1. Combinational Circuits

You know that with AND, OR, and NOT gates alone, you can express any Boolean function whatsoever. The set of those gates is universal. In this question you will show how a two-to-one multiplexer is also universal by showing how each of AND, OR, and NOT can be made from one two-to-one multiplexer.

(a) Make a truth table for a two-to-one multiplexer. This device has three inputs, which we will label S, D0 and D1. The select input S controls which of D0 and D1 is copied to the output: when S is 0, the output equals D0, and when S is 1, the output equals D1.

(b) Show how to make each traditional logic gate from one multiplexer by setting each of the 3 inputs to x, y, 1, or 0.
2. Regular Expressions, Deterministic Finite State Automata

We have the three letter alphabet \{ a, b, c \} and the language of all strings that start and end with a.

Here are some examples of strings, and whether they are in the language:

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td>empty string</td>
</tr>
<tr>
<td>abca</td>
<td></td>
<td>abc</td>
</tr>
<tr>
<td>abacaa</td>
<td></td>
<td>baca</td>
</tr>
</tbody>
</table>

a) Which one of these Regular Expressions generates all strings that start and end with a? Circle the roman numeral that goes with your answer.

i) \( a^* (a \mid b \mid c)^* a \)

ii) \( a (a \mid b \mid c)^* a \)

iii) \( a ((b \mid c)^* a)^* \)

iv) \( a^* ((b \mid c)^* a)^* \)

v) \( a (b \mid c)^* a \)
2. RE, DFA continued

b) Which one of the following DFA accepts all strings that start and end with a? Circle the roman numeral that goes with your answer.

i) 

- N → Y
  - a
- Y → N
  - a
- N
  - b,c
- N
  - a,b,c
- Y
  - a,b,c

ii) 

- Y → N
  - b
- N → Y
  - b,c
- N
  - c
- Y
  - a
- b,c

iii) 

- N → Y
  - a
- Y → N
  - b,c
- N
  - b,c
- N
  - a
- Y
  - a
- b,c
3. **Linked Lists** Assume you have access to the private Node class:

```java
private class Node {
    double value;
    Node next;
}
```

Consider the following method which operates on linked lists:

```java
public boolean linky_dink (Node head) {
    Node a,b;
    a = head;
    if (a == null) return true;
    b = a.next;

    while ( b != null && b != a ) {
        b = b.next;
        if (b == null) return true;
        b = b.next;
        a = a.next;
    }

    return (b == null);
}
```

(a) What does `linky_dink` return on the following lists? Circle your answer.

i) `head` → `null`
   - returns true
   - returns false
   - does not return

ii) `head` → `null`
   - returns true
   - returns false
   - does not return
3. Linked Lists continued

(b) What does linky_dink do?

(c) If your linked list has $N$ nodes, what is the complexity of linky_dink? Circle your answer.

$N \quad N \log N \quad N^2 \quad 2^N$
4. Turing Machine

a) The Turing Machine above starts in the leftmost state. If this Turing Machine is run on the tape below, with the tape head starting at the position marked by the arrow, what will be the contents of the tape when it halts, AND where will the head be?

Write your answer in the empty tape below.

```
...0003/0/0110XX...```

b) What computation does this Turing Machine perform?
5. **Data Structures** Circle your answer.

Circle the data structure that is the most appropriate choice for the described problem.

(a) Store and retrieve student records, which have unique usernames.

- Array
- Linked List
- Queue
- Symbol Table

(b) Store all student grades and retrieve all grades higher than 90.

- Linked List
- Binary Search Tree
- Symbol Table
- Stack

(c) Implement the Back button in a browser

- Queue
- Binary Search Tree
- Stack
- Circular Linked List

6. **Theory True or False** Circle your answer.

T F (a) P is the set of search problems solvable in Polynomial time by a deterministic Turing Machine.

T F (b) NP is the set of search problems not solvable in Polynomial time by a deterministic Turing Machine.

T F (c) For proper encapsulation, instance variables should always be declared public.

T F (d) Because the Halting Problem is unsolvable, it is impossible to tell if your TSP program for Assignment 6 has an infinite loop.

T F (e) A Universal Turing Machine can compute anything that any other Turing Machine could possibly compute.

T F (g) If P equals NP, then the Traveling Salesperson Problem can be solved in polynomial time by a deterministic Turing Machine.

T F (h) If P does not equal NP, then there is no case of the Traveling Salesperson Problem for which you can find the optimal tour in polynomial time.

T F (j) Factoring is known to be in NP but has not been proven to be NP-complete, so the discovery of a polynomial-time algorithm for factoring would mean that P equals NP.

T F (k) Factoring is known to be in NP but has not been proven to be NP-complete, so no polynomial-time algorithm for factoring is possible.