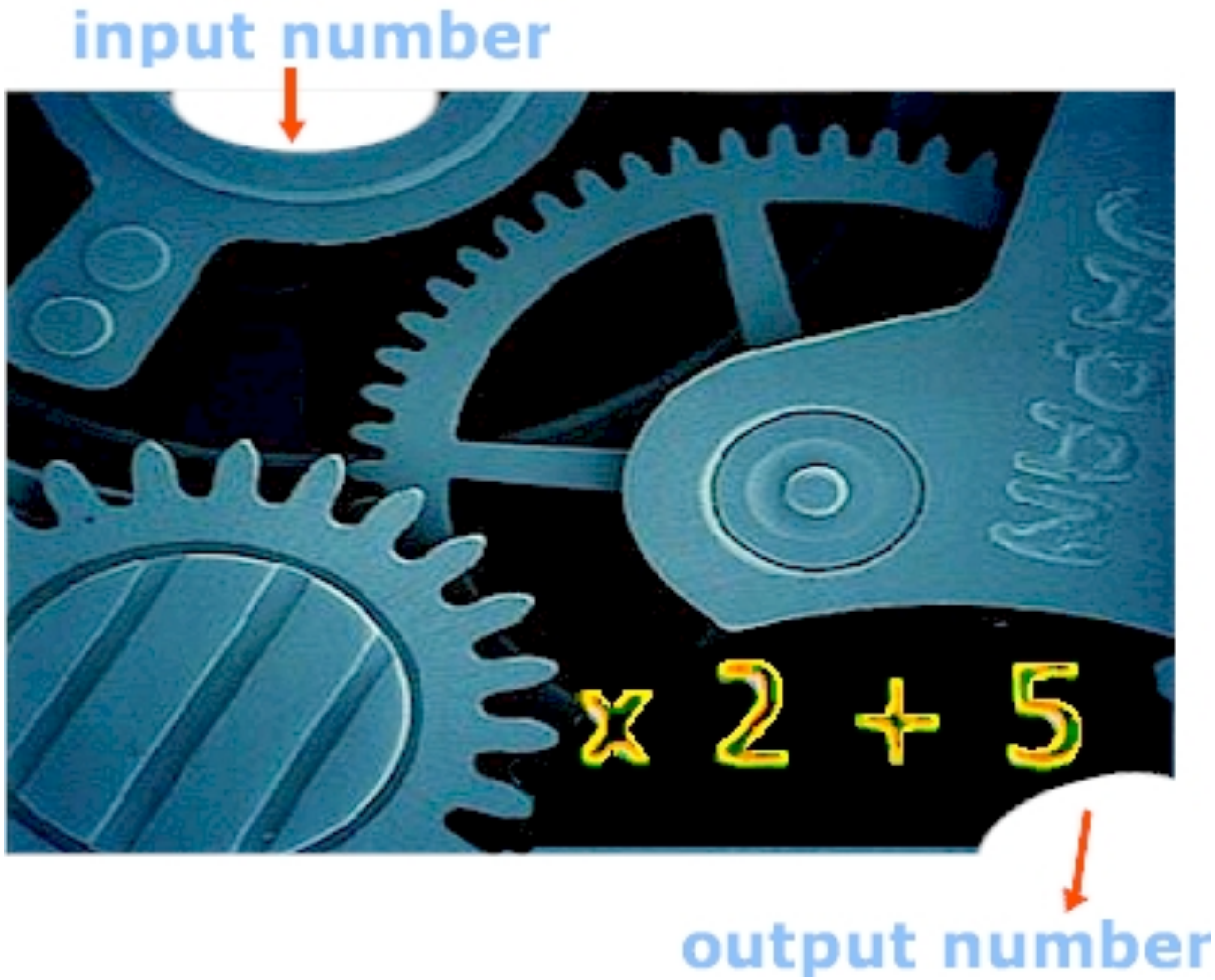
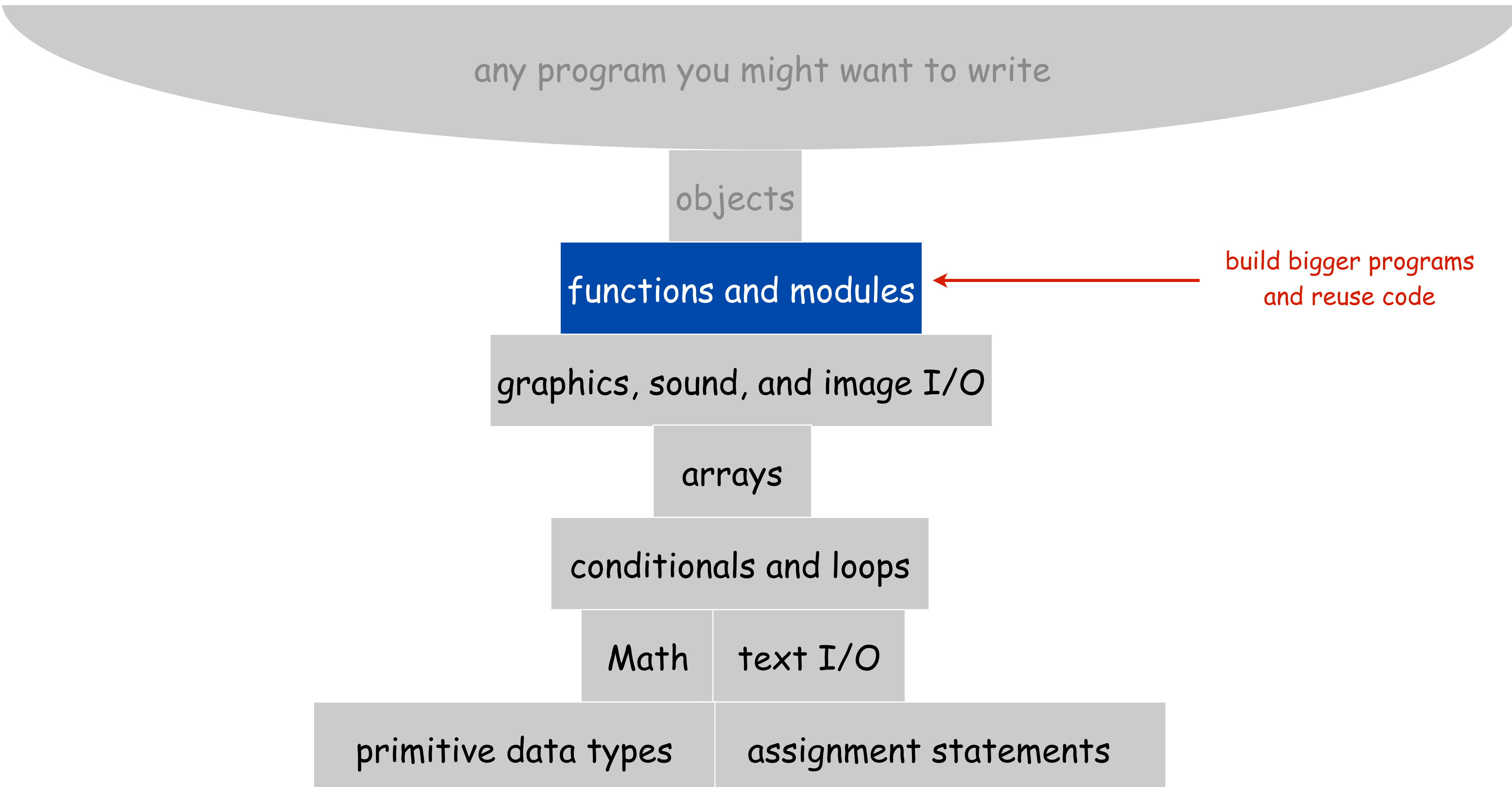
2.1 Functions



A Foundation for Programming



Functions (Static Methods)

Java function.

- Takes zero or more input arguments.
- Returns zero or one output value.
- May cause side effects (e.g., output to standard draw).

more general than mathematical functions

Applications.

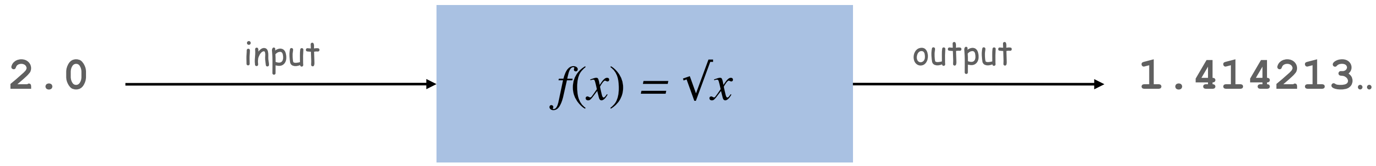
- Scientists use mathematical functions to calculate formulas.
- Programmers use functions to build modular programs.
- You use functions for both.

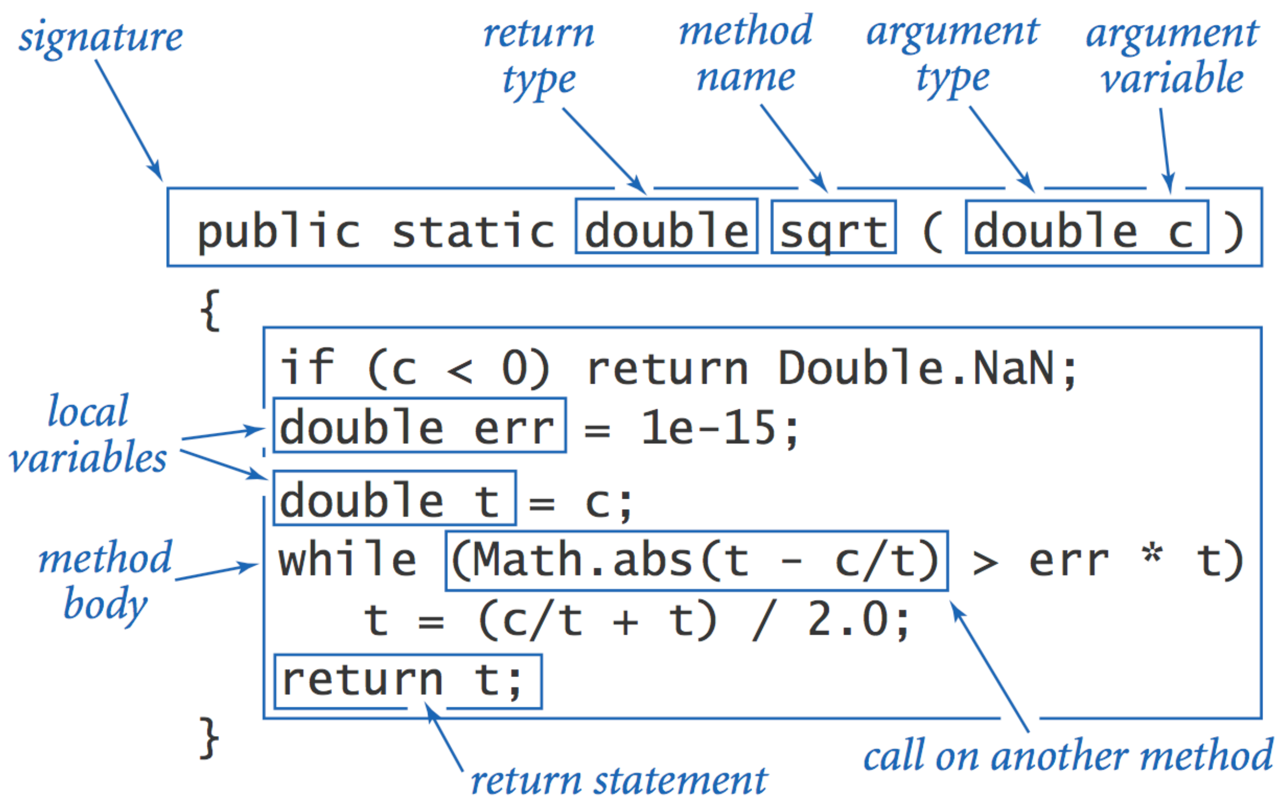
Examples.

- Built-in functions: `Math.random()`, `Math.abs()`, `Integer.parseInt()`.
- Our I/O libraries: `StdIn.readInt()`, `StdDraw.line()`, `StdAudio.play()`.
- User-defined functions: `main()`.

Anatomy of a Java Function

Java functions. Easy to write your own.





Mumbojumbo Demystification, Part 2

```
public class Gambler {
   public static void main(String[] args)  {
      int stake  = Integer.parseInt(args[0]);
      int goal   = Integer.parseInt(args[1]);
      int trials = Integer.parseInt(args[2]);
      . . .
      . . .

}
```

Flow of Control

Key point. Functions provide a new way to control the flow of execution.

```
public class Newton
{
   public static double sqrt(double c)
   {
      double epsilon = 1e-15;
      if (c < 0) return Double.NaN;
      double t = c;
      while (Math.abs(t - c/t) > epsilon * t)
         t = (c/t + t) / 2.0;
      return t;
   }

   public static void main(String[] args)
   {
      double[] a = new double[args.length];
      for (int i = 0; i < args.length; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < a.length; i++)
      {
         double x = sqrt(a[i]);
         StdOut.println(x);
      }
   }
}
```

Flow of Control

Key point. Functions provide a new way to control the flow of execution.

Summary of what happens when a function is called:

- Control transfers to the function code.
- Argument variables are assigned the values given in the call.
- Function code is executed.
- Return value is assigned in place of the function name in the calling code.
- Control transfers back to the calling code.

Note. This technique (standard in Java) is known as "pass by value".

other languages may use different methods

Scope

Scope (of a name). The code that can refer to that name.

Def. A variable's scope is code following the declaration in its block.

```
public class Newton
{
   public static double sqrt(double c)
   {
      double epsilon = 1e-15;
      if (c < 0) return Double.NaN;
      double t = c;
      while (Math.abs(t - c/t) > epsilon * t)
         t = (c/t + t) / 2.0;
      return t;
   }

   public static void main(String[] args)
   {
      double[] a = new double[args.length];
      for (int i = 0; i < args.length; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < a.length; i++)
         System.out.println(sqrt(a[i]));
   }
}
```

scope of **c**

scope of **epsilon**

scope of **t**

scope of **a**

two different variables with the same name **i** each with two lines of scope

Best practice: declare variables so as to limit their scope.

Function Call Trace

```
public class Newton
{
   public static double sqrt(double c)
   {
      double epsilon = 1e-15;
      if (c < 0) return Double.NaN;
      double t = c;
      while (Math.abs(t - c/t) > epsilon * t)
         t = (c/t + t) / 2.0;
      return t;
   }

   public static void main(String[] args)
   {
      double[] a = new double[args.length];
      for (int i = 0; i < args.length; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < a.length; i++)
         System.out.println(sqrt(a[i]));
   }
}
```

Functions Challenge 1

What happens when you compile and run the following code?

```
public class Cubes1
{
   public static int cube(int i)
   {
      int j = i * i * i;
      return j;
   }
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

Functions Challenge 2

What happens when you compile and run the following code?

```
public class Cubes2
{
   public static int cube(int i)
   {
      int i = i * i * i;
      return i;
   }
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

Functions Challenge 3

What happens when you compile and run the following code?

```
public class Cubes3
{
   public static int cube(int i)
   {
      i = i * i * i;
   }
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

Functions Challenge 4

What happens when you compile and run the following code?

```
public class Cubes4
{
   public static int cube(int i)
   {
      i = i * i * i;
      return i;
   }
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

Functions Challenge 5

What happens when you compile and run the following code?

```
public class Cubes5
{
   public static int cube(int i)
   {
      return i * i * i;
   }
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

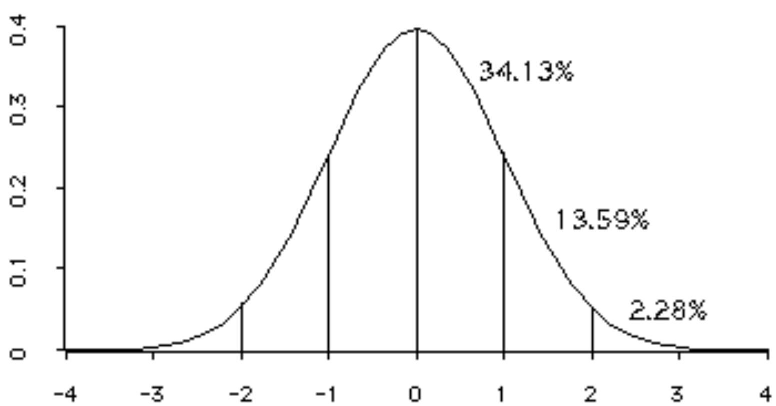
Example: Gaussian Distribution

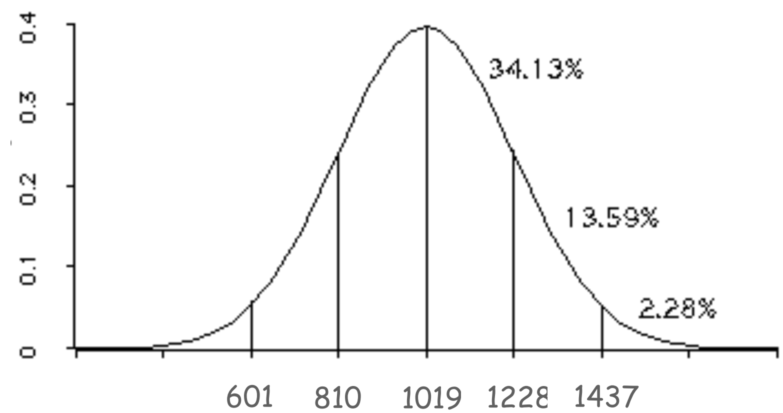
Gaussian Distribution

Standard Gaussian distribution.

- "Bell curve."
- Basis of most statistical analysis in social and physical sciences.

Ex. 2000 SAT scores follow a Gaussian distribution with mean μ = 1019, stddev σ = 209.





$$\phi(x) = \frac{1}{\sqrt{2\pi}} \; e^{-x^2/2}$$

$$\phi(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-(x-\mu)^2 / 2\sigma^2} = \phi\left(\frac{x-\mu}{\sigma}\right) / \sigma$$

Java Function for $\phi(x)$

Mathematical functions. Use built-in functions when possible; build your own when not available.

```
public class Gaussian
{

   public static double phi(double x)
   {
      return Math.exp(-x*x / 2) / Math.sqrt(2 * Math.PI);
   }

   public static double phi(double x, double mu, double sigma)
   {
      return phi((x - mu) / sigma) / sigma;
   }
}
```

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \; e^{-x^2/2}$$

$$\phi(x, \mu, \sigma) \; = \; \phi\left(\frac{x-\mu}{\sigma}\right) / \sigma$$

Overloading. Functions with different signatures are different.

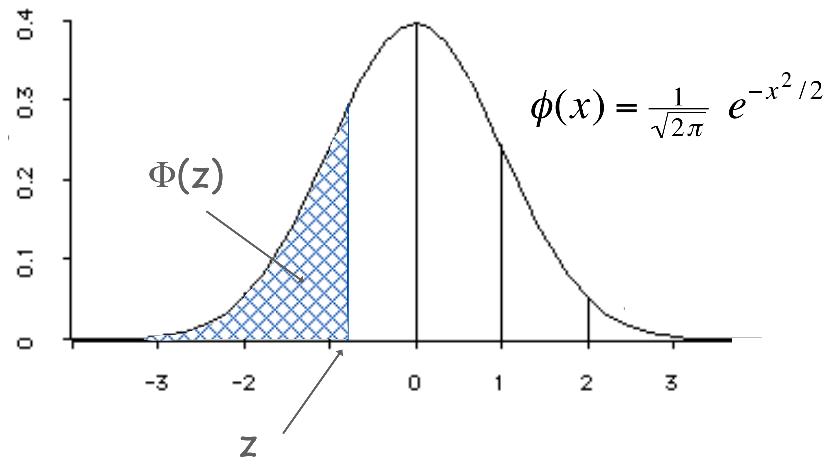
Multiple arguments. Functions can take any number of arguments.

Calling other functions. Functions can call other functions. ← library or user-defined

Gaussian Cumulative Distribution Function

Goal. Compute Gaussian cdf $\Phi(z)$.

Challenge. No "closed form" expression and not in Java library.



Taylor series

$$\Phi(z) = \int_{-\infty}^{z} \phi(x)\,dx$$

$$= \frac{1}{2} + \phi(z)\left(z + \frac{z^3}{3} + \frac{z^5}{3\cdot 5} + \frac{z^7}{3\cdot 5\cdot 7} + \ldots\right)$$

Bottom line. 1,000 years of mathematical formulas at your fingertips.

Java function for $\Phi(z)$

```
public class Gaussian
{

   public static double phi(double x)
      // as before

   public static double Phi(double z)
   {
      if (z < -8.0) return 0.0;
      if (z >  8.0) return 1.0;
      double sum = 0.0, term = z;
      for (int i = 3; sum + term != sum; i += 2) {
         sum  = sum + term;
         term = term * z * z / i;
      }
      return 0.5 + sum * phi(z);
   }

   public static double Phi(double z, double mu, double sigma)
   {
      return Phi((z - mu) / sigma);
   }
}
```

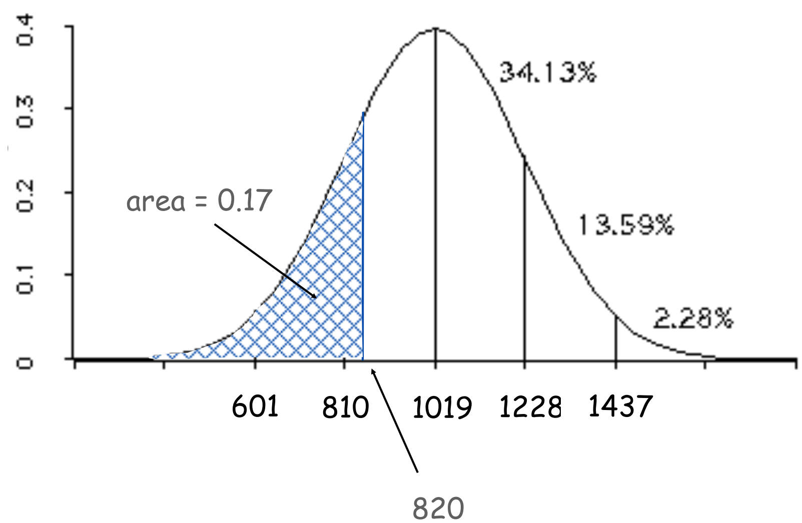
accurate with absolute error less than $8 * 10^{-16}$

$$\Phi(z,\ \mu,\ \sigma) \ = \ \int_{-\infty}^{z} \phi(z, \mu, \sigma) \ = \ \Phi((z-\mu)\ /\ \sigma)$$

SAT Scores

Q. NCAA requires at least 820 for Division I athletes.
What fraction of test takers in 2000 did not qualify?

A. $\Phi(820, \mu, \sigma) \approx 0.17051$. [approximately 17%]




```
double fraction = Gaussian.Phi(820, 1019, 209);
```

Gaussian Distribution

Q. Why relevant in mathematics?

A. Central limit theorem: under very general conditions, average of a set of variables tends to the Gaussian distribution.

Q. Why relevant in the sciences?

A. Models a wide range of natural phenomena and random processes.

- Weights of humans, heights of trees in a forest.
- SAT scores, investment returns.

Caveat.

> Everybody believes in the exponential law of errors: the experimenters, because they think it can be proved by mathematics; and the mathematicians, because they believe it has been established by observation. - M. Lippman in a letter to H. Poincaré

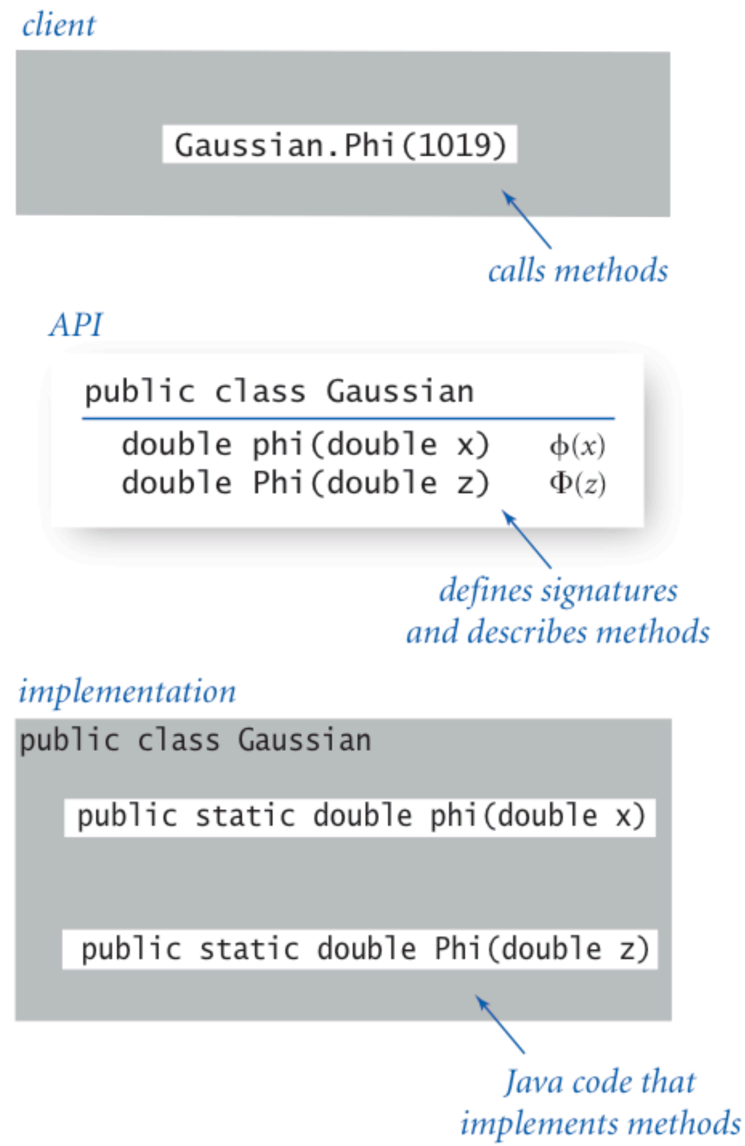
Libraries

Library. A module (class) whose methods are primarily intended for use by many other programs.

Client. Program that calls library method(s).

API. Contract between client and implementation.

Implementation. Program that implements the methods of an API (i.e., contains the code).



Why use libraries?

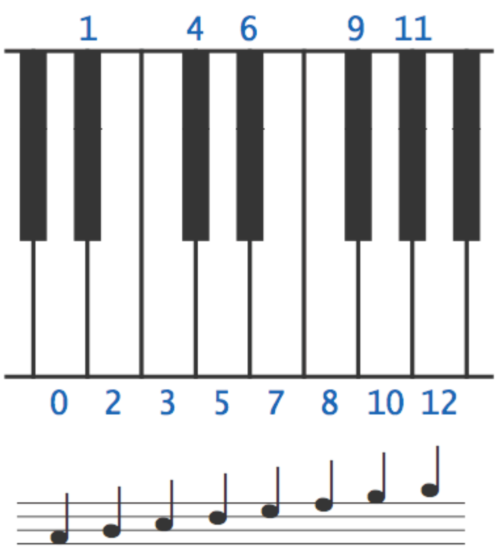
- Makes code easier to understand.
- Makes code easier to debug.
- Makes code easier to maintain and improve.
- Makes code easier to reuse.

Digital Audio

Sound. Perception of the vibration of molecules in our eardrums.

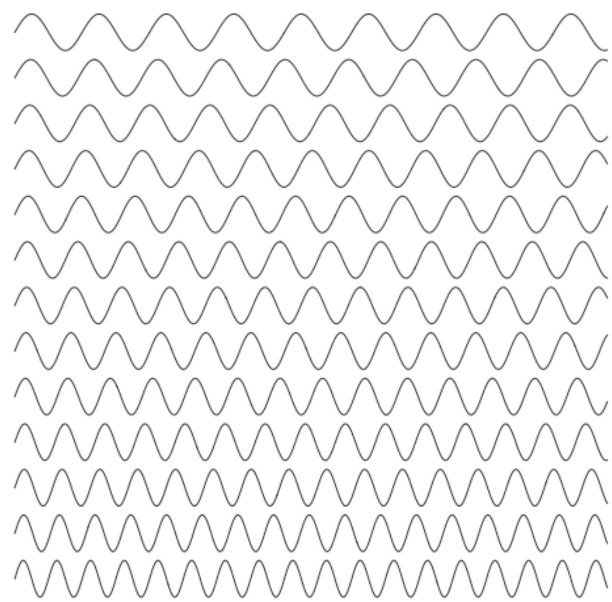
Concert A. Sine wave, scaled to oscillated at 440Hz.

Other notes. 12 notes on chromatic scale, divided logarithmically.



note	*i*	*frequency*
A	0	440.00
A♯ *or* B♭	1	466.16
B	2	493.88
C	3	523.25
C♯ *or* D♭	4	554.37
D	5	587.33
D♯ *or* E♭	6	622.25
E	7	659.26
F	8	698.46
F♯ *or* G♭	9	739.99
G	10	783.99
G♯ *or* A♭	11	830.61
A	12	880.00

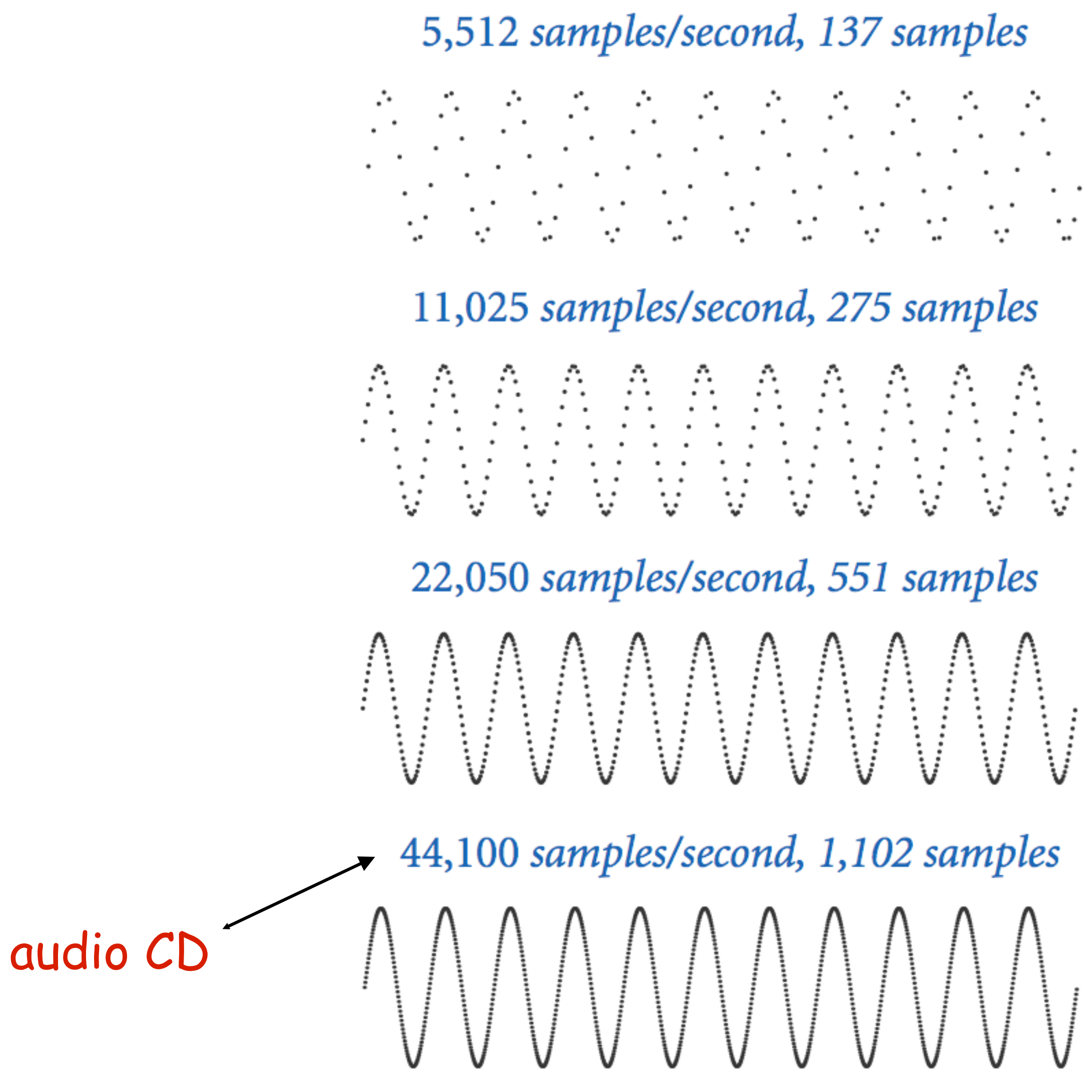
$440 \times 2^{i/12}$



Notes, numbers, and waves

Digital Audio

Sampling. Represent curve by sampling it at regular intervals.



$$y(i) \;=\; \sin\left(\frac{2\pi \cdot i \cdot 440}{44{,}100}\right)$$

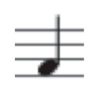
Warmup: Musical Tone

Musical tone. Create a music tone of a given frequency and duration.

```
public class Tone
{
   public static void main(String[] args)
   {
      int sps = 44100;
      double hz       = Double.parseDouble(args[0]);
      double duration = Double.parseDouble(args[1]);
      int N = (int) (sps * duration);
      double[] a = new double[N+1];
      for (int i = 0; i <= N; i++)
         a[i] = Math.sin(2 * Math.PI * i * hz / sps);
      StdAudio.play(a);
   }
}
```

$$y(i) \;=\; \sin\left(\frac{2\pi \cdot i \cdot hz}{44{,}100}\right)$$

```
% java Tone 440 1.5
[ concert A for 1.5 seconds]
```



Play That Tune

Goal. Play pitches and durations from standard input on standard audio.

```
public class PlayThatTune
{
   public static void main(String[] args)
   {
      int sps = 44100;
      while (!StdIn.isEmpty())
      {
            int pitch = StdIn.readInt();
            double duration = StdIn.readDouble();
            double hz = 440 * Math.pow(2, pitch / 12.0);
            int N = (int) (sps * duration);
            double[] a = new double[N+1];
            for (int i = 0; i <= N; i++)
               a[i] = Math.sin(2 * Math.PI * i * hz / sps);
            StdAudio.play(a);
      }
   }
}
```

```
% more elise.txt
7 .125
6 .125
7 .125
6 .125
7 .125
2 .125
5 .125
3 .125
0 .25
...
```

```
% java PlayThatTune < elise.txt
```

Musical Tone Function

Musical tone. Create a music tone of a given frequency and duration.

```
public static double[] tone(double hz, double seconds)
{
   int SAMPLE_RATE = 44100;
   int N = (int) (seconds * SAMPLE_RATE);
   double[] a = new double[N+1];
   for (int i = 0; i <= N; i++)
      a[i] = Math.sin(2 * Math.PI * i * hz / SAMPLE_RATE);
   return a;
}
```

$$y(i) \;=\; \sin\left(\frac{2\pi \cdot i \cdot hz}{44{,}100}\right)$$

Remark. Can use arrays as function return value and/or argument.

Standard audio. Library for playing digital audio.

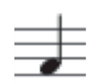
`public class StdAudio`

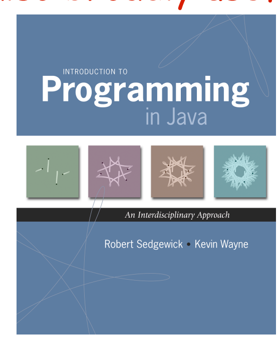
void	play(String file)	*play the given .wav file*
void	play(double[] a)	*play the given sound wave*
void	play(double x)	*play sample for 1/44100 second*
void	save(String file, double[] a)	*save to a .wav file*
double[]	read(String file)	*read from a .wav file*

library developed for this course (also broadly useful)

Concert A. Play concert A for `1.5` seconds using **`StdAudio`**.

```
double[] a = tone(440, 1.5);
StdAudio.play(a);
```



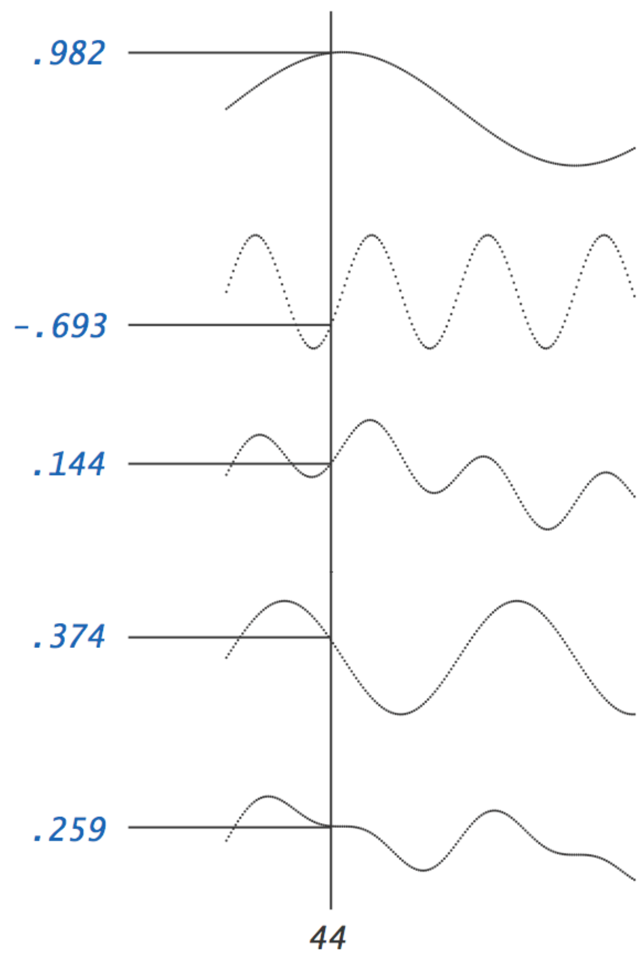


Remark. Java arrays passed "by reference" (no copy made).

Harmonics

Concert A with harmonics. Obtain richer sound by adding tones one octave above and below concert A.

880 Hz (above), 220 Hz (below), 440 Hz (concert A)




```
lo = tone(220, .0041);
lo[44] = .982

hi = tone(880, .0041);
hi[44] = -.693

h = sum(hi, lo, .5, .5);
h[44] = .5*lo[44]+.5*hi[44];
      = .5*.982 - .5*.693 = .144

A = tone(440, .0041);
A[44] = .374

sum(A, h, .5, .5);
A[44] + h[44] = .5*.144 + .5*.374
              = .259
```

Harmonics

```
public class PlayThatTuneDeluxe                    // improved version with Harmonics
{
   // Return weighted sum of two arrays.
   public static double[] sum(double[] a, double[] b, double awt, double bwt) {
      double[] c = new double[a.length];
      for (int i = 0; i < a.length; i++)
         c[i] = a[i]*awt + b[i]*bwt;
      return c;
   }

   // Return a note of given pitch and duration.
   public static double[] note(int pitch, double duration) {
      double hz = 440.0 * Math.pow(2, pitch / 12.0);
      double[] a  = tone(1.0 * hz, duration);
      double[] hi = tone(2.0 * hz, duration);
      double[] lo = tone(0.5 * hz, duration);
      double[] h  = sum(hi, lo, .5, .5);
      return sum(a, h, .5, .5);
   }

   public static double[] tone(double hz, double t)
      // see previous slide

   public static void main(String[] args)
      // see next slide
}
```

Harmonics

Play that tune (deluxe version). Read in pitches and durations from standard input, and play using standard audio.

```
public static void main(String[] args)
{
   while (!StdIn.isEmpty())
   {
      int pitch = StdIn.readInt();
      double duration = StdIn.readDouble();
      double[] a = note(pitch, duration);
      StdAudio.play(a);
   }
}
```

```
% more elise.txt
7 .125
6 .125
7 .125
6 .125
7 .125
2 .125
5 .125
3 .125
0 .25
```

```
% java PlayThatTune < elise.txt
```

```java
public class PlayThatTune
{
   public static double[] sum(double[] a, double[] b,
                              double awt, double bwt)
   {
      double[] c = new double[a.length];
      for (int i = 0; i < a.length; i++)
         c[i] = a[i]*awt + b[i]*bwt;
      return c;
   }

   public static double[] tone(double hz, double t)
   {
      int sps = 44100;
      int N = (int) (sps * t);
      double[] a = new double[N+1];
      for (int i = 0; i <= N; i++)
         a[i] = Math.sin(2 * Math.PI * i * hz / sps);
      return a;
   }

   public static double[] note(int pitch, double t)
   {
      double hz = 440.0 * Math.pow(2, pitch / 12.0);
      double[] a  = tone(hz, t);
      double[] hi = tone(2*hz, t);
      double[] lo = tone(hz/2, t);
      double[] h  = sum(hi, lo, .5, .5);
      return sum(a, h, .5, .5);
   }

   public static void main(String[] args)
   {
      while (!StdIn.isEmpty())
      {
         int pitch = StdIn.readInt();
         double duration = StdIn.readDouble();
         double[] a = note(pitch, duration);
         StdAudio.play(a);
      }
   }
}
```