3.2 Creating Data Types
Data Types

Data type. Set of values and operations on those values.

Basic types.

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Set of Values</th>
<th>Some Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>boolean</td>
<td>true, false</td>
<td>not, and, or, xor</td>
</tr>
<tr>
<td>int</td>
<td>$-2^{31}$ to $2^{31} - 1$</td>
<td>add, subtract, multiply</td>
</tr>
<tr>
<td>String</td>
<td>sequence of Unicode characters</td>
<td>concatenate, compare</td>
</tr>
</tbody>
</table>

Last time. Write programs that use data types.

Today. Write programs to create our own data types.
Defining Data Types in Java

To define a data type, define:

- Set of values.
- Operations defined on them.

Java class. Allows us to define data types by specifying:

- **Instance variables.** (set of values)
- **Methods.** (operations defined on them)
- **Constructors.** (create and initialize new objects)
Point Charge Data Type

**Goal.** Create a data type to manipulate point charges.

**Set of values.** Three real numbers. [position and electrical charge]

**Operations.**
- Create a new point charge at \((r_x, r_y)\) with electric charge \(q\).
- Determine electric potential \(V\) at \((x, y)\) due to point charge.
- Convert to String.

\[
V = k \frac{q}{r}
\]

\[r = \sqrt{(r_x - x)^2 + (r_y - y)^2}\]

\(k = \text{electrostatic constant} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\)
Point Charge Data Type

**Goal.** Create a data type to manipulate point charges.

**Set of values.** Three real numbers. [position and electrical charge]

**API:**

```java
public class Charge {
    public Charge(double x0, double y0, double q0)
    public double potentialAt(double x, double y)  // electric potential at (x, y) due to charge
    public String toString()  // string representation
}
```
Charge Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```java
public static void main(String[] args)
{
    double x = Double.parseDouble(args[0]);
    double y = Double.parseDouble(args[1]);
    Charge c1 = new Charge(.51, .63, 21.3);
    Charge c2 = new Charge(.13, .94, 81.9);
    double v1 = c1.potentialAt(x, y);
    double v2 = c2.potentialAt(x, y);
    StdOut.println(c1);
    StdOut.println(c2);
    StdOut.println(v1 + v2);
}
```

% java Charge .50 .50
21.3 at (0.51, 0.63)
81.9 at (0.13, 0.94)
2.74936907085912e12
**Anatomy of Instance Variables**

**Instance variables.** Specifies the set of values.
- Declare outside any method.
- Always use access modifier `private`.
- Use modifier `final` with instance variables that never change.

```java
public class Charge {
    private final double rx, ry;
    private final double q;
}
```

- Makes data type abstract
- Makes objects immutable (stay tuned)
Anatomy of a Constructor

**Constructor.** Specifies what happens when you create a new object.

Invoking a constructor. Use `new` operator to create a new object.

```java
public Charge(double x0, double y0, double q0) {
    rx = x0;
    ry = y0;
    q = q0;
}
```

```java
Charge c1 = new Charge(.51, .63, 21.3);
Charge c2 = new Charge(.13, .94, 81.9);
```
**Method.** Define operations on instance variables.

```java
public double potentialAt(double x, double y) {
    double k = 8.99e09;
    double dx = x - rx;
    double dy = y - ry;
    return k * q / Math.sqrt(dx*dx + dy*dy);
}
```

**Invoking a method.** Use dot operator to invoke a method in client code.

```java
double v1 = c1.potentialAt(x, y);
double v2 = c2.potentialAt(x, y);
```
Anatomy of a Class

```java
public class Charge {
    private final double rx, ry;
    private final double q;

    public Charge(double x0, double y0, double q0) {
        rx = x0; ry = y0; q = q0;
    }

    public double potentialAt(double x, double y) {
        double k = 8.99e9;
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString() {
        return q + " at " + "(" + rx + ", " + ry + ")";
    }

    public static void main(String[] args) {
        double x = Double.parseDouble(args[0]);
        double y = Double.parseDouble(args[1]);
        Charge c1 = new Charge(.51, .63, 21.3);
        Charge c2 = new Charge(.13, .94, 81.9);
        double v1 = c1.potentialAt(x, y);
        double v2 = c2.potentialAt(x, y);
        StdOut.printf("%.1e\n", (v1 + v2));
    }
}
```
Charge Client Example: Potential Visualization

Potential visualization. Read in N point charges from a file; compute total potential at each point in unit square.

```plaintext
% more charges.txt
9
.51 .63 -100
.50 .50  40
.50 .72  10
.33 .33   5
.20 .20  -10
.70 .70  10
.82 .72   20
.85 .23   30
.90 .12  -50

% java Potential < charges.txt
```
Arrays of objects. Allocate memory for the array; then allocate memory for each individual object.

```java
// Read in the data.
int N = StdIn.readInt();
Charge[] a = new Charge[N];
for (int i = 0; i < N; i++)
{
    double x0 = StdIn.readDouble();
    double y0 = StdIn.readDouble();
    double q0 = StdIn.readDouble();
    a[i] = new Charge(x0, y0, q0);
}
```
// Plot the data.
int SIZE = 512;
Picture pic = new Picture(SIZE, SIZE);
for (int col = 0; col < SIZE; col++)
  for (int row = 0; row < SIZE; row++)
  {
    double V = 0.0;
    for (int i = 0; i < N; i++)
    {
      double x = 1.0 * col / SIZE;
      double y = 1.0 * row / SIZE;
      V += a[i].potentialAt(x, y);
    }
    Color color = getColor(V); // Arbitrary double-Color map.
    pic.set(col, SIZE-1-row, color);
  }
pic.show();
Fix the serious bug in the following code.

```java
public class Charge {
    private double rx, ry;
    private double q;
    public Charge (double x0, double y0, double q0) {
        double rx = x0;
        double ry = y0;
        double q = q0;
    }
}
```
Turtle Graphics
Turtle Graphics

**Goal.** Create a data type to manipulate a turtle moving in the plane.
**Set of values.** Location and orientation of turtle.

**API.**

```java
public class Turtle {
    Turtle(double x0, double y0, double a0)  // create a new turtle at (x0, y0) facing a0 degrees counterclockwise from the x-axis
    void turnLeft(double delta)  // rotate delta degrees counterclockwise
    void goForward(double step)  // move distance step, drawing a line
}
```

// Draw a square.
Turtle turtle = new Turtle(0.0, 0.0, 0.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
public class Turtle
{
    private double x, y; // turtle is at (x, y)
    private double angle; // facing this direction

    public Turtle(double x0, double y0, double a0)
    {
        x = x0;
        y = y0;
        angle = a0;
    }

    public void turnLeft(double delta)
    {
        angle += delta;
    }

    public void goForward(double d)
    {
        double oldx = x;
        double oldy = y;
        x += d * Math.cos(Math.toRadians(angle));
        y += d * Math.sin(Math.toRadians(angle));
        StdDraw.line(oldx, oldy, x, y);
    }
}
public class Ngon
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        double angle = 360.0 / N;
        double step = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < N; i++)
        {
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
public class Spiral
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        double decay = Double.parseDouble(args[1]);
        double angle = 360.0 / N;
        double step = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < 10 * N; i++)
        {
            step /= decay;
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
Spira Mirabilis in Nature
Complex Numbers

Be rational

Get real.

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Complex Number Data Type

Goal. Create a data type to manipulate complex numbers.
Set of values. Two real numbers: real and imaginary parts.

API. 

```java
public class Complex

    Complex(double real, double imag)

    Complex plus(Complex b)  // sum of this number and b
    Complex times(Complex b) // product of this number and b
    double abs()            // magnitude
    String toString()       // string representation
```

\[ a = 3 + 4i, \quad b = -2 + 3i \]
\[ a + b = 1 + 7i \]
\[ a \times b = -18 + i \]
\[ |a| = 5 \]
Applications of Complex Numbers

Relevance. A quintessential mathematical abstraction.

Applications.
• Fractals.
• Impedance in RLC circuits.
• Signal processing and Fourier analysis.
• Control theory and Laplace transforms.
• Quantum mechanics and Hilbert spaces.
• . . .
Complex Number Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```java
public static void main(String[] args)
{
    Complex a = new Complex(3.0, 4.0);
    Complex b = new Complex(-2.0, 3.0);
    Complex c = a.times(b);
    StdOut.println("a = " + a);
    StdOut.println("b = " + b);
    StdOut.println("c = " + c);
}
```

```
% java TestClient
a = 3.0 + 4.0i
b = -2.0 + 3.0i
c = -18.0 + 1.0i
```

Remark. Can't write \( a = b \times c \) since no operator overloading in Java.
public class Complex {

    private final double re;  // instance variables
    private final double im;

    public Complex(double real, double imag) {  // constructor
        re = real;
        im = imag;
    }

    public String toString() {  // methods
        return re + " + " + im + "i";
    }

    public double abs() {  // methods
        return Math.sqrt(re*re + im*im);
    }

    public Complex plus(Complex b) {  // methods
        double real = re + b.re;
        double imag = im + b.im;
        return new Complex(real, imag);
    }

    public Complex times(Complex b) {  // methods
        double real = re * b.re - im * b.im;
        double imag = re * b.im + im * b.re;
        return new Complex(real, imag);
    }

}
Mandelbrot Set

**Mandelbrot set.** A particular set of complex numbers.

**Plot.** Plot \((x, y)\) black if \(z = x + y \, i\) is in the set, and white otherwise.

- No simple formula describes which complex numbers are in set.
- Instead, describe using an algorithm.
Mandelbrot Set

**Mandelbrot set.** Is complex number $z_0$ in set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, then $z_0$ not in set; otherwise $z_0$ is in set.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$Z_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-1/2 + 0i$</td>
</tr>
<tr>
<td>1</td>
<td>$-1/4 + 0i$</td>
</tr>
<tr>
<td>2</td>
<td>$-7/16 + 0i$</td>
</tr>
<tr>
<td>3</td>
<td>$-79/256 + 0i$</td>
</tr>
<tr>
<td>4</td>
<td>$-26527/65536 + 0i$</td>
</tr>
<tr>
<td>5</td>
<td>$-1443801919/4294967296 + 0i$</td>
</tr>
</tbody>
</table>

$z = -1/2$ is in Mandelbrot set

<table>
<thead>
<tr>
<th>$t$</th>
<th>$Z_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1 + i$</td>
</tr>
<tr>
<td>1</td>
<td>$1 + 3i$</td>
</tr>
<tr>
<td>2</td>
<td>$-7 + 7i$</td>
</tr>
<tr>
<td>3</td>
<td>$1 - 97i$</td>
</tr>
<tr>
<td>4</td>
<td>$-9407 - 193i$</td>
</tr>
<tr>
<td>5</td>
<td>$88454401 + 3631103i$</td>
</tr>
</tbody>
</table>

$z = 1 + i$ not in Mandelbrot set
Plotting the Mandelbrot Set

Practical issues.
- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.

Approximate solution.
- Sample from an $N$-by-$N$ grid of points in the plane.
- Fact: if $|z_t| > 2$ for any $t$, then $z$ not in Mandelbrot set.
- Pseudo-fact: if $|z_{255}| \leq 2$ then $z$ "likely" in Mandelbrot set.
Mandelbrot function with complex numbers.
• Is $z$ in the Mandelbrot set?
• Returns white (definitely no) or black (probably yes).

```java
public static Color mand(Complex z0)
{
    Complex z = z0;
    for (int t = 0; t < 255; t++)
    {
        if (z.abs() > 2.0) return Color.WHITE;
        z = z.times(z);
        z = z.plus(z0);
    }
    return Color.BLACK;
}
```

More dramatic picture: replace `Color.WHITE` with grayscale or color.
new `Color(255-t, 255-t, 255-t)`
Complex Number Data Type: Another Client

Plot the Mandelbrot set in gray scale.

```java
public static void main(String[] args) {
    double xc = Double.parseDouble(args[0]);
    double yc = Double.parseDouble(args[1]);
    double size = Double.parseDouble(args[2]);
    int N = 512;
    Picture pic = new Picture(N, N);

    for (int x = 0; x < N; x++)
        for (int y = 0; y < N; y++)
            {
            double x0 = xc - size/2 + size*x/N;
            double y0 = yc - size/2 + size*y/N;
            Complex z0 = new Complex(x0, y0);
            Color color = mand(z0);
            pic.set(x, N-1-y, color);
            }
    pic.show();
}
```
Mandelbrot Set

% java Mandelbrot -.5 0 2

% java Mandelbrot .1045 -.637 .01
Mandelbrot Set

```java
java ColorMandelbrot -1.5 0 2 < mandel.txt
```

-1.5 0 .02

-1.5 0 .002