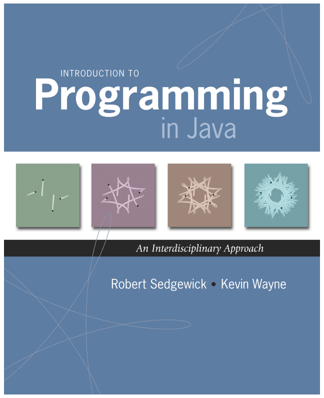
4.3 Stacks and Queues



Data Types and Data Structures

Data types. Set of values and operations on those values.

- Some are built into the Java language: int, double[], String, …
- Most are not: `Complex, Picture, Stack, Queue, ST, Graph,` …

this lecture next lecture

Data structures.

- Represent data or relationships among data.
- Some are built into Java language: arrays.
- Most are not: linked list, circular list, tree, sparse array, graph, …

this lecture TSP assignment next lecture

Collections

Fundamental data types.

- Set of operations (add, remove, test if empty) on generic data.
- Intent is clear when we insert.
- Which item do we remove?

Stack. [LIFO = last in first out] ⟵ this lecture

- Remove the item most recently added.
- Ex: cafeteria trays, Web surfing.

Queue. [FIFO = first in, first out] ⟵ this lecture

- Remove the item least recently added.
- Ex: Hoagie Haven line.

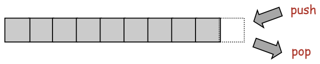
Symbol table. ⟵ next lecture

- Remove the item with a given key.
- Ex: Phone book.

Stacks

Stack API

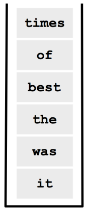
public class *StackOfStrings		
	*StackOfStrings()	*create an empty stack*
boolean	isEmpty()	*is the stack empty?*
void	push(String item)	*push a string onto the stack*
String	pop()	*pop the stack*



Stack Client Example 1: Reverse

```
public class Reverse {
   public static void main(String[] args) {
      StackOfStrings stack = new StackOfStrings();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         stack.push(s);
      }
      while (!stack.isEmpty()) {
         String s = stack.pop();
         StdOut.println(s);
      }
   }
}
```

```
% more tiny.txt
it was the best of times

% java Reverse < tiny.txt
times of best the was it
```

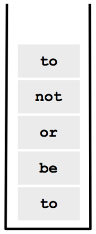


stack contents when standard input is empty

Stack Client Example 2: Test Client

```
public static void main(String[] args) {
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty()) {
      String s = StdIn.readString();
      if (s.equals("-"))
         StdOut.println(stack.pop());
      else
         stack.push(s);
   }
}
```

```
% more test.txt
to be or not to – be – – that – – – is

% java StackOfStrings < test.txt
to be not that or be
```



stack contents just before first pop operation

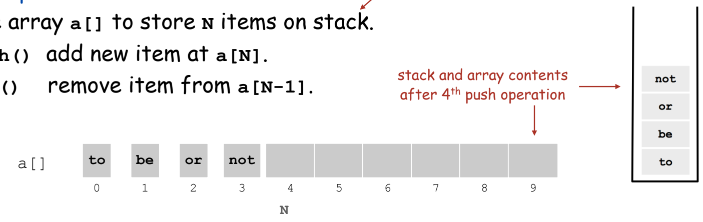
Stack: Array Implementation

Array implementation of a stack.

- Use array a[] to store N items on stack.
- push() add new item at a[N].
- pop() remove item from a[N-1].

how big to make array? [stay tuned]

stack and array contents after 4th push operation



```
public class ArrayStackOfStrings {
   private String[] a;
   private int N = 0;

   public ArrayStackOfStrings(int max) { a = new String[max]; }
   public boolean isEmpty()        { return (N == 0);  }
   public void push(String item)   { a[N++] = item;    }
   public String pop()             { return a[--N];    }
}
```

temporary solution: make client provide capacity

Array Stack: Test Client Trace

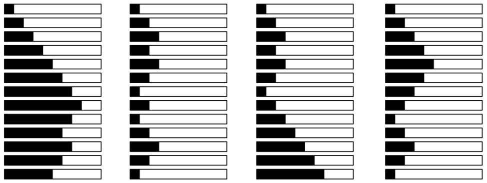
	StdIn	StdOut	N	a[] 0	1	2	3	4
			0					
push	to		1	to				
	be		2	to	be			
	or		3	to	be	or		
	not		4	to	be	or	not	
	to		5	to	be	or	not	to
pop	-	to	4	to	be	or	not	to
	be		5	to	be	or	not	be
	-	be	4	to	be	or	not	be
	-	not	3	to	be	or	not	be
	that		4	to	be	or	that	be
	-	that	3	to	be	or	that	be
	-	or	2	to	be	or	that	be
	-	be	1	to	be	or	that	be
	is		2	to	is	or	not	to

Array Stack: Performance

Running time. Push and pop take constant time.

Memory. Proportional to client-supplied capacity, not number of items.

Problem.
- Original API does not take capacity as argument (bad to change API).
- Client might not know what capacity to use.
- Client might use multiple stacks.



Challenge. Stack where capacity is not known ahead of time.

Linked Lists

Sequential vs. Linked Allocation

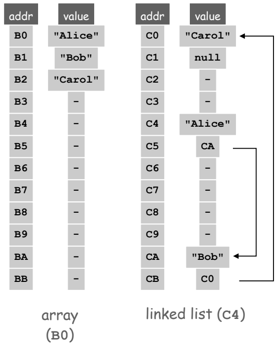
Sequential allocation. Put items one after another.
- TOY: consecutive memory cells.
- Java: array of objects.

Linked allocation. Include in each object a link to the next one.
- TOY: link is memory address of next item.
- Java: link is reference to next item.

Key distinctions.
- Array: random access, fixed size.
- Linked list: sequential access, variable size.

get i^{th} item

get next item



addr	value
B0	"Alice"
B1	"Bob"
B2	"Carol"
B3	-
B4	-
B5	-
B6	-
B7	-
B8	-
B9	-
BA	-
BB	-

array (B0)

addr	value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0

linked list (C4)

Linked Lists

Linked list.

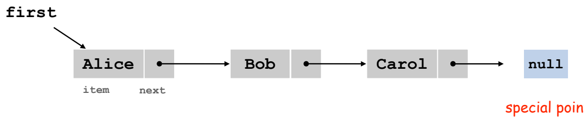
- A recursive data structure.
- An item plus a pointer to another linked list (or empty list).
- Unwind recursion: linked list is a sequence of items.

why private?
stay tuned

Node data type.

- A reference to a `String`.
- A reference to another `Node`.

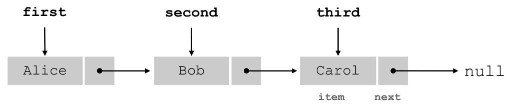
```
private class Node {
   private String item;
   private Node next;
}
```

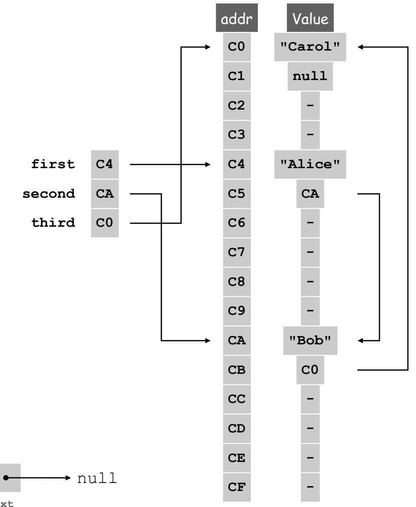


special pointer value `null` terminates list

Building a Linked List

```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```





first	C4
second	CA
third	C0

addr	Value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0
CC	-
CD	-
CE	-
CF	-

main memory

Linked List Demo

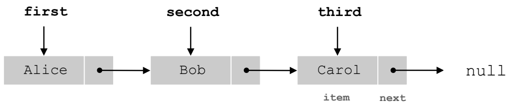
```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```

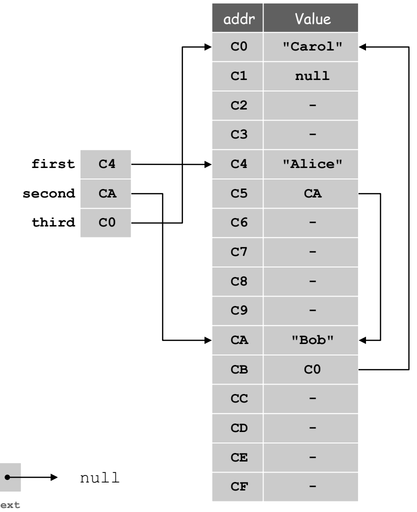
addr	Value
C0	-
C1	-
C2	-
C3	-
C4	-
C5	-
C6	-
C7	-
C8	-
C9	-
CA	-
CB	-
CC	-
CD	-
CE	-
CF	-

main memory

Linked List Demo

```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```





first	C4
second	CA
third	C0

addr	Value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0
CC	-
CD	-
CE	-
CF	-

main memory

List Processing Challenge 1

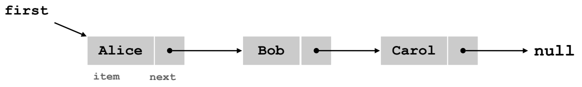
Q. What does the following code fragment do?

```
Node last = new Node();
last.item = StdIn.readString();
last.next = null;
Node first = last;
while (!StdIn.isEmpty()) {
   last.next = new Node();
   last = last.next;
   last.item = StdIn.readString();
   last.next = null;
}
```

List Processing Challenge 2

Q. What does the following code fragment do?

```
for (Node x = first; x != null; x = x.next) {
   StdOut.println(x.item);
}
```

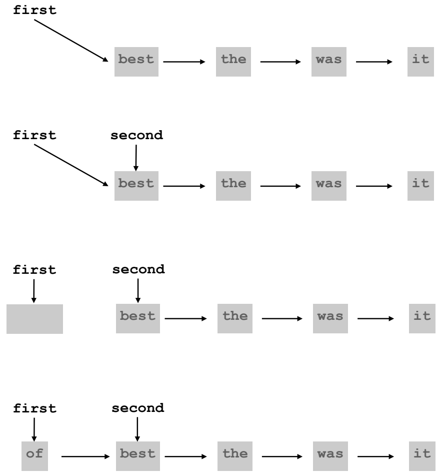


Enough with the Idioms

How about this idea:

- Use a linked list to implement a stack

Stack Push: Linked List Implementation

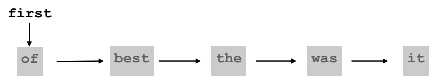


```
Node second = first;
```

```
first = new Node();
```

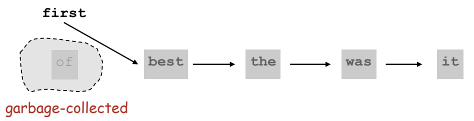
```
first.item = "of";
first.next = second;
```

Stack Pop: Linked List Implementation

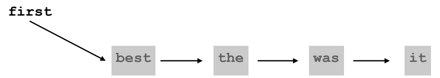


"of"

```
String item = first.item;
```



```
first = first.next;
```



```
return item;
```

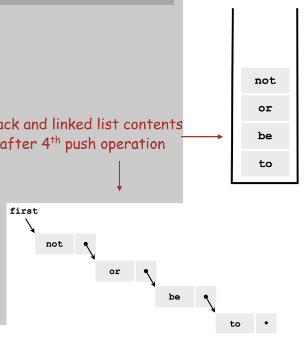
Stack: Linked List Implementation

```
public class LinkedStackOfStrings {
   private Node first = null;

   private class Node {
      private String item;
      private Node next;
   }

   public boolean isEmpty() { return first == null; }

   public void push(String item) {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop() {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

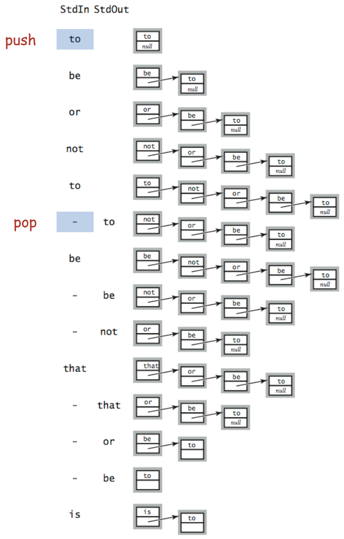
special reserved name

"inner class"

stack and linked list contents after 4th push operation



Linked List Stack: Test Client Trace

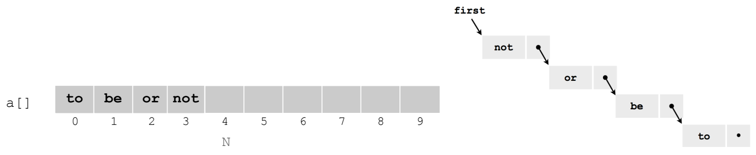


Stack Data Structures: Tradeoffs

Two data structures to implement `Stack` data type.

Array.
- Every push/pop operation take constant time.
- But... must fix maximum capacity of stack ahead of time.

Linked list.
- Every push/pop operation takes constant time.
- Memory is proportional to number of items on stack.
- But... uses extra space and time to deal with references.



Parameterized Data Types

Stack: Linked List Implementation

```
public class LinkedStackOfStrings {
   private Node first = null;

   private class Node {
      private String item;
      private Node next;
   }                    "inner class"

   public boolean isEmpty() { return first == null; }

   public void push(String item) {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop() {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

Parameterized Data Types

We just implemented: `StackOfStrings`.

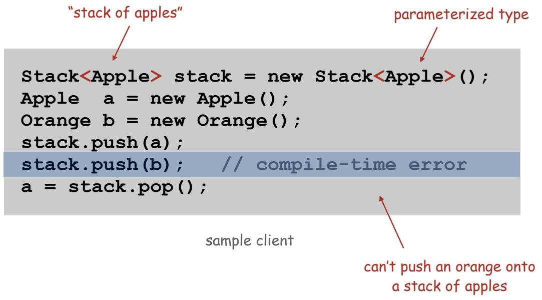
We also want: `StackOfInts`, `StackOfURLs`, `StackOfVans`, …

Strawman. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

Generics

Generics. Parameterize stack by a single type.



```
Stack<Apple> stack = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
stack.push(a);
stack.push(b);    // compile-time error
a = stack.pop();
```

sample client

Generic Stack: Linked List Implementation

```
public class Stack<Item> {
   private Node first = null;

   private class Node {
      private Item item;
      private Node next;
   }

   public boolean isEmpty() { return first == null; }

   public void push(Item item) {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public Item pop() {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

parameterized type name
(chosen by programmer)

Autoboxing

Generic stack implementation. Only permits reference types.

Wrapper type.
- Each primitive type has a wrapper reference type.
- Ex: `Integer` is wrapper type for `int`.

Autoboxing. Automatic cast from primitive type to wrapper type.
Autounboxing. Automatic cast from wrapper type to primitive type.

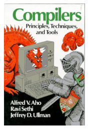
```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);        // autobox     (int -> Integer)
int a = stack.pop();   // auto-unbox (Integer -> int)
```

Stack Applications

Real world applications.
- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

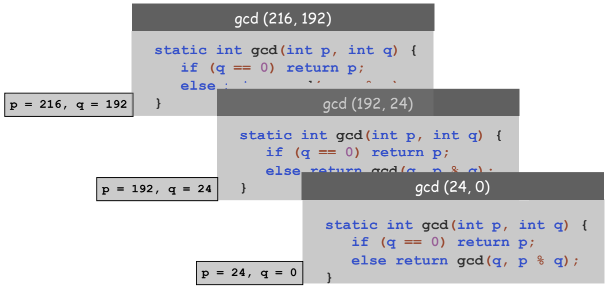
Java

Adobe PostScript



Function Calls

How a compiler implements functions.
- Function call: push local environment and return address.
- Return: pop return address and local environment.



```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

Recursive function. Function that calls itself.
Note. Can always use an explicit stack to remove recursion.

Arithmetic Expression Evaluation

Goal. Evaluate infix expressions.

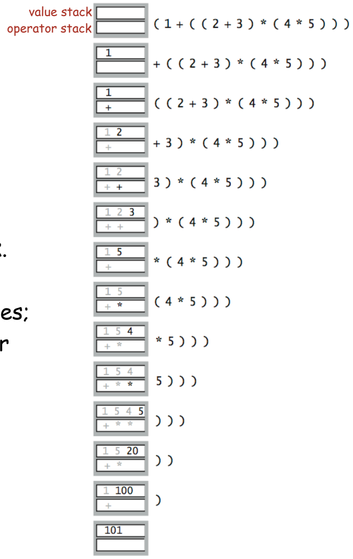
```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

operand operator

Two stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the value stack.

Context. An interpreter!



Arithmetic Expression Evaluation

```
public class Evaluate {
   public static void main(String[] args) {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")")) {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Why correct? When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

So it's as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity, whitespace.

```
1 + (2 - 3 - 4) * 5 * sqrt(6*6 + 7*7)
```

Stack-Based Programming Languages

Observation 1. Remarkably, the 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

Observation 2. All of the parentheses are redundant!

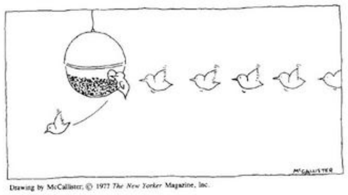
```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, …

Queues

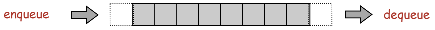


Drawing by McCallister; © 1977 The New Yorker Magazine, Inc.

Queue API

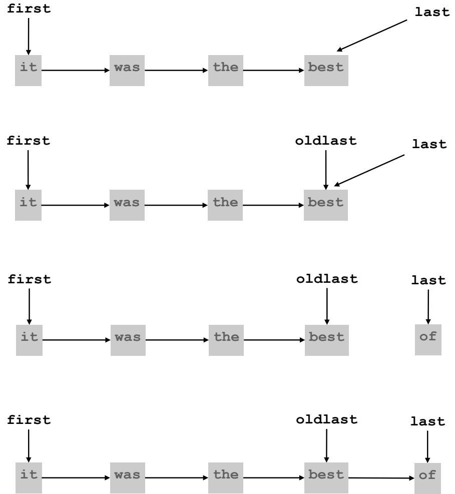
public class Queue<Item>

	Queue<Item>()	*create an empty queue*
boolean	isEmpty()	*is the queue empty?*
void	enqueue(Item item)	*enqueue an item*
Item	dequeue()	*dequeue an item*
int	length()	*queue length*



```
public static void main(String[] args) {
   Queue<String> q = new Queue<String>();
   q.enqueue("Vertigo");
   q.enqueue("Just Lose It");
   q.enqueue("Pieces of Me");
   q.enqueue("Pieces of Me");
   while(!q.isEmpty())
      StdOut.println(q.dequeue());
}
```

Enqueue: Linked List Implementation

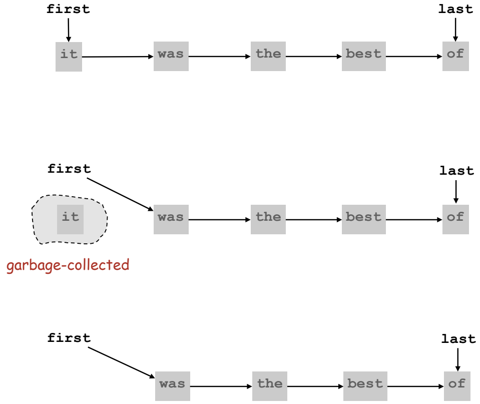


```
Node oldlast = last;
```

```
last = new Node();
last.item = "of";
last.next = null;
```

```
oldlast.next = last;
```

Dequeue: Linked List Implementation



```
String item = first.item;
```

```
first = first.next;
```

```
return item;
```

Queue: Linked List Implementation

```
public class Queue<Item> {
   private Node first, last;

   private class Node { Item item; Node next; }

   public boolean isEmpty() { return first == null; }

   public void enqueue(Item item) {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public Item dequeue() {
      Item item = first.item;
      first     = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```

Queue Applications

Some applications.

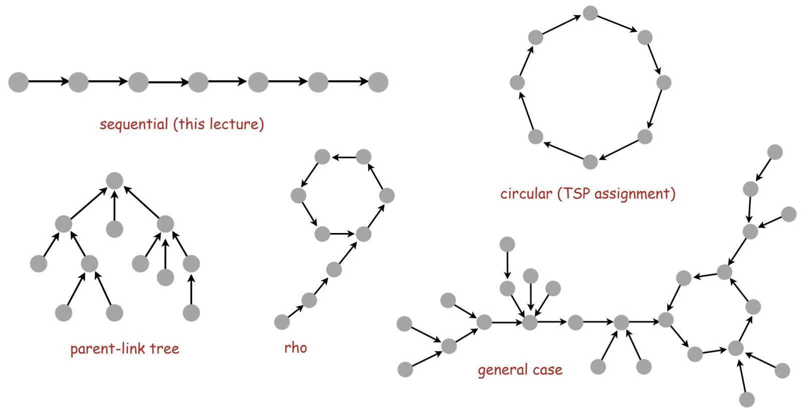
- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Guitar string.
- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.

Singly-Linked Data Structures

From the point of view of a particular object:
all of these structures look the same. ●→



Multiply-linked data structures. Many more possibilities.

Conclusions

Sequential allocation: supports indexing, fixed size.
Linked allocation: variable size, supports sequential access.

Linked structures are a central programming tool.

- Linked lists.
- Binary trees.
- Graphs.
- Sparse matrices.