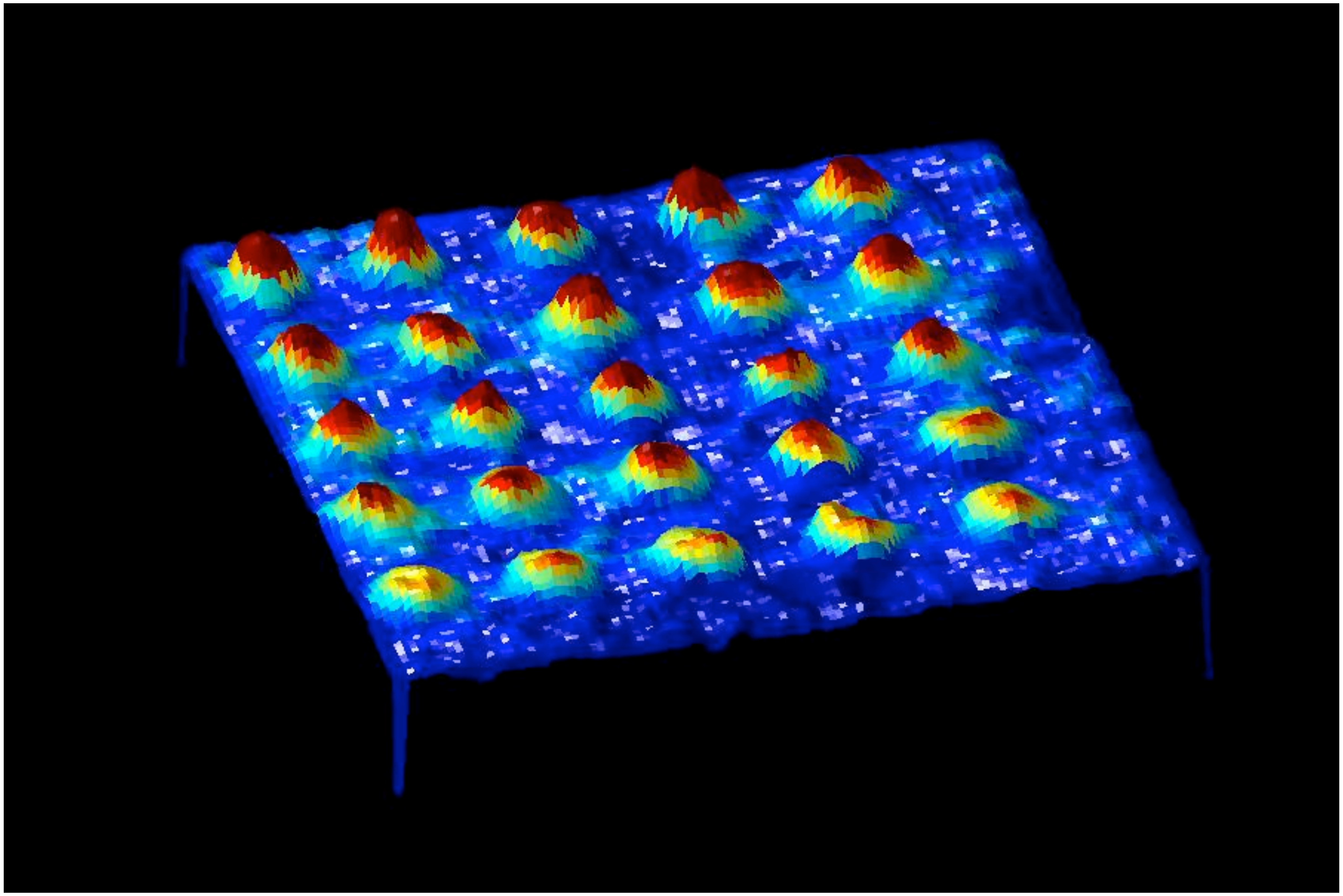
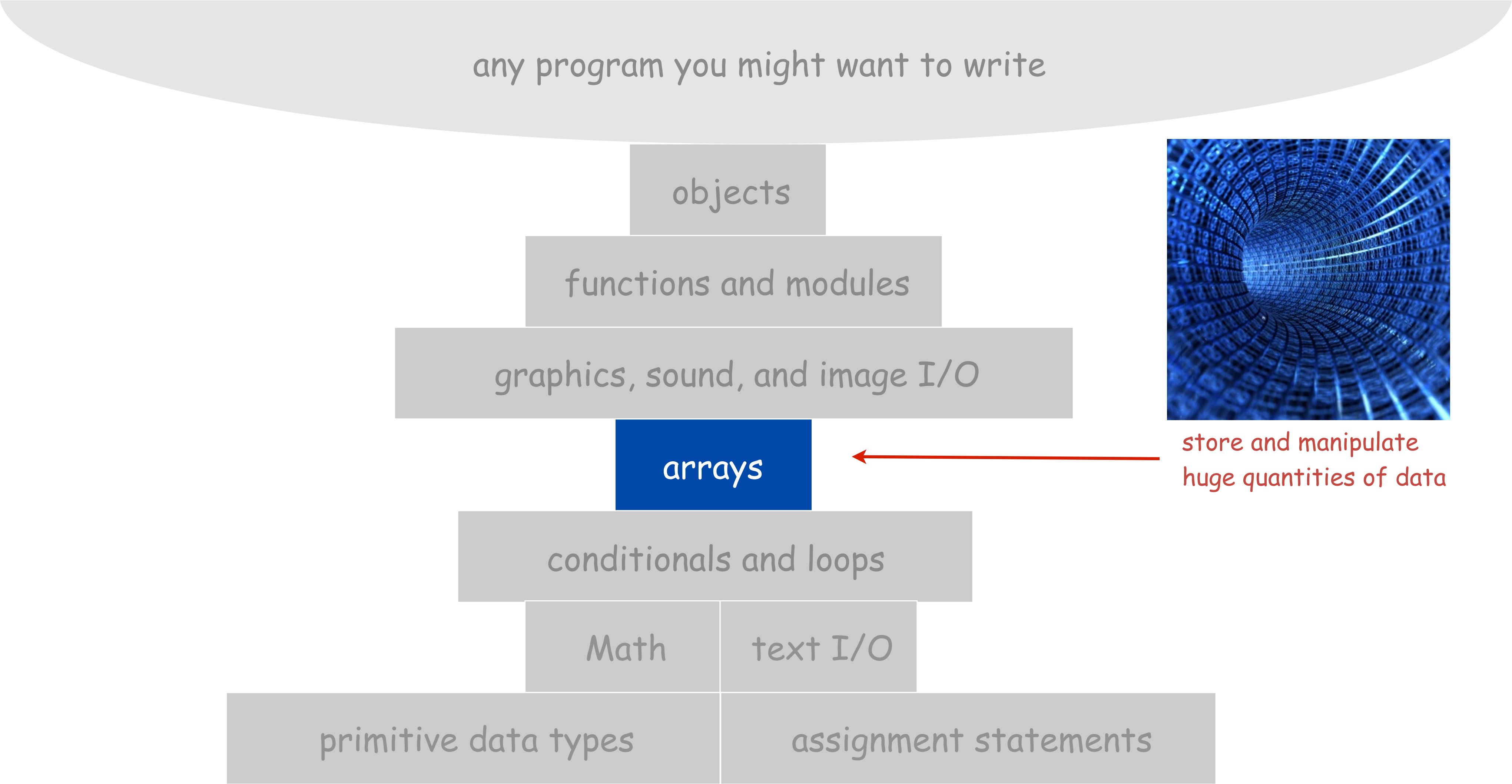
1.4 Arrays



A Foundation for Programming



any program you might want to write

objects

functions and modules

graphics, sound, and image I/O

arrays

conditionals and loops

Math

text I/O

primitive data types

assignment statements

store and manipulate
huge quantities of data

Arrays

This lecture. Store and manipulate huge quantities of data.

Array. Indexed sequence of values of the same type.

Examples.

- 52 playing cards in a deck.
- 5 thousand undergrads at Princeton.
- 1 million characters in a book.
- 10 million audio samples in an MP3 file.
- 4 billion nucleotides in a DNA strand.
- 73 billion Google queries per year.
- 50 trillion cells in the human body.
- 6.02×10^{23} particles in a mole.

index	value
0	doug
1	dp6
2	dgabai
3	pprzytyc
4	funk
5	achaney
6	vivek
7	sidch
8	snadimpa
9	eberinge
10	kevinlee
11	siyuliu
12	jlisrael

Many Variables of the Same Type

Goal. 10 variables of the same type.

```
// Tedious and error-prone code.
double a0, a1, a2, a3, a4, a5, a6, a7, a8, a9;
a0 = 0.0;
a1 = 0.0;
a2 = 0.0;
a3 = 0.0;
a4 = 0.0;
a5 = 0.0;
a6 = 0.0;
a7 = 0.0;
a8 = 0.0;
a9 = 0.0;
...
a4 = 3.0;
...
a8 = 8.0;
...
double x = a4 + a8;
```

Many Variables of the Same Type

Goal. 10 variables of the same type.

```
// Easy alternative.
double[] a = new double[10];
...
a[4] = 3.0;
...
a[8] = 8.0;
...
double x = a[4] + a[8];
```

declares, creates, and initializes
[stay tuned for details]

Many Variables of the Same Type

Goal. 1 million variables of the same type.

```
// Scales to handle large arrays.
double[] a = new double[1000000];
...
a[234567] = 3.0;
...
a[876543] = 8.0;
...
double x = a[234567] + a[876543];
```

Arrays in Java

Java has special language support for arrays.

- To make an array: declare, create, and initialize it.
- To access element `i` of array named `a`, use `a[i]`.
- Array indices start at `0`.

```
int N = 1000;
double[] a;                  // declare the array
a = new double[N];           // create the array
for (int i = 0; i < N; i++)  // initialize the array
   a[i] = 0.0;               // all to 0.0
```

Arrays in Java

Java has special language support for arrays.

- To make an array: declare, create, and initialize it.
- To access element `i` of array named `a`, use `a[i]`.
- Array indices start at `0`.

```
int N = 1000;
double[] a;                   // declare the array
a = new double[N];            // create the array
for (int i = 0; i < N; i++)   // initialize the array
   a[i] = 0.0;                // all to 0.0
```

Compact alternatives: Declare, create, and initialize in one statement.

- Default: all entries automatically set to 0.

```
double[] a = new double[1000];
```

- Initialize to literal values

compact version

```
double[] x = { 0.3, 0.6, 0.1 };
```

equivalent code

```
double[] x = new double[3];
x[0] = 0.3; x[1] = 0.6; x[2] = 0.1;
```

Sample Array Code: Vector Dot Product

Dot product. Given two vectors **x[]** and **y[]** of length **N**, their dot product is the sum of the products of their corresponding components.

```
double[] x = { 0.3, 0.6, 0.1 };
double[] y = { 0.5, 0.1, 0.4 };

double sum = 0.0;
for (int i = 0; i < N; i++)
   sum += x[i]*y[i];
```

i	x[i]	y[i]	x[i]*y[i]	sum
				0
0	.30	.50	.15	.15
1	.60	.10	.06	.21
2	.10	.40	.04	.25
				.25

Array Processing Examples

```
double[] a = new double[N];
for (int i = 0; i < N; i++)
   a[i] = Math.random();
```

create an array with N random values

```
double[] b = new double[N];
for (int i = 0; i < N; i++)
   b[i] = a[i];
```

copy to another array

```
for (int i = 0; i < N; i++)
   System.out.println(a[i]);
```

print the array values, one per line

```
double sum = 0.0;
for (int i = 0; i < N; i++)
   sum += a[i];
double average = sum / N;
```

compute the average of the array values

```
double max = Double.NEGATIVE_INFINITY;
for (int i = 0; i < N; i++)
   if (a[i] > max) max = a[i];
```

find the maximum of the array values

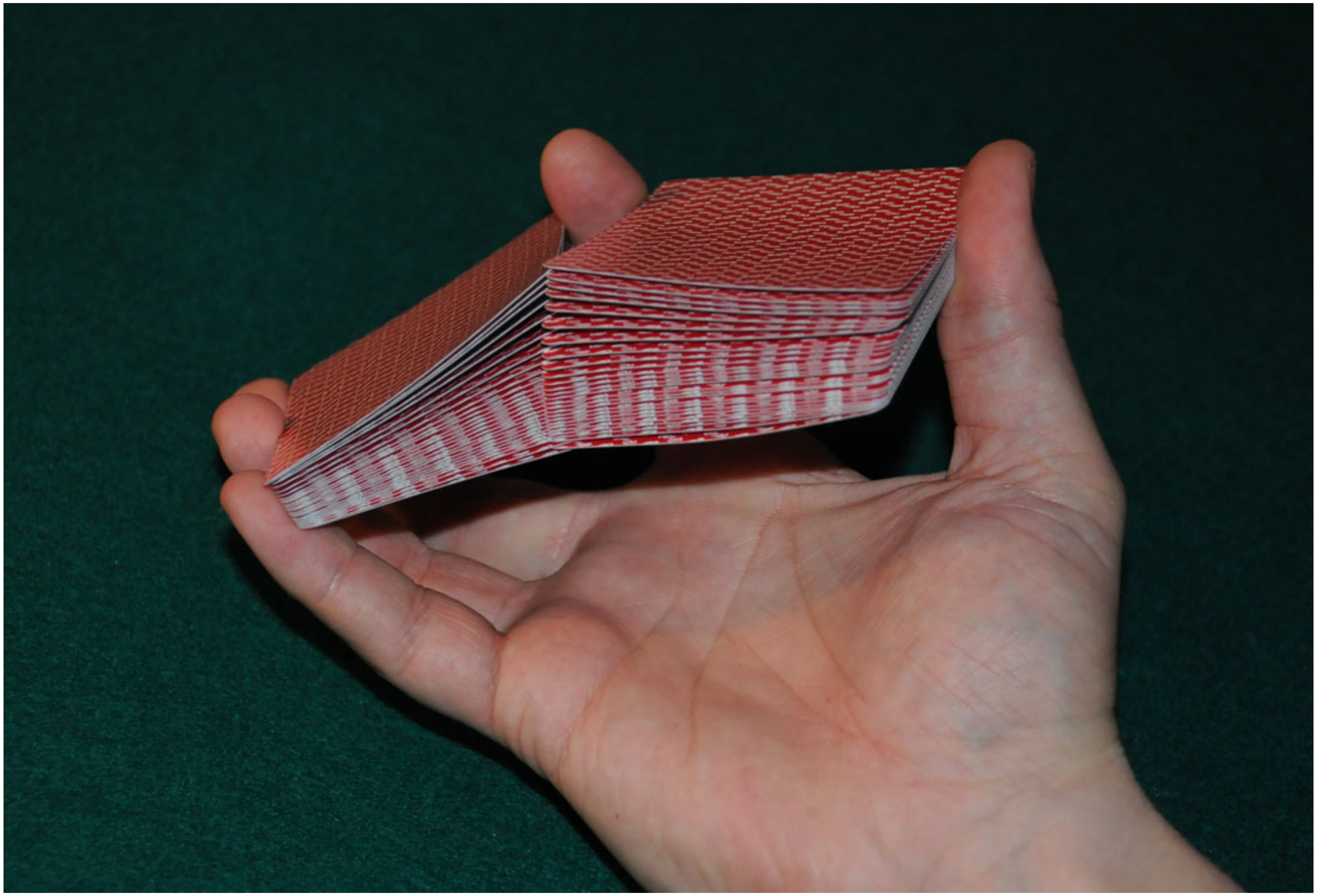
```
for (int i = 0; i < N/2; i++)
{
   double temp = b[i];
   b[i] = b[N-1-i];
   b[N-i-1] = temp;
}
```

reverse the elements within the array

Mumbo-Jumbo Demystification, Part 1

```
public class Gambler {
   public static void main(String[] args) {
      int stake  = Integer.parseInt(args[0]);
      int goal   = Integer.parseInt(args[1]);
      int trials = Integer.parseInt(args[2]);
      . . .
      . . .
   }
}
```

Shuffling a Deck



Setting Array Values at Compile Time

Ex. Print a random card.

```
String[] rank =
{
   "2", "3", "4", "5", "6", "7", "8", "9",
   "10", "Jack", "Queen", "King", "Ace"
};


String[] suit =
{
   "Clubs", "Diamonds", "Hearts", "Spades"
};

int i = (int) (Math.random() * 13);  // between 0 and 12
int j = (int) (Math.random() * 4);   // between 0 and 3

System.out.println(rank[i] + " of " + suit[j]);
```

Array Challenge 1

The following code sets array values to the 52 card values and prints them.

In which order are they printed?

```
String[] rank = { "2", "3" ..., "King", "Ace" };
String[] suit =
   { "clubs", "diamonds", "hearts", "spades" };

String[] deck = new String[52];
for (int i = 0; i < 13; i++)
   for (int j = 0; j < 4; j++)
      deck[4*i + j] = rank[i] + " of " + suit[j];

for (int i = 0; i < 52; i++)
   System.out.println(deck[i]);
```

typical array processing code changes values at runtime

A.

```
2 of clubs
2 of diamonds
2 of hearts
2 of spades
3 of clubs
...
```

B.

```
2 of clubs
3 of clubs
4 of clubs
5 of clubs
6 of clubs
...
```

Array Challenge 2

Swap the **for** statements: **rank** index in inner loop, **suit** index in outer loop.

Now, in which order are they printed?

```
String[] rank = { "2", "3" ..., "King", "Ace" };
String[] suit =
  { "clubs", "diamonds", "hearts", "spades" };

String[] deck = new String[52];
for (int j = 0; j < 4; j++)
   for (int i = 0; i < 13; i++)
      deck[4*i + j] = rank[i] + " of " + suit[j];

for (int i = 0; i < 52; i++)
   System.out.println(deck[i]);
```

these lines swapped
from previous slide

A.

```
2 of clubs
2 of diamonds
2 of hearts
2 of spades
3 of clubs
...
```

B.

```
2 of clubs
3 of clubs
4 of clubs
5 of clubs
6 of clubs
...
```

Array Challenge 3

The following code sets array values to the 52 card values and prints them.

What change to the code will produce the “B” order ?

```
String[] rank = { "2", "3" ..., "King", "Ace" };
String[] suit =
  { "clubs", "diamonds", "hearts", "spades" };

String[] deck = new String[52];
for (int i = 0; i < 13; i++)
   for (int j = 0; j < 4; j++)
      deck[4*i + j] = rank[i] + " of " + suit[j];

for (int i = 0; i < 52; i++)
   System.out.println(deck[i]);
```

A.

```
2 of clubs
2 of diamonds
2 of hearts
2 of spades
3 of clubs
...
```

B.

```
2 of clubs
3 of clubs
4 of clubs
5 of clubs
6 of clubs
...
```

Shuffling

Goal. Given an array, rearrange its elements in random order.

Shuffling algorithm.

- In iteration `i`, pick random card from `deck[i]` through `deck[N-1]`, with each card equally likely.
- Exchange it with `deck[i]`.

```
int N = deck.length;
for (int i = 0; i < N; i++)
{
   int r = i + (int) (Math.random() * (N-i));
   String t = deck[r];
   deck[r] = deck[i];
   deck[i] = t;
}
```

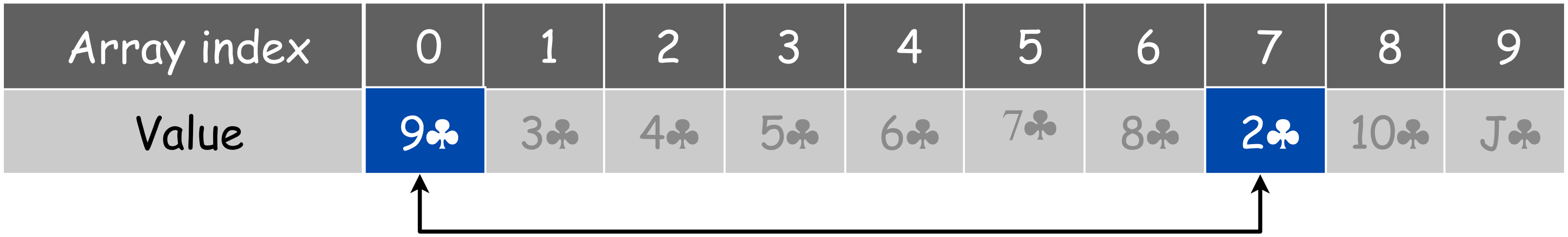
swap idiom

between i and N-1

Shuffle an Array

Shuffle a deck of cards.

- In i^{th} iteration, put a random element from remainder of deck at index i.
  - choose random integer r between i and N-1
  - swap values in positions r and i



Array index	0	1	2	3	4	5	6	7	8	9
Value	9♣	3♣	4♣	5♣	6♣	7♣	8♣	2♣	10♣	J♣

random integer = 7

Shuffling a Deck of Cards

```
public class Deck
{
   public static void main(String[] args)
   {
      String[] suit = { "Clubs", "Diamonds", "Hearts", "Spades" };
      String[] rank = { "2", "3", "4", "5", "6", "7", "8", "9",
                        "10", "Jack", "Queen", "King", "Ace"    };
      int SUITS = suit.length;
      int RANKS = rank.length;
      int N = SUITS * RANKS;
```

avoid "hardwired" constants like 52, 4, and 13.

build the deck

```
      String[] deck = new String[N];
      for (int i = 0; i < RANKS; i++)
         for (int j = 0; j < SUITS; j++)
            deck[SUITS*i + j] = rank[i] + " of " + suit[j];
```

shuffle

```
      for (int i = 0; i < N; i++)
      {
         int r = i + (int) (Math.random() * (N-i));
         String t = deck[r];
         deck[r] = deck[i];
         deck[i] = t;
      }
```

print shuffled deck

```
      for (int i = 0; i < N; i++)
         System.out.println(deck[i]);
   }
}
```

Shuffling a Deck of Cards

```
% java Deck
5 of Clubs
Jack of Hearts
9 of Spades
10 of Spades
9 of Clubs
7 of Spades
6 of Diamonds
7 of Hearts
7 of Clubs
4 of Spades
Queen of Diamonds
10 of Hearts
5 of Diamonds
Jack of Clubs
Ace of Hearts
...
5 of Spades
```

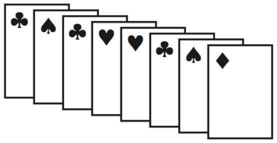
```
% java Deck
10 of Diamonds
King of Spades
2 of Spades
3 of Clubs
4 of Spades
Queen of Clubs
2 of Hearts
7 of Diamonds
6 of Spades
Queen of Spades
3 of Spades
Jack of Diamonds
6 of Diamonds
8 of Spades
9 of Diamonds
...
10 of Spades
```

Coupon Collector

Coupon Collector Problem

Coupon collector problem. Given N different card types, how many do you have to collect before you have (at least) one of each type?

assuming each possibility is equally likely for each card that you collect



Simulation algorithm. Repeatedly choose an integer **`i`** between **`0`** and **`N-1`**. Stop when we have at least one card of every type.

Q. How to check if we've seen a card of type `i`?

A. Maintain a boolean array so that **`found[i]`** is **`true`** if we've already collected a card of type **`i`**.

Coupon Collector: Java Implementation

```
public class CouponCollector
{
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      int cardcnt = 0;   // number of cards collected
      int valcnt = 0;    // number of distinct cards

      // Do simulation.
      boolean[] found = new boolean[N];
      while (valcnt < N)
      {
         int val = (int) (Math.random() * N);
         cardcnt++;
         if (!found[val])
         {
            valcnt++;
            found[val] = true;
         }
      }

      // all N distinct cards found
      System.out.println(cardcnt);
   }
}
```

type of next card
(between 0 and N-1)

Coupon Collector: Debugging

Debugging. Add code to print contents of all variables.

val	found						valcnt	cardcnt
	0	1	2	3	4	5		
	F	F	F	F	F	F	0	0
2	F	F	T	F	F	F	1	1
0	T	F	T	F	F	F	2	2
4	T	F	T	F	T	F	3	3
0	T	F	T	F	T	F	3	4
1	T	T	T	F	T	F	4	5
2	T	T	T	F	T	F	4	6
5	T	T	T	F	T	T	5	7
0	T	T	T	F	T	T	5	8
1	T	T	T	F	T	T	5	9
3	T	T	T	T	T	T	6	10

Challenge. Debugging with arrays requires tracing many variables.

Coupon Collector: Mathematical Context

Coupon collector problem. Given N different possible cards, how many do you have to collect before you have (at least) one of each type?

Fact. About N (1 + 1/2 + 1/3 + … + 1/N) ~ N ln N

see ORF 245 or COS 341

Ex. N = 30 baseball teams. Expect to wait ≈ 120 years before all teams win a World Series.

under idealized assumptions

Coupon Collector: Scientific Context

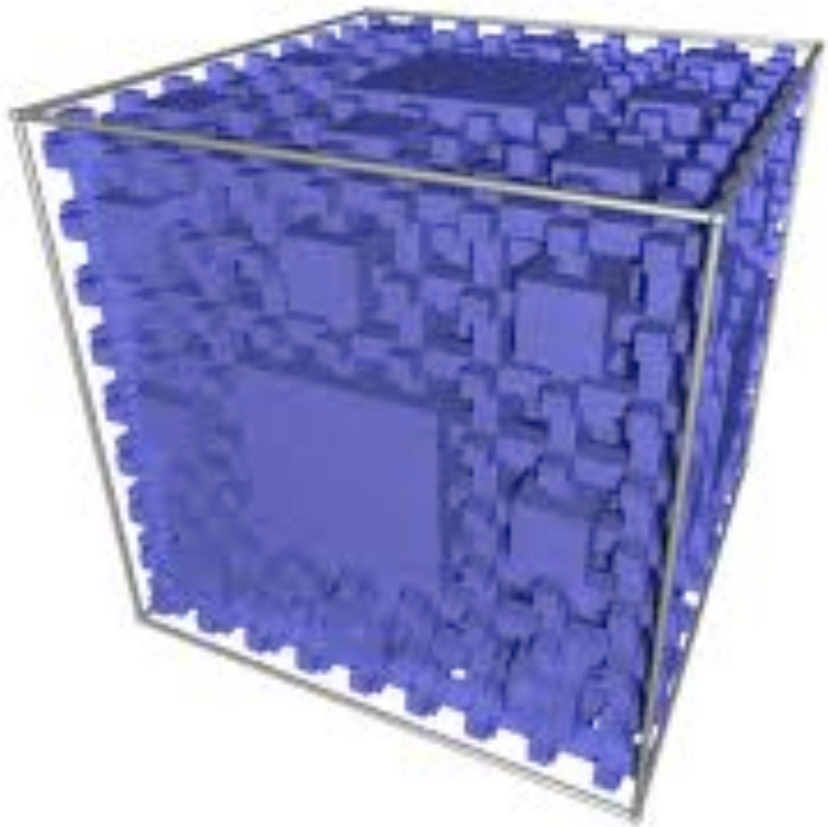
Q. Given a sequence from nature, does it have same characteristics as a random sequence?

A. No easy answer - many tests have been developed.

Coupon collector test. Compare number of elements that need to be examined before all values are found against the corresponding answer for a random sequence.

TOUR OF ACCOUNTING
OVER HERE WE HAVE OUR RANDOM NUMBER GENERATOR.
NINE NINE NINE NINE NINE NINE
10/25/01 © 2001 United Feature Syndicate, Inc.
ARE YOU SURE THAT'S RANDOM?
THAT'S THE PROBLEM WITH RANDOMNESS: YOU CAN NEVER BE SURE.

Multidimensional Arrays



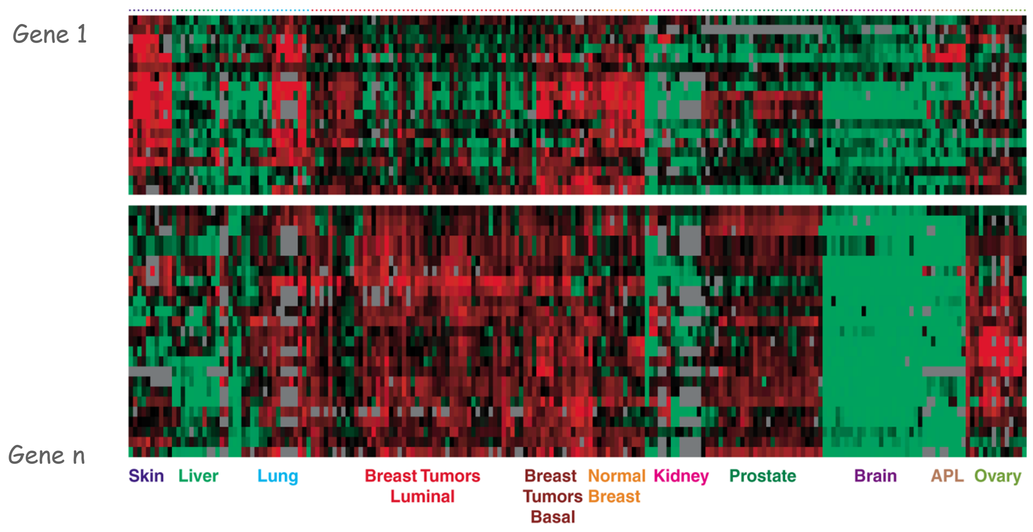
Two Dimensional Arrays

Two dimensional arrays.

- Table of data for each experiment and outcome.
- Table of grades for each student and assignments.
- Table of grayscale values for each pixel in a 2D image.

Mathematical abstraction. Matrix.

Java abstraction. 2D array.



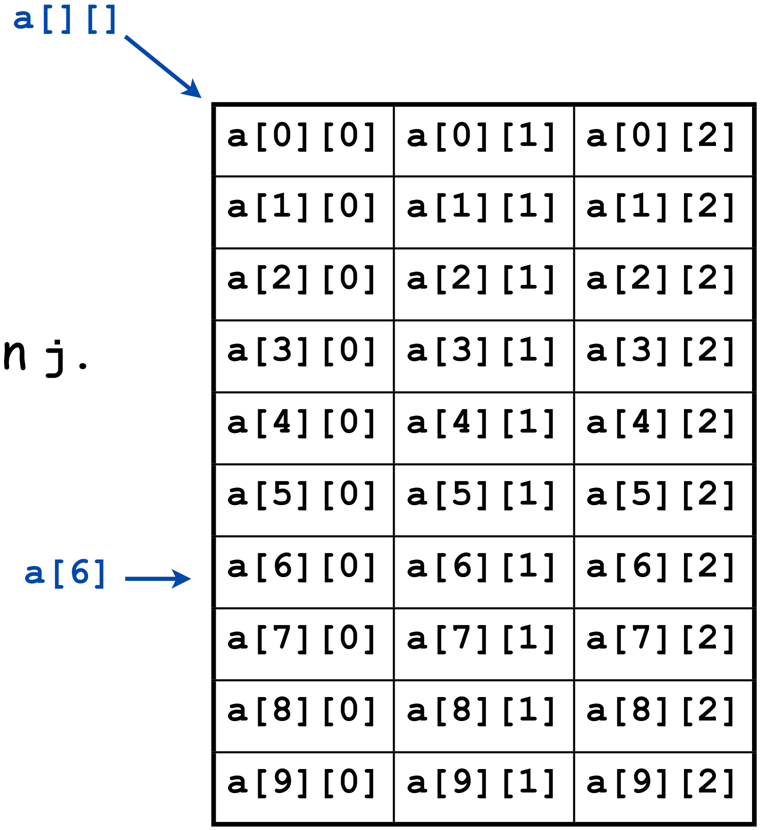
Reference: Botstein & Brown group

🟥 gene expressed
🟩 gene not expressed

Two Dimensional Arrays in Java

Declare, create, initialize. Like 1D, but add another pair of brackets.

```
int M = 10;
int N = 3;
double[][] a = new double[M][N];
```



a[0][0]	a[0][1]	a[0][2]
a[1][0]	a[1][1]	a[1][2]
a[2][0]	a[2][1]	a[2][2]
a[3][0]	a[3][1]	a[3][2]
a[4][0]	a[4][1]	a[4][2]
a[5][0]	a[5][1]	a[5][2]
a[6][0]	a[6][1]	a[6][2]
a[7][0]	a[7][1]	a[7][2]
a[8][0]	a[8][1]	a[8][2]
a[9][0]	a[9][1]	a[9][2]

A 10-by-3 array

Array access.

Use `a[i][j]` to access entry in row `i` and column `j`.

Indices start at `0`.

Initialize.

This code is implicit (sets all entries to 0).

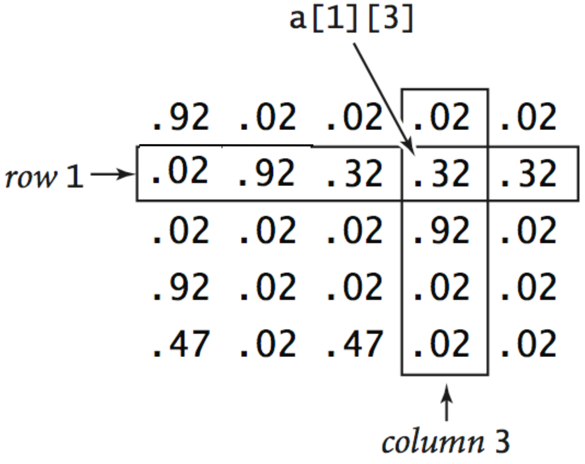
```
for (int i = 0; i < M; i++)
   for (int j = 0; j < N; j++)
      a[i][j] = 0.0;
```

Warning. This implicit code might slow down your program for big arrays.

Setting 2D Array Values at Compile Time

Initialize 2D array by listing values.

```
double[][] p =
{
   { .02, .92, .02, .02, .02 },
   { .02, .02, .32, .32, .32 },
   { .02, .02, .02, .92, .02 },
   { .92, .02, .02, .02, .02 },
   { .47, .02, .47, .02, .02 },
};
```

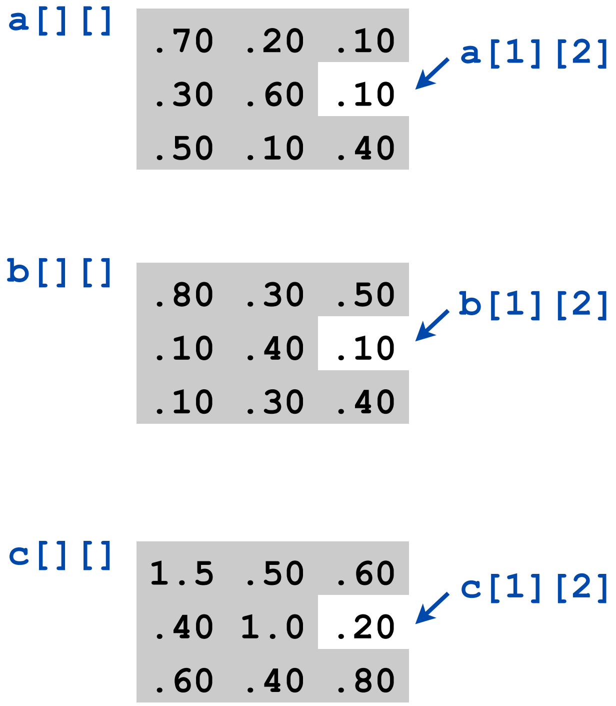


.92	.02	.02	.02	.02
.02	.92	.32	.32	.32
.02	.02	.02	.92	.02
.92	.02	.02	.02	.02
.47	.02	.47	.02	.02

Matrix Addition

Matrix addition. Given two N-by-N matrices `a` and `b`, define `c` to be the N-by-N matrix where `c[i][j]` is the sum `a[i][j] + b[i][j]`.

```
double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
   for (int j = 0; j < N; j++)
      c[i][j] = a[i][j] + b[i][j];
```



`a[][]`

$$\begin{array}{ccc} .70 & .20 & .10 \\ .30 & .60 & \boxed{.10} \\ .50 & .10 & .40 \end{array}$$

`b[][]`

$$\begin{array}{ccc} .80 & .30 & .50 \\ .10 & .40 & \boxed{.10} \\ .10 & .30 & .40 \end{array}$$

`c[][]`

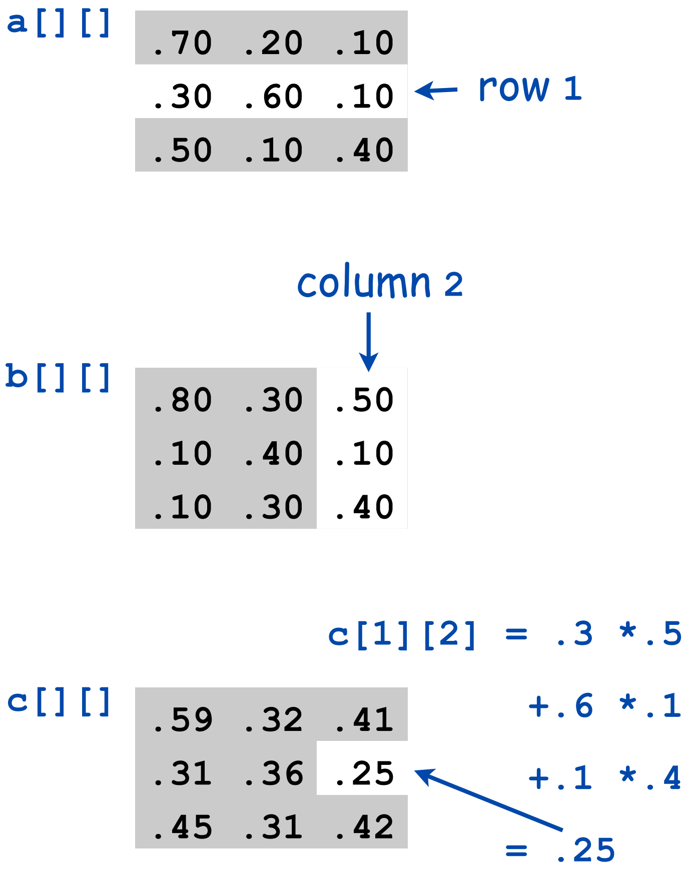
$$\begin{array}{ccc} 1.5 & .50 & .60 \\ .40 & 1.0 & \boxed{.20} \\ .60 & .40 & .80 \end{array}$$

Matrix Multiplication

Matrix multiplication. Given two N-by-N matrices a and b, define c to be the N-by-N matrix where `c[i][j]` is the dot product of the `i`th row of `a` and the `j`th row of `b`.

all values initialized to 0

```
double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
   for (int j = 0; j < N; j++)
      for (int k = 0; k < N; k++)
         c[i][j] += a[i][k] * b[k][j];
```



a[][]

.70	.20	.10
.30	.60	.10
.50	.10	.40

row 1

column 2

b[][]

.80	.30	.50
.10	.40	.10
.10	.30	.40

c[][]

.59	.32	.41
.31	.36	.25
.45	.31	.42

```
c[1][2] = .3 *.5
        +.6 *.1
        +.1 *.4
        = .25
```

Array Challenge 4

How many multiplications to multiply two N-by-N matrices?

```
double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
   for (int j = 0; j < N; j++)
      for (int k = 0; k < N; k++)
         c[i][j] += a[i][k] * b[k][j];
```

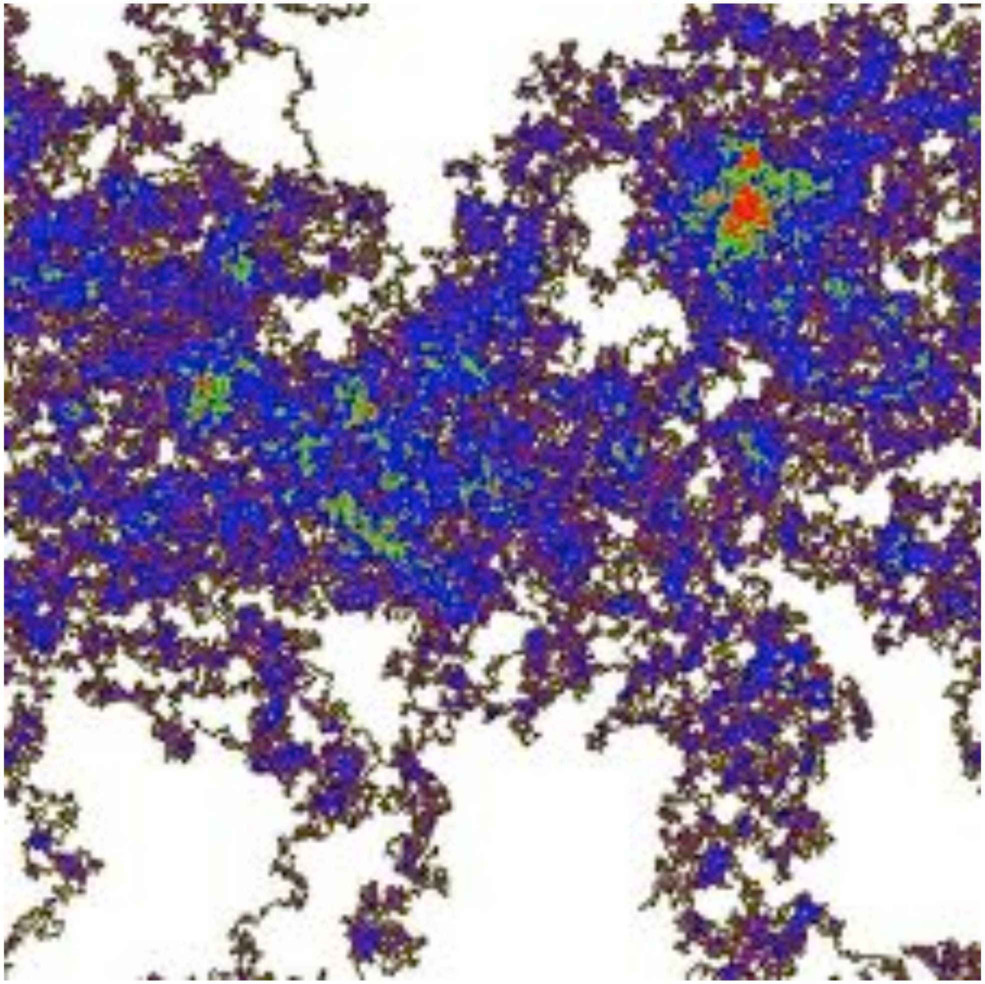
A. N

B. N^2

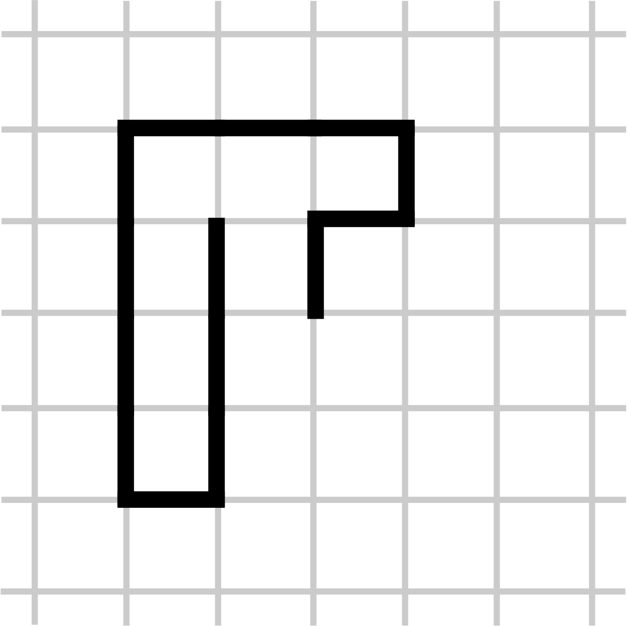
C. N^3

D. N^4

Application: 2D Random Walks



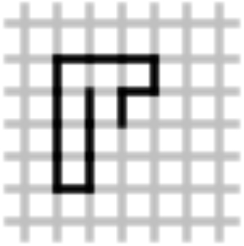
Application: Self-Avoiding Walks



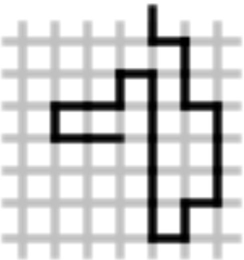
Self-Avoiding Walk

Model.

- N-by-N lattice.
- Start in the middle.
- Randomly move to a neighboring intersection, avoiding all previously visited intersections.
- Two possible outcomes: escape and dead end



dead end



escape

Applications. Polymers, statistical mechanics, etc.

Q. What fraction of time will you escape in an 5-by-5 lattice?

Q. In an N-by-N lattice?

Q. In an N-by-N-by-N lattice?

Self-Avoiding Walk: Implementation

```
public class SelfAvoidingWalk
{
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);     // lattice size
      int T = Integer.parseInt(args[1]);     // number of trials
      int deadEnds = 0;                      // trials ending at dead end

      for ( each trial )
      {
         boolean[][] a = new boolean[N][N];   // intersections visited
         int x = N/2, y = N/2;                // current position

         while ( you're still inside the lattice )
         {

            if ( you're at a dead end )
            {  deadEnds++;  break; }

            a[x][y] = true;                  // mark as visited

            double r = Math.random();
            if      (r < 0.25) { if (!a[x+1][y]) x++; }
            else if (r < 0.50) { if (!a[x-1][y]) x--; }
            else if (r < 0.75) { if (!a[x][y+1]) y++; }
            else if (r < 1.00) { if (!a[x][y-1]) y--; }
         }
      }
      System.out.println(100*deadEnds/T + "% dead ends");
   }
}
```

dead end

take a random step to a new intersection

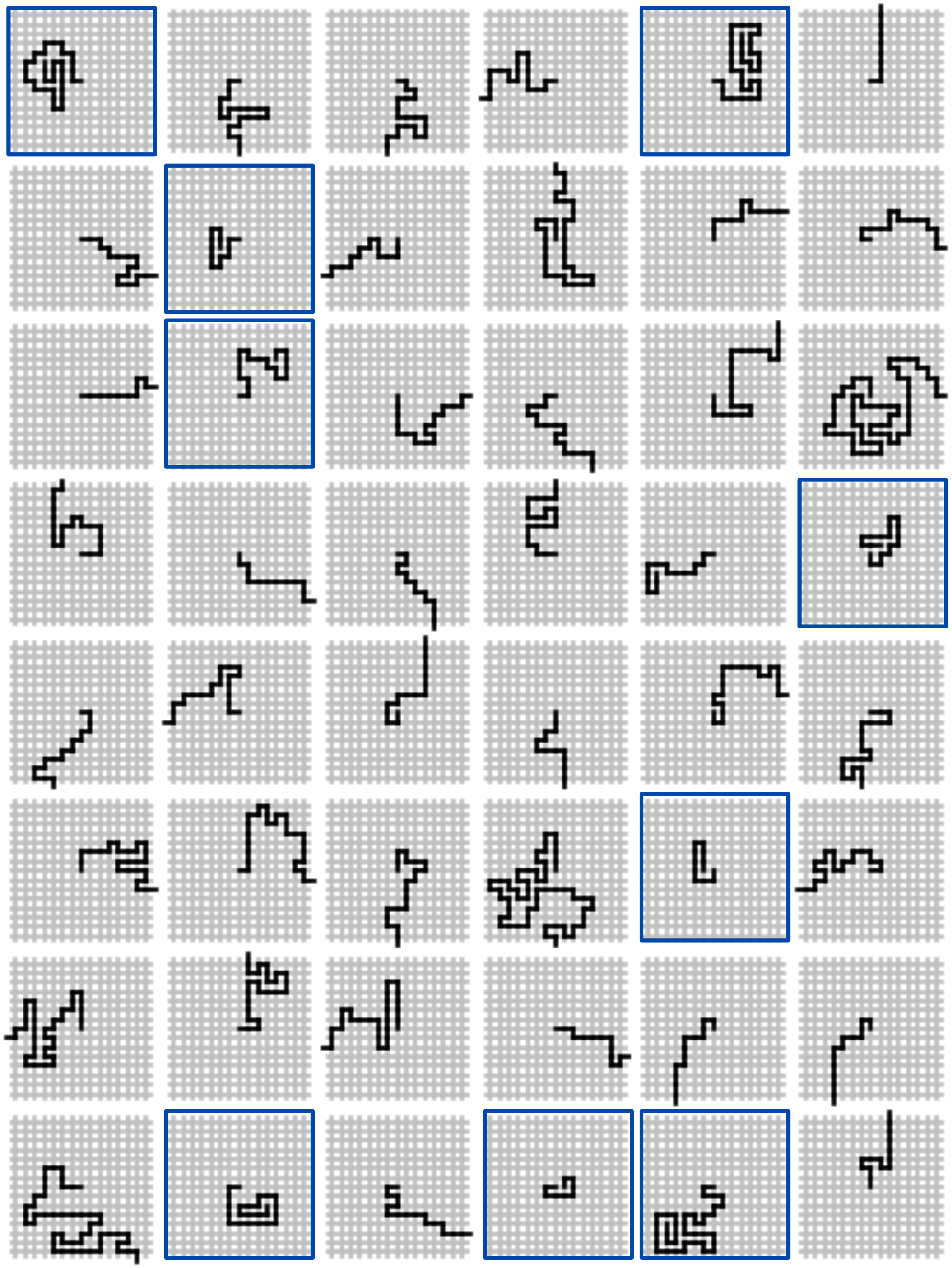
Self-Avoiding Walk: Implementation

```java
public class SelfAvoidingWalk
{
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);    // lattice size
      int T = Integer.parseInt(args[1]);    // number of trials
      int deadEnds = 0;                     // trials ending at dead end

      for (int t = 0; t < T; t++)
      {
         boolean[][] a = new boolean[N][N];  // intersections visited
         int x = N/2, y = N/2;               // current position

         while (x > 0 && x < N-1 && y > 0 && y < N-1)
         {

            if (a[x-1][y] && a[x+1][y] && a[x][y-1] && a[x][y+1])
            {  deadEnds++;  break; }

            a[x][y] = true;                 // mark as visited

            double r = Math.random();
            if      (r < 0.25) { if (!a[x+1][y]) x++; }
            else if (r < 0.50) { if (!a[x-1][y]) x--; }
            else if (r < 0.75) { if (!a[x][y+1]) y++; }
            else if (r < 1.00) { if (!a[x][y-1]) y--; }
         }
      }
      System.out.println(100*deadEnds/T + "% dead ends");
   }
}
```

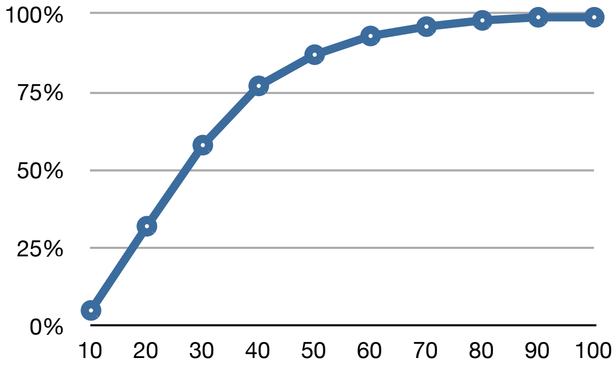
dead end

take a random step to a new intersection

Self-Avoiding Walks



```
% java SelfAvoidingWalk 10 100000
5% dead ends
% java SelfAvoidingWalk 20 100000
32% dead ends
% java SelfAvoidingWalk 30 100000
58% dead ends
% java SelfAvoidingWalk 40 100000
77% dead ends
% java SelfAvoidingWalk 50 100000
87% dead ends
% java SelfAvoidingWalk 60 100000
93% dead ends
% java SelfAvoidingWalk 70 100000
96% dead ends
% java SelfAvoidingWalk 80 100000
98% dead ends
% java SelfAvoidingWalk 90 100000
99% dead ends
% java SelfAvoidingWalk 100 100000
99% dead ends
```



Summary

Arrays.

- Organized way to store huge quantities of data.
- Almost as easy to use as primitive types.
- Can directly access an element given its index.

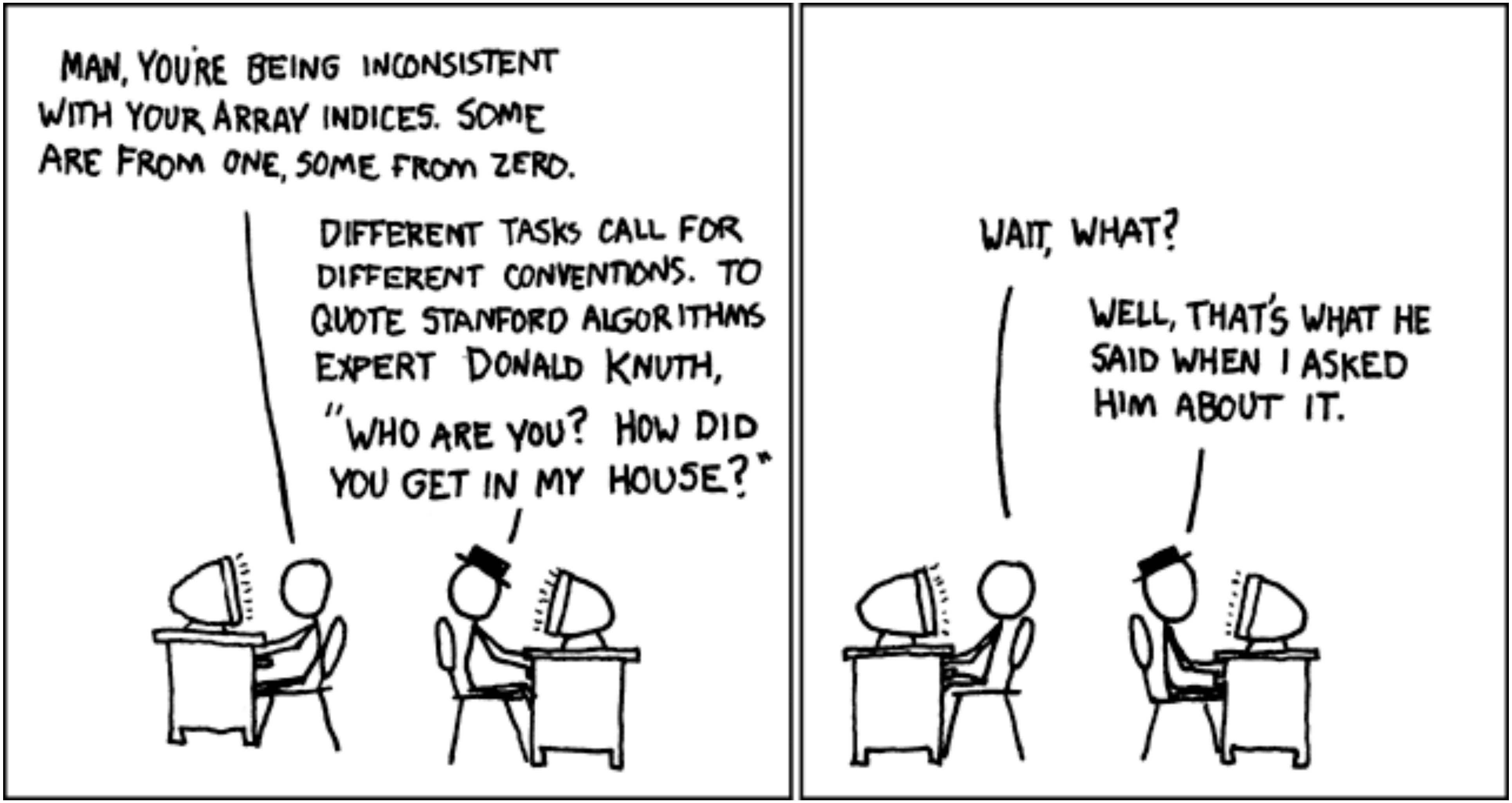
Caveats:

- Need to fix size of array ahead of time.
- Don't forget to allocate memory with new.
- Indices start at 0 not 1.
- Out-of-bounds to access a[-1] or a[N] of N element array.
  - in Java: ArrayIndexOutOfBoundsException
  - in C: "ghastly error"

Ahead. Reading in large quantities of data from a file into an array.

Off by One

"You're always off by 1 in this business." - J. Morris


MAN, YOU'RE BEING INCONSISTENT WITH YOUR ARRAY INDICES. SOME ARE FROM ONE, SOME FROM ZERO.
DIFFERENT TASKS CALL FOR DIFFERENT CONVENTIONS. TO QUOTE STANFORD ALGORITHMS EXPERT DONALD KNUTH, "WHO ARE YOU? HOW DID YOU GET IN MY HOUSE?"
WAIT, WHAT?
WELL, THAT'S WHAT HE SAID WHEN I ASKED HIM ABOUT IT.

http://imgs.xkcd.com/comics/donald_knuth.png