3.2 Creating Data Types

Data Types

Data type. Set of values and operations on those values.

Basic types.

Data Type	Set of Values	Some Operations
`boolean`	`true, false`	not, and, or, xor
`int`	-2^{31} to $2^{31} - 1$	add, subtract, multiply
`String`	sequence of Unicode characters	concatenate, compare

Last time. Write programs that use data types.
Today. Write programs to create our own data types.

Defining Data Types in Java

To define a data type, define:
- Set of values.
- Operations defined on them.

Java class. Allows us to define data types by specifying:
- Instance variables. (set of values)
- Methods. (operations defined on them)
- Constructors. (create and initialize new objects)

Point Charge Data Type

Goal. Create a data type to manipulate point charges.

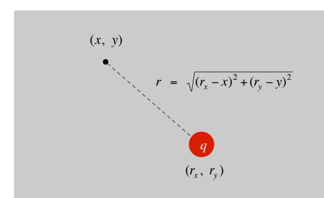
Set of values. Three real numbers. [position and electrical charge]

Operations.
- Create a new point charge at (r_x, r_y) with electric charge q.
- Determine electric potential V at (x, y) due to point charge.
- Convert to `String`.

$$V = k \frac{q}{r}$$

r = distance between (x, y) and (r_x, r_y)

k = electrostatic constant = $8.99 \times 10^9 \ \mathrm{N \cdot m^2 / C^2}$



Point Charge Data Type

Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

API:

`public class Charge`

	`Charge(double x0, double y0, double q0)`	
`double`	`potentialAt(double x, double y)`	*electric potential at (x, y) due to charge*
`String`	`toString()`	*string representation*

Charge Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args)
{
   double x = Double.parseDouble(args[0]);
   double y = Double.parseDouble(args[1]);
   Charge c1 = new Charge(.51, .63, 21.3);
   Charge c2 = new Charge(.13, .94, 81.9);
   double v1 = c1.potentialAt(x, y);
   double v2 = c2.potentialAt(x, y);
   StdOut.println(c1);
   StdOut.println(c2);
   StdOut.println(v1 + v2);
}
```

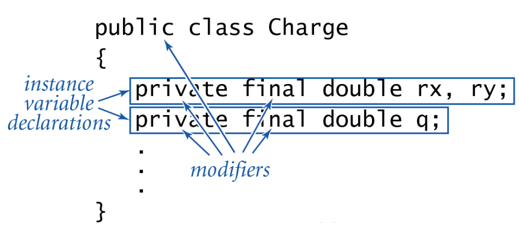
automagically invokes the `toString()` method

```
% java Charge .50 .50
21.3 at (0.51, 0.63)
81.9 at (0.13, 0.94)
2.74936907085912e12
```


Anatomy of Instance Variables

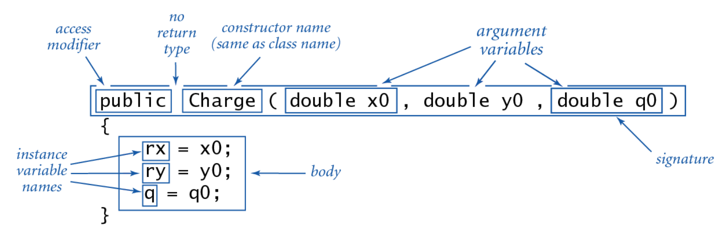
Instance variables. Specifies the set of values.

- Declare outside any method.
- Always use access modifier **private**. ← makes data type abstract
- Use modifier **final** with instance variables that never change. ← makes objects immutable (stay tuned)



Anatomy of a Constructor

Constructor. Specifies what happens when you create a new object.



Invoking a constructor. Use `new` operator to create a new object.

```
Charge c1 = new Charge(.51, .63, 21.3);
Charge c2 = new Charge(.13, .94, 81.9);
```

invoke constructor

Anatomy of an Instance Method

Method. Define operations on instance variables.

```
public double potentialAt(double x, double y)
{
   double k  = 8.99e09;
   double dx = x - rx;
   double dy = y - ry;
   return k * q  / Math.sqrt(dx*dx + dy*dy);
}
```

access modifier; return type; method name; argument variables; signature; local variables; argument variable name; instance variable name; call on a static method; local variable name

Invoking a method. Use dot operator to invoke a method in client code.

```
double v1 = c1.potentialAt(x, y);
double v2 = c2.potentialAt(x, y);
```

object name; invoke method

Anatomy of a Class

```
public class Charge
{
   private final double rx, ry;
   private final double q;

   public Charge(double x0, double y0, double q0)
   {  rx = x0; ry = y0; q = q0;  }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }

   public String toString()
   {  return q +" at " + "("+ rx + ", " + ry +")";  }

   public static void main(String[] args)
   {
      double x = Double.parseDouble(args[0]);
      double y = Double.parseDouble(args[1]);
      Charge c1 = new Charge(.51, .63, 21.3);
      Charge c2 = new Charge(.13, .94, 81.9);
      double v1 = c1.potentialAt(x, y);
      double v2 = c2.potentialAt(x, y);
      StdOut.printf("%.1e\n", (v1 + v2));
   }
}
```

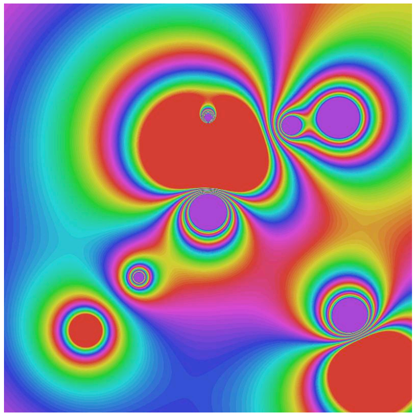
class name; instance variables; constructor; instance methods; instance variable names; test client; create and initialize object; invoke constructor; object name; invoke method

Charge Client Example: Potential Visualization

Potential visualization. Read in N point charges from a file; compute total potential at each point in unit square.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
% java Potential < charges.txt
```



Potential Visualization

Arrays of objects. Allocate memory for the array; then allocate memory for each individual object.

```
// Read in the data.
int N = StdIn.readInt();
Charge[] a = new Charge[N];
for (int i = 0; i < N; i++)
{
   double x0 = StdIn.readDouble();
   double y0 = StdIn.readDouble();
   double q0 = StdIn.readDouble();
   a[i] = new Charge(x0, y0, q0);
}
```

Potential Visualization

```
// Plot the data.
int SIZE = 512;
Picture pic = new Picture(SIZE, SIZE);
for (int col = 0; col < SIZE; col++)
   for (int row = 0; row < SIZE; row++)
   {
      double V = 0.0;
      for (int i = 0; i < N; i++)
      {
         double x = 1.0 * col / SIZE;
         double y = 1.0 * row / SIZE;
         V += a[i].potentialAt(x, y);
      }
      Color color = getColor(V);  // Arbitrary double-Color map.
      pic.set(col, SIZE-1-row, color);
   }
pic.show();
```

$$V = \sum_i (k\, q_i \,/\, r_i)$$

(0, 0) is upper left

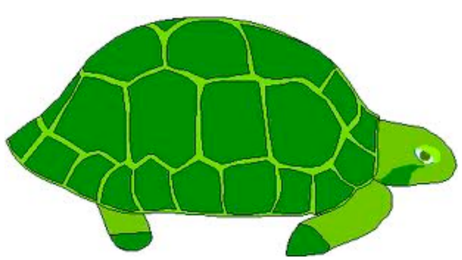
Data Type Challenge

[easy if you read Exercise 3.2.5]

Fix the serious bug in the following code.

```
public class Charge
{
   private double rx, ry;
   private double q;
   public Charge (double x0, double y0, double q0)
   {
      double rx = x0;
      double ry = y0;
      double q = q0;
   }
}
```


Turtle Graphics



Turtle Graphics

Goal. Create a data type to manipulate a turtle moving in the plane.
Set of values. Location and orientation of turtle.

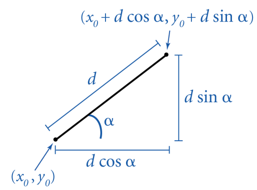
API. `public class Turtle`

	`Turtle(double x0, double y0, double a0)`	*create a new turtle at (x_0, y_0) facing a_0 degrees counterclockwise from the x-axis*
`void`	`turnLeft(double delta)`	*rotate* `delta` *degrees counterclockwise*
`void`	`goForward(double step)`	*move distance* `step`*, drawing a line*

```
// Draw a square.
Turtle turtle = new Turtle(0.0, 0.0, 0.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
```

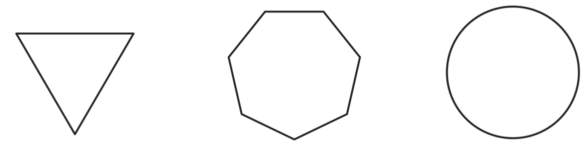
Turtle Graphics Implementation

```
public class Turtle
{
   private double x, y;    // turtle is at (x, y)
   private double angle;   // facing this direction

   public Turtle(double x0, double y0, double a0)
   {
      x = x0;
      y = y0;
      angle = a0;
   }

   public void turnLeft(double delta)
   {
      angle += delta;
   }

   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
}
```



Turtle client example: N-gon

```
public class Ngon
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++)
      {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```



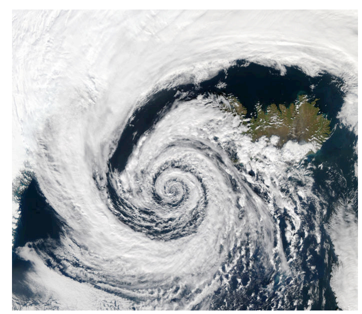
3 7 1440

Turtle client example: Spira Mirabilis

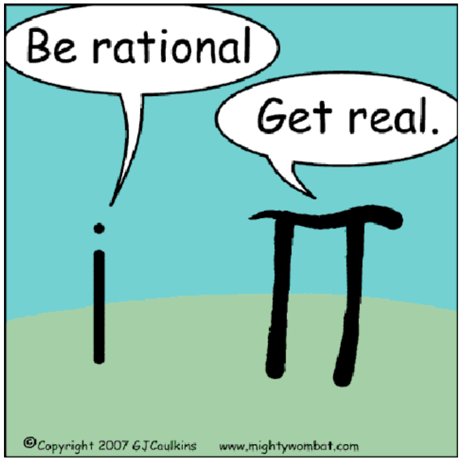
```
public class Spiral
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double decay = Double.parseDouble(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++)
      {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

3 1.0 3 1.2 1440 1.00004 1440 1.0004

Spira Mirabilis in Nature



Complex Numbers



Complex Number Data Type

Goal. Create a data type to manipulate complex numbers.
Set of values. Two real numbers: real and imaginary parts.

API.

`public class Complex`		
	`Complex(double real, double imag)`	
`Complex`	`plus(Complex b)`	*sum of this number and* `b`
`Complex`	`times(Complex b)`	*product of this number and* `b`
`double`	`abs()`	*magnitude*
`String`	`toString()`	*string representation*

$a = 3 + 4i,\ b = -2 + 3i$

$a + b = 1 + 7i$

$a \times b = -18 + i$

$|a| = 5$

Applications of Complex Numbers

Relevance. A quintessential mathematical abstraction.

Applications.
- Fractals.
- Impedance in RLC circuits.
- Signal processing and Fourier analysis.
- Control theory and Laplace transforms.
- Quantum mechanics and Hilbert spaces.
- . . .

Complex Number Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args)
{
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   Complex c = a.times(b);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("c = " + c);
}
```

result of `c.toString()`

```
% java TestClient
a = 3.0 + 4.0i
b = -2.0 + 3.0i
c = -18.0 + 1.0i
```

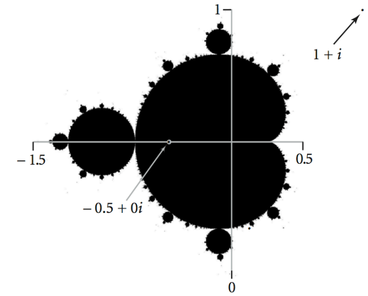
Remark. Can't write `a = b*c` since no operator overloading in Java.

Complex Number Data Type: Implementation

```
public class Complex
{
   private final double re;                         instance variables
   private final double im;

   public Complex(double real, double imag)         constructor
   {
      re = real;
      im = imag;
   }

   public String toString()                         methods
   {  return re + " + " + im + "i"; }

   public double abs()
   {  return Math.sqrt(re*re + im*im); }

   public Complex plus(Complex b)
   {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }

   public Complex times(Complex b)
   {                                               refers to b's instance variables
      double real = re * b.re – im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }

}
```


Mandelbrot Set

Mandelbrot set. A particular set of complex numbers.
Plot. Plot (x, y) black if $z = x + y\,i$ is in the set, and white otherwise.

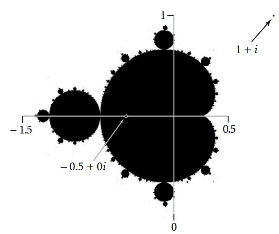


- No simple formula describes which complex numbers are in set.
- Instead, describe using an algorithm.

Mandelbrot Set

Mandelbrot set. Is complex number z_0 in set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, then z_0 not in set; otherwise z_0 is in set.



t	z_t
0	-1/2 + 0i
1	-1/4 + 0i
2	-7/16 + 0i
3	-79/256 + 0i
4	-26527/65536 + 0i
5	-1443801919/4294967296 + 0i

z = -1/2 is in Mandelbrot set

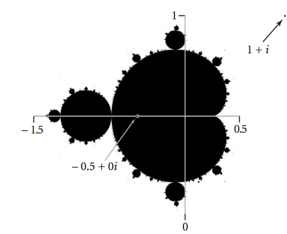
t	z_t
0	1 + i
1	1 + 3i
2	-7 + 7i
3	1 - 97i
4	-9407 - 193i
5	88454401 + 3631103i

z = 1 + i not in Mandelbrot set

Plotting the Mandelbrot Set

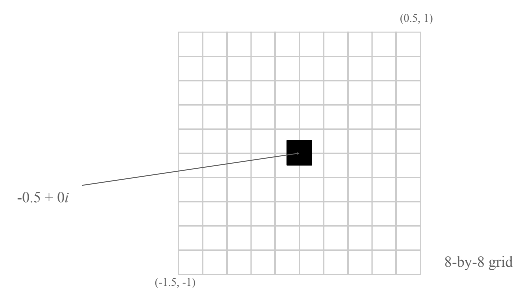
Practical issues.

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.



Approximate solution.

- Sample from an N-by-N grid of points in the plane.
- Fact: if $|z_t| > 2$ for any t, then z not in Mandelbrot set.
- Pseudo-fact: if $|z_{255}| \le 2$ then z "likely" in Mandelbrot set.



Complex Number Data Type: Another Client

Mandelbrot function with complex numbers.

- Is z in the Mandelbrot set?
- Returns white (definitely no) or black (probably yes).

```
public static Color mand(Complex z0)
{
   Complex z = z0;
   for (int t = 0; t < 255; t++)
   {
      if (z.abs() > 2.0) return Color.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return Color.BLACK;
}
```

$z = z^2 + z_0$

More dramatic picture: replace `Color.WHITE` with grayscale or color.

`new Color(255-t, 255-t, 255-t)`

Complex Number Data Type: Another Client

Plot the Mandelbrot set in gray scale.

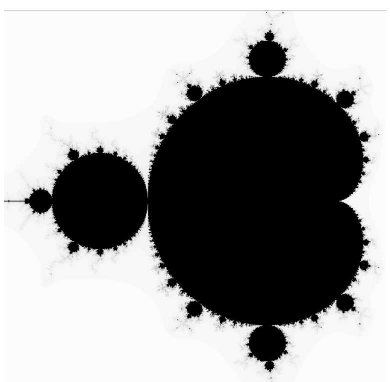
```
public static void main(String[] args)
{
   double xc   = Double.parseDouble(args[0]);
   double yc   = Double.parseDouble(args[1]);
   double size = Double.parseDouble(args[2]);
   int N = 512;
   Picture pic = new Picture(N, N);

   for (int x = 0; x < N; x++)
      for (int y = 0; y < N; y++)
      {
         double x0 = xc - size/2 + size*x/N;
         double y0 = yc - size/2 + size*y/N;
         Complex z0 = new Complex(x0, y0);
         Color color = mand(z0);
         pic.set(x, N-1-y, color);
      }
   pic.show();
}
```

scale to screen coordinates

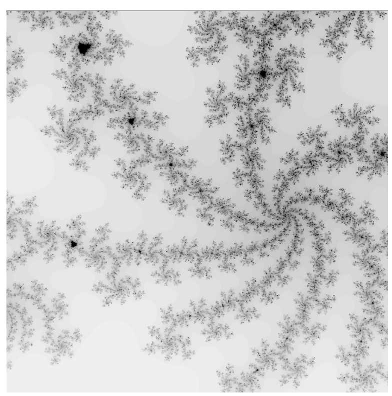
(0, 0) is upper left

Mandelbrot Set

`% java Mandelbrot -.5 0 2`

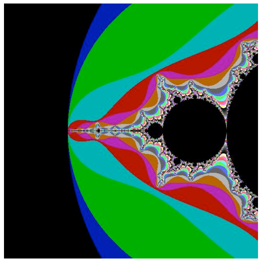


`% java Mandelbrot .1045 -.637 .01`

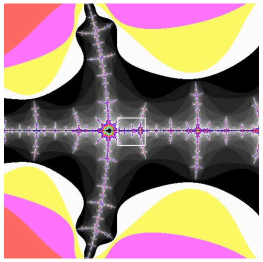


Mandelbrot Set

`% java ColorMandelbrot -1.5 0 2 < mandel.txt`



`-1.5 0 .02`



`-1.5 0 .002`