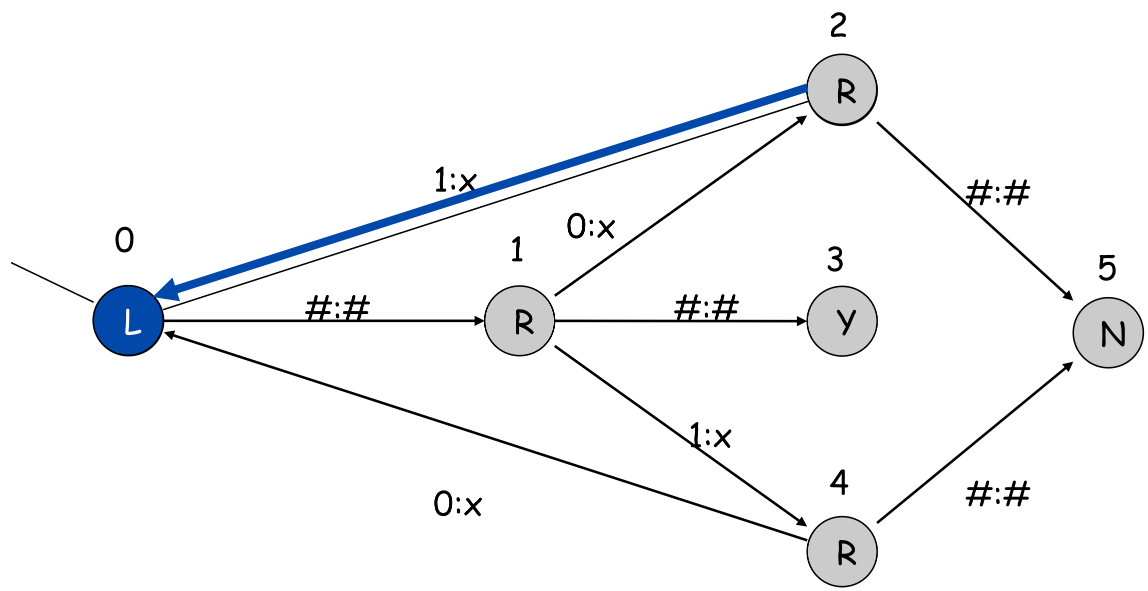
Turing Machine: Execution

States.

- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

State transition diagram.

- Ex. if in state 2 and input symbol is 1 then: overwrite the 1 with x, move to state 0, move tape head to left.



After

...	#	#	x	x	x	x	1	0	#	#	...

Turing Machine: Initialization and Termination

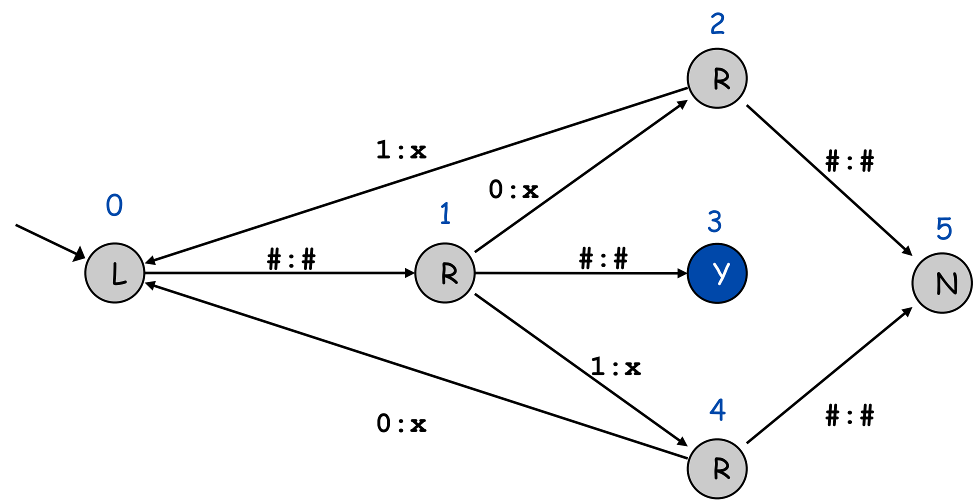
Initialization.

- Set input on some portion of tape.
- Set tape head position.
- Set initial state.

...	#	#	0	0	1	1	1	0	#	#	...

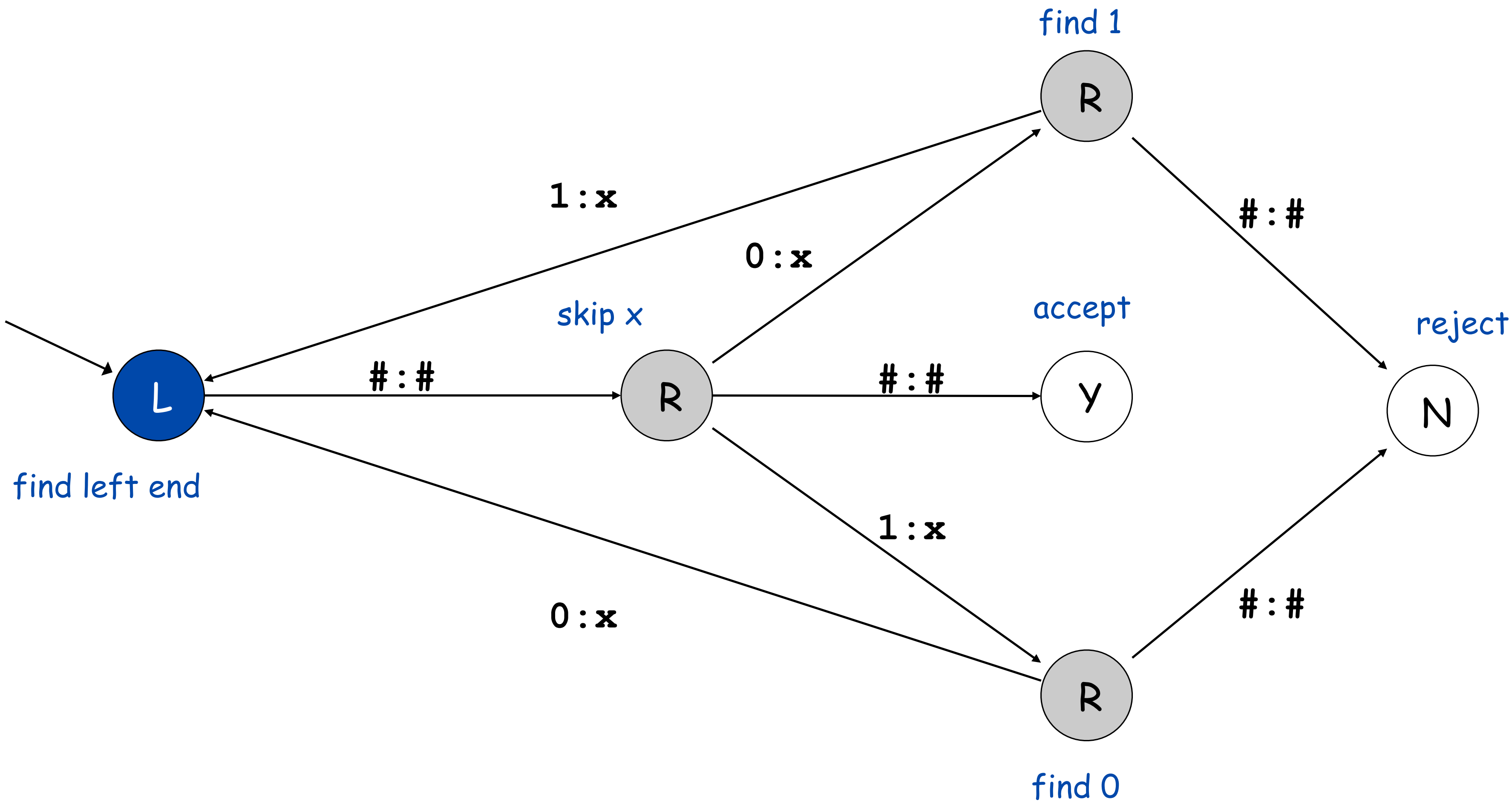
Termination.

- Stop if enter `yes`, `no`, or `halt` state.
- Infinite loop possible.
  - (definitely stay tuned !)



...	#	#	x	x	x	x	x	x	#	#	...

Example: Equal Number of 0's and 1's



...	#	#	0	0	1	1	1	0	#	#	...

Turing Machine Summary

Goal: simplest machine that is "as powerful" as conventional computers.

Surprising Fact 1. Such machines are very simple: TM is enough!
Surprising Fact 2. Some problems cannot be solved by ANY computer.

next lecture

Consequences.

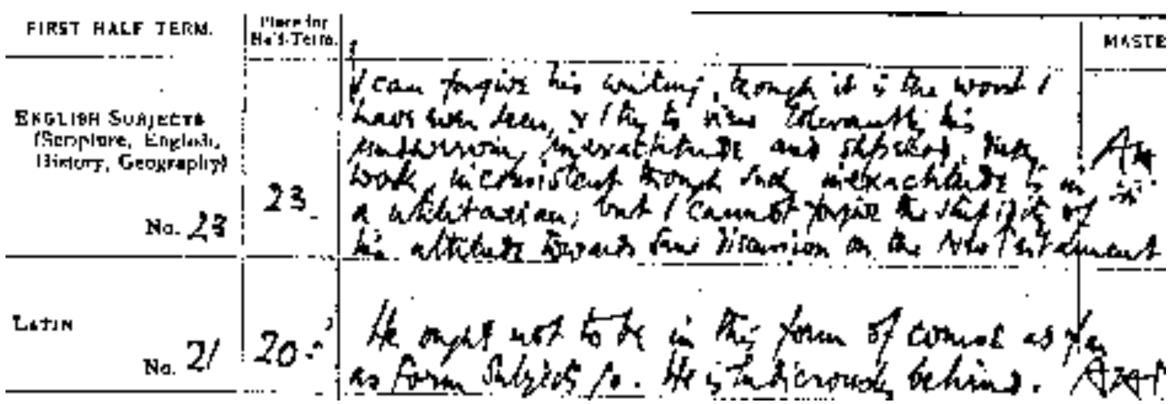
- Precursor to general purpose programmable machines.
- Exposes fundamental limitations of all computers.
- Enables us to study the physics and universality of computation.
- No need to seek more powerful machines!

Variations

- Instead of just recognizing strings, TM's can produce output: the contents of the tape.
- Instead of Y and N states, TM's can have a plain Halt state.

Alan Turing (1912-1954).

- Father of computer science.
- Computer Science's "Nobel Prize" is called the Turing Award.



FIRST HALF TERM.	Place for Half-Term.		MASTE
English Subjects (Scripture, English, History, Geography) No. 23	23	I can forgive his writing, though it is the worst I have ever seen, & I try to view tolerantly his unshakable inexactitude and slipshod, dirty, work, inconsistent though such inexactitude is in a utilitarian; but I cannot forgive the stupidity of his attitude towards sane discussion on the New Testament	ATM
Latin No. 21	20	He ought not to be in this form of course as far as form subjects go. He is far behind.	AJ

Alan's report card at 14.

Alan Turing and his elder brother.