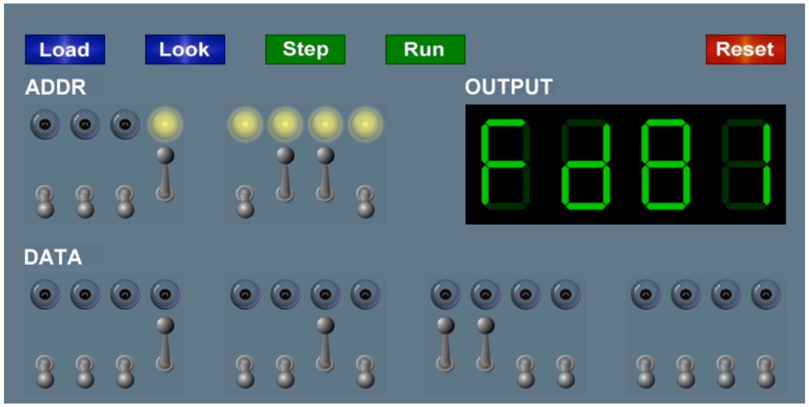
5. The TOY Machine

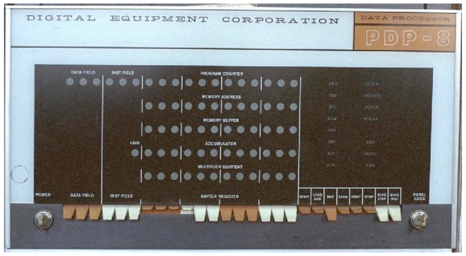


Introduction to Computer Science · Sedgewick and Wayne · Copyright © 2007 · http://www.cs.Princeton.EDU/IntroCS

What is TOY?

An imaginary machine similar to:

- Ancient computers.
- Today's microprocessors.
- And practically everything in between !



Why Study TOY?

Machine language programming.

- How do Java programs relate to computer?
- Key to understanding Java references.
- Still situations today where it is really necessary.

multimedia, computer games, embedded devices, scientific computing, MMX, Altivec

Computer architecture.

- How does it work?
- How is a computer put together?

TOY machine. Optimized for simplicity, not cost or performance.

Inside the Box

Switches. Input data and programs.

Lights. View data.

Memory.

- Stores data and programs.
- 256 16-bit "words."
- Special word for stdin / stdout.

Program counter (PC).

- An extra 8-bit register.
- Keeps track of next instruction to be executed.

Registers.

- Fastest form of storage.
- Scratch space during computation.
- 16 16-bit registers.
- Register 0 is always 0.

Arithmetic-logic unit (ALU). Manipulate data stored in registers.

Standard input, standard output. Interact with outside world.

Data and Programs Are Encoded in Binary

Each bit consists of two states:

- 1 or 0; true or false.
- Switch is on or off; wire has high voltage or low voltage.

Everything stored in a computer is a sequence of bits.

- Data and programs.
- Text, documents, pictures, sounds, movies, executables, …

$\text{M} = 77_{10} = 01001101_2 = 4\text{D}_{16}$
$\text{O} = 79_{10} = 01001111_2 = 4\text{F}_{16}$
$\text{M} = 77_{10} = 01001101_2 = 4\text{D}_{16}$

Binary Encoding

How to represent integers?

- Use binary encoding.
- Ex: $6375_{10} = 0001100011100111_2$

Dec	Bin	Dec	Bin
0	0000	8	1000
1	0001	9	1001
2	0010	10	1010
3	0011	11	1011
4	0100	12	1100
5	0101	13	1101
6	0110	14	1110
7	0111	15	1111

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

$$6375_{10} = +2^{12} +2^{11} +2^{7} +2^{6} +2^{5} +2^{2} +2^{1} +2^{0}$$

$$= 4096 +2048 +128 +64 +32 +4 +2 +1$$

Hexadecimal Encoding

How to represent integers?

- Use hexadecimal encoding.
- Binary code, four bits at a time.
- Ex: 6375_{10} $= 0001100011100111_2$
  $= 18\text{E}7_{16}$

Dec	Bin	Hex	Dec	Bin	Hex
0	0000	0	8	1000	8
1	0001	1	9	1001	9
2	0010	2	10	1010	A
3	0011	3	11	1011	B
4	0100	4	12	1100	C
5	0101	5	13	1101	D
6	0110	6	14	1110	E
7	0111	7	15	1111	F

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1
1				8				E				7			

$$6375_{10} = 1 \times 16^3 + 8 \times 16^2 + 14 \times 16^1 + 7 \times 16^0$$

$$= 4096 + 2048 + 224 + 7$$

Machine "Core" Dump

Machine contents at a particular place and time.

- Record of what program has done.
- Completely determines what machine will do.

Registers

R0	R1	R2	R3
0000	0000	0000	0000
R4	**R5**	**R6**	**R7**
0000	0000	0000	0000
R8	**R9**	**RA**	**RB**
0000	0000	0000	0000
RC	**RD**	**RE**	**RF**
0000	0000	0000	0000

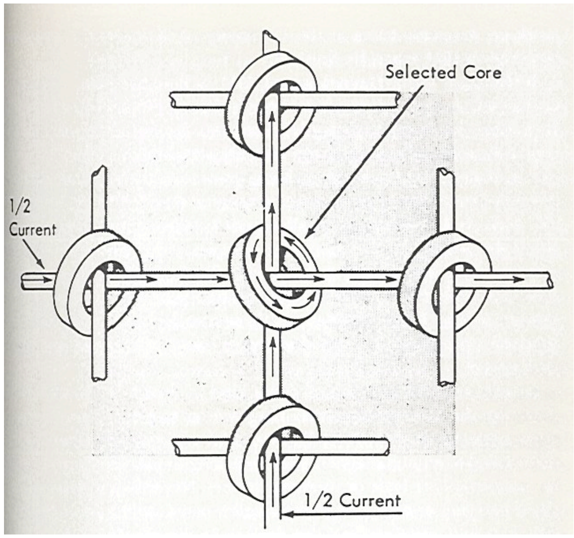
pc

10

Main Memory

	0/8	1/9	2/A	1/B	3/C	4/D	5/E	6/f
00:	0008	0005	0000	0000	0000	0000	0000	0000
08:	0000	0000	0000	0000	0000	0000	0000	0000
10:	8A00	8B01	1CAB	9C02	0000	0000	0000	0000
18:	0000	0000	0000	0000	0000	0000	0000	0000
20:	0000	0000	0000	0000	0000	0000	0000	0000
28:	0000	0000	0000	0000	0000	0000	0000	0000
.								
.								
E8:	0000	0000	0000	0000	0000	0000	0000	0000
F0:	0000	0000	0000	0000	0000	0000	0000	0000
F8:	0000	0000	0000	0000	0000	0000	0000	0000

Why do They Call it "Core"?



http://www.columbia.edu/acis/history/core.html

A Sample Program

A sample program. Adds `0008 + 0005 = 000D`.

RA	RB	RC
0000	0000	0000

Registers

pc
10

Program counter

TOY memory (program and data)

comments

```
00: 0008   8
01: 0005   5
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

TOY code to compute 0008_{10} + 0005_{10}

A Sample Program

Program counter. The `pc` is initially `10`, so the machine interprets `8A00` as an instruction.

RA	RB	RC
0000	0000	0000

Registers

pc
10

Program counter

index of next instruction to execute

```
00: 0008   8
01: 0005   5
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

Load

Load. [opcode 8]

- Loads the contents of some memory location into a register.
- `8A00` means load the contents of memory cell `00` into register `A`.

RA	RB	RC
0000	0000	0000

Registers

pc
10

Program counter

```
00: 0008   8
01: 0005   5
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
8_{16}				A_{16}				00_{16}							
opcode				dest d				addr							

Load

Load. [opcode 8]

- Loads the contents of some memory location into a register.
- `8B01` means load the contents of memory cell `01` into register `B`.

RA	RB	RC
0008	0000	0000

Registers

pc
11

Program counter

```
00: 0008    8
01: 0005    5
02: 0000    0

10: 8A00    RA ← mem[00]
11: 8B01    RB ← mem[01]
12: 1CAB    RC ← RA + RB
13: 9C02    mem[02] ← RC
14: 0000    halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1
8_{16}				B_{16}				01_{16}							
opcode				dest d				addr							

Add

Add. [opcode 1]

- Add contents of two registers and store sum in a third.
- `1CAB` means add the contents of registers `A` and `B` and put the result into register `C`.

RA	RB	RC
0008	0005	0000

Registers

pc
12

Program counter

```
00: 0008    8
01: 0005    5
02: 0000    0

10: 8A00    RA ← mem[00]
11: 8B01    RB ← mem[01]
12: 1CAB    RC ← RA + RB
13: 9C02    mem[02] ← RC
14: 0000    halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	1	0	0	1	0	1	0	1	0	1	1
1_{16}				C_{16}				A_{16}				B_{16}			
opcode				dest d				source s				source t			

Store

Store. [opcode 9]

- Stores the contents of some register into a memory cell.
- `9C02` means store the contents of register `C` into memory cell `02`.

RA	RB	RC
0008	0005	000D

Registers

pc
13

Program counter

```
00: 0008    8
01: 0005    5
02: 0000    0

10: 8A00    RA ← mem[00]
11: 8B01    RB ← mem[01]
12: 1CAB    RC ← RA + RB
13: 9C02    mem[02] ← RC
14: 0000    halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0
9_{16}				C_{16}				02_{16}							
opcode				dest d				addr							

Halt

Halt. [opcode 0]

- Stop the machine.

RA	RB	RC
0008	0005	000D

Registers

pc
14

Program counter

```
00: 0008    8
01: 0005    5
02: 000D    D

10: 8A00    RA ← mem[00]
11: 8B01    RB ← mem[01]
12: 1CAB    RC ← RA + RB
13: 9C02    mem[02] ← RC
14: 0000    halt
```

add.toy

TOY code to compute 0008_{10} + 0005_{10}

Same Program, Different Data

Program. Sequence of instructions.

Instruction. `10`, `11`, `12`, `13`, and `14` (executed when pc points to it).

Data. `00`, `01`, and `02` (used and changed by instructions).

RA	RB	RC
0000	0000	0000

Registers

pc
10

Program counter

data

```
00: 1CAB   7339
01: 1CAB   7339
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

instruction

add.toy

TOY code to compute $7339_{10} + 7339_{10}$

$$1CAB_{16} = 1 \times 16^3 + 12 \times 16^2 + 10 \times 16^1 + 11 \times 16^0 = 4096 + 3072 + 160 + 11 = 7339_{10}$$

Load

Load. [opcode 8]

- Loads the contents of some memory location into a register.
- `8A00` means load the contents of memory cell `00` into register `A`.

RA	RB	RC
0000	0000	0000

Registers

pc
10

Program counter

```
00: 1CAB   7339
01: 1CAB   7339
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
8_{16}				A_{16}				00_{16}							
opcode				dest d				addr							

Load

Load. [opcode 8]

- Loads the contents of some memory location into a register.
- `8B01` means load the contents of memory cell `01` into register `B`.

RA	RB	RC
1CAB	0000	0000

Registers

pc
11

Program counter

```
00: 1CAB   7339
01: 1CAB   7339
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1
8_{16}				B_{16}				01_{16}							
opcode				dest d				addr							

Add

Add. [opcode 1]

- Add contents of two registers and store sum in a third.
- `1CAB` means add the contents of registers `A` and `B` and put the result into register `C`.

RA	RB	RC
1CAB	1CAB	0000

Registers

pc
12

Program counter

```
00: 1CAB   7339
01: 1CAB   7339
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	1	0	0	1	0	1	0	1	0	1	1
1_{16}				C_{16}				A_{16}				B_{16}			
opcode				dest d				source s				source t			

Store

Store. [opcode 9]
- Stores the contents of some register into a memory cell.
- `9C02` means store the contents of register `C` into memory cell `02`.

RA	RB	RC
1CAB	1CAB	3956

Registers

pc
13

Program counter

```
00: 1CAB   7339
01: 1CAB   7339
02: 0000   0

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0
9_{16}				C_{16}				02_{16}							
opcode				dest d				addr							

Halt

Halt. [opcode 0]
- Stop the machine.

RA	RB	RC
1CAB	1CAB	3956

Registers

pc
14

Program counter

```
00: 1CAB    7339
01: 1CAB    7339
02: 3956   14678

10: 8A00   RA ← mem[00]
11: 8B01   RB ← mem[01]
12: 1CAB   RC ← RA + RB
13: 9C02   mem[02] ← RC
14: 0000   halt
```

add.toy

Program and Data

Program. Sequence of 16-bit integers, interpreted one way.

Data. Sequence of 16-bit integers, interpreted another way.

Program counter (pc). Holds memory address of the "next instruction" and determines which integers get interpreted as instructions.

16 instruction types. Changes contents of registers, memory, and pc in specified, well-defined ways.

Instructions

→	0:	halt
→	1:	add
	2:	subtract
	3:	and
	4:	xor
	5:	shift left
	6:	shift right
	7:	load address
→	8:	load
→	9:	store
	A:	load indirect
	B:	store indirect
	C:	branch zero
	D:	branch positive
	E:	jump register
	F:	jump and link

TOY Instruction Set Architecture

TOY instruction set architecture (ISA).
- Interface that specifies behavior of machine.
- 16 register, 256 words of main memory, 16-bit words.
- 16 instructions.

Each instruction consists of 16 bits.
- Bits 12-15 encode one of 16 instruction types or opcodes.
- Bits 8-11 encode destination register `d`.
- Bits 0-7 encode:
  - [Format 1] source registers `s` and `t`
  - [Format 2] 8-bit memory address or constant

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	1	0	1	1	1	0	1	0	0	0	0	0	0	1	0	0
Format 1	opcode				dest d				source s				source t			
Format 2	opcode				dest d				addr							

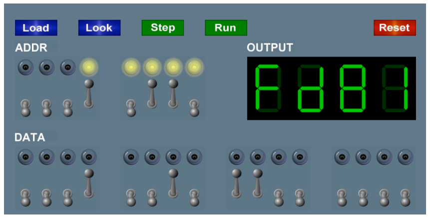
Interfacing with the TOY Machine

To enter a program or data:

- Set 8 memory address switches.
- Set 16 data switches.
- Press `Load`: data written into addressed word of memory.

To view the results of a program:

- Set 8 memory address switches.
- Press `Look`: contents of addressed word appears in lights.



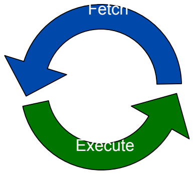
Interfacing with the TOY Machine

To execute the program:

- Set 8 memory address switches to address of first instruction.
- Press `Look` to set `pc` to first instruction.
- Press `Run` to repeat fetch-execute cycle until halt opcode.

Fetch-execute cycle.

- Fetch: get instruction from memory.
- Execute: update `pc`, move data to or from memory and registers, perform calculations.



Flow Control

Flow control.

- To harness the power of TOY, need loops and conditionals.
- Manipulate `pc` to control program flow.

Branch if zero. [opcode C]

- Changes `pc` depending on whether value of some register is zero.
- Used to implement: `for`, `while`, `if-else`.

Branch if positive. [opcode D]

- Changes `pc` depending on whether value of some register is positive.
- Used to implement: `for`, `while`, `if-else`.

An Example: Multiplication

Multiply. Given integers `a` and `b`, compute `c` = `a` × `b`.

TOY multiplication. No direct support in TOY hardware.

Brute-force multiplication algorithm:

- Initialize `c` to `0`.
- Add `b` to `c`, `a` times.

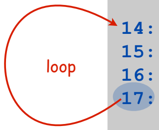
```
int a = 3;
int b = 9;
int c = 0;

while (a != 0) {
   c = c + b;
   a = a - 1;
}
```

brute force multiply in Java

Issues ignored. Slow, overflow, negative numbers.

Multiply

```
0A: 0003   3   ←  inputs
0B: 0009   9
0C: 0000   0   ←  output

0D: 0000   0   ←  constants
0E: 0001   1


10: 8A0A   RA ← mem[0A]             a
11: 8B0B   RB ← mem[0B]             b
12: 8C0D   RC ← mem[0D]             c = 0

13: 810E   R1 ← mem[0E]             always 1

14: CA18   if (RA == 0) pc ← 18     while (a != 0) {
15: 1CCB   RC ← RC + RB                c = c + b
16: 2AA1   RA ← RA - R1                a = a - 1
17: C014   pc ← 14                  }

18: 9C0C   mem[0C] ← RC
19: 0000   halt
```



multiply.toy

Step-By-Step Trace

		R1	RA	RB	RC
10: 8A0A	RA ← mem[0A]		0003		
11: 8B0B	RB ← mem[0B]			0009	
12: 8C0D	RC ← mem[0D]				0000
13: 810E	R1 ← mem[0E]	0001			
14: CA18	if (RA == 0) pc ← 18				
15: 1CCB	RC ← RC + RB				0009
16: 2AA1	RA ← RA – R1		0002		
17: C014	pc ← 14				
14: CA18	if (RA == 0) pc ← 18				
15: 1CCB	RC ← RC + RB				0012
16: 2AA1	RA ← RA – R1		0001		
17: C014	pc ← 14				
14: CA18	if (RA == 0) pc ← 18				
15: 1CCB	RC ← RC + RB				001B
16: 2AA1	RA ← RA – R1		0000		
17: C014	pc ← 14				
14: CA18	if (RA == 0) pc ← 18				
18: 9C0C	mem[0C] ← RC				
19: 0000	halt				

multiply.toy

An Efficient Multiplication Algorithm

Inefficient multiply.

- Brute force multiplication algorithm loops a times.
- In worst case, 65,535 additions!

"Grade-school" multiplication.

- Always 16 additions to multiply 16-bit integers.

Decimal

```
        1 2 3 4
      * 1 5 1 2
        2 4 6 8
      1 2 3 4
    6 1 7 0
  1 2 3 4
0 1 8 6 5 8 0 8
```

Binary

```
        1 0 1 1
      * 1 1 0 1
        1 0 1 1
      0 0 0 0
    1 0 1 1
  1 0 1 1
1 0 0 0 1 1 1 1
```

Binary Multiplication

Grade school binary multiplication algorithm to compute c = a × b.

- Initialize c = 0.
- Loop over i bits of b.
  - if b_i = 0, do nothing ← b_i = i^{th} bit of b
  - if b_i = 1, shift a left i bits and add to c

```
        1 0 1 1     a
      * 1 1 0 1     b
        1 0 1 1     a << 0
      0 0 0 0
    1 0 1 1         a << 2
  1 0 1 1           a << 3
1 0 0 0 1 1 1 1     c
```

Implement with built-in TOY shift instructions.

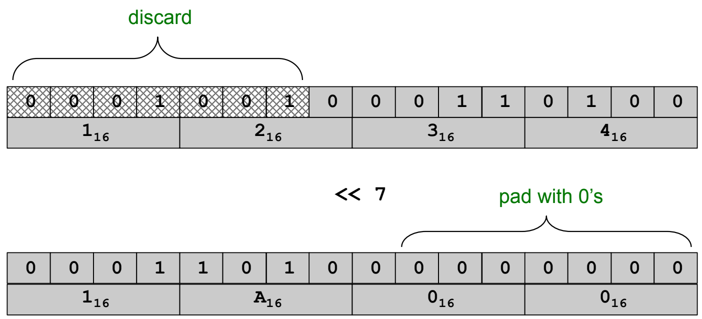
```
int c = 0;
for (int i = 15; i >= 0; i--)
   if (((b >> i) & 1) == 1)
      c = c + (a << i);
```

← b_i = i^{th} bit of b

Shift Left

Shift left. (opcode 5)

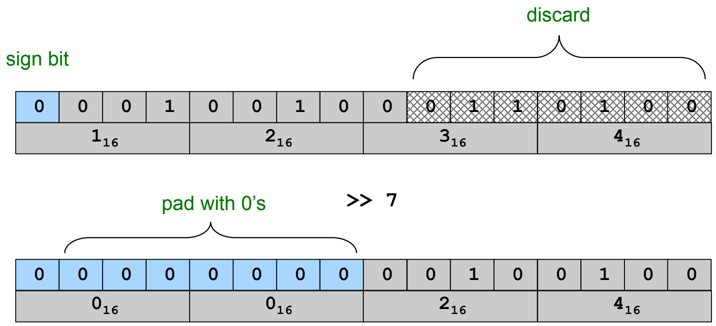
- Move bits to the left, padding with zeros as needed.
- 1234_{16} << 7_{16} = $1A00_{16}$



Shift Right

Shift right. (opcode 6)

- Move bits to the right, padding with sign bit as needed.
- 1234_{16} >> 7_{16} = 0024_{16}

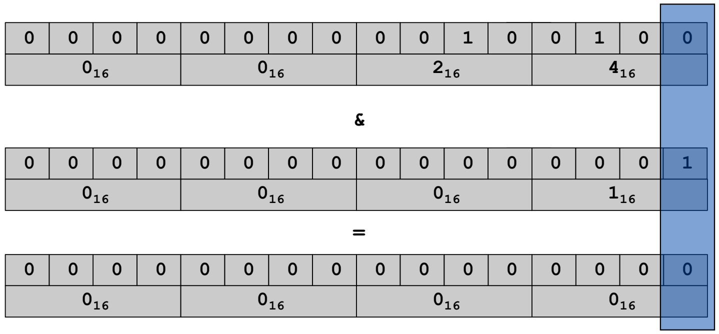


Bitwise AND

Logical AND. (opcode B)

- Logic operations are BITWISE.
- 0024_{16} & 0001_{16} = 0000_{16}

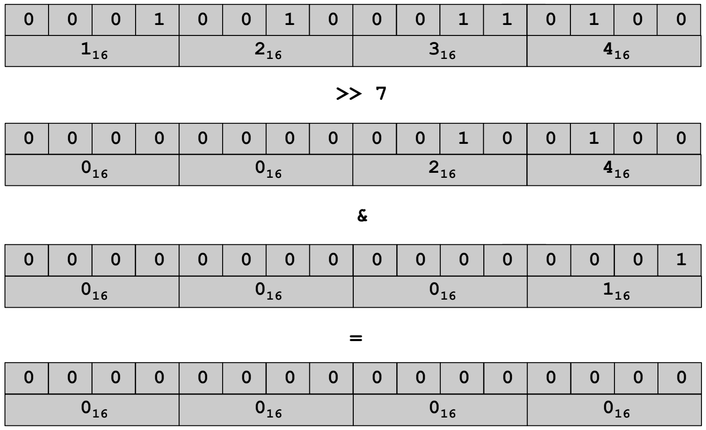
x	y	&
0	0	0
0	1	0
1	0	0
1	1	1



Shifting and Masking

Shift and mask: get the 7th bit of 1234.

- Compute 1234_{16} >> 7_{16} = 0024_{16}.
- Compute 0024_{16} & 1_{16} = 0_{16}.



Binary Multiplication

```
0A: 0003     3    ⇐  inputs
0B: 0009     9
0C: 0000     0    ⇐  output
0D: 0000     0
0E: 0001     1    ⇐  constants
0F: 0010    16

10: 8A0A    RA ← mem[0A]           a
11: 8B0B    RB ← mem[0B]           b
12: 8C0D    RC ← mem[0D]           c = 0
13: 810E    R1 ← mem[0E]           always 1
14: 820F    R2 ← mem[0F]           i = 16      ⇐ 16 bit words

loop                               do {
15: 2221    R2 ← R2 -  R1             i--
16: 53A2    R3 ← RA << R2             a << i
17: 64B2    R4 ← RB >> R2             b >> i
branch
18: 3441    R4 ← R4 &  R1             b_i = ith bit of b
19: C41B    if (R4 == 0) goto 1B      if b_i is 1
1A: 1CC3    RC ← RC + R3                 add a << i to sum
1B: D215    if (R2 > 0) goto 15    } while (i > 0);

1C: 9C0C    mem[0C] ← RC
```

multiply-fast.toy

TOY Reference Card

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Format 1	opcode				dest d				source s				source t			
Format 2	opcode				dest d				addr							

#	Operation	Fmt	Pseudocode
0:	halt	1	`exit(0)`
1:	add	1	`R[d] ← R[s] + R[t]`
2:	subtract	1	`R[d] ← R[s] - R[t]`
3:	and	1	`R[d] ← R[s] & R[t]`
4:	xor	1	`R[d] ← R[s] ^ R[t]`
5:	shift left	1	`R[d] ← R[s] << R[t]`
6:	shift right	1	`R[d] ← R[s] >> R[t]`
7:	load addr	2	`R[d] ← addr`
8:	load	2	`R[d] ← mem[addr]`
9:	store	2	`mem[addr] ← R[d]`
A:	load indirect	1	`R[d] ← mem[R[t]]`
B:	store indirect	1	`mem[R[t]] ← R[d]`
C:	branch zero	2	`if (R[d] == 0) pc ← addr`
D:	branch positive	2	`if (R[d] > 0) pc ← addr`
E:	jump register	2	`pc ← R[d]`
F:	jump and link	2	`R[d] ← pc; pc ← addr`

Register 0 always 0.
Loads from `mem[FF]` from stdin.
Stores to `mem[FF]` to stdout.

Useful TOY "Idioms"

Jump absolute.

- Jump to a fixed memory address.
  - branch if zero with destination
  - register 0 is always 0

```
17: C014   pc ← 14
```

Register assignment.

- No instruction that transfers contents of one register into another.
- Pseudo-instruction that simulates assignment:
  - add with register 0 as one of two source registers

```
17: 1230   R[2] ← R[3]
```

No-op.

- Instruction that does nothing.
- Plays the role of whitespace in C programs.
  - numerous other possibilities!

```
17: 1000   no-op
```

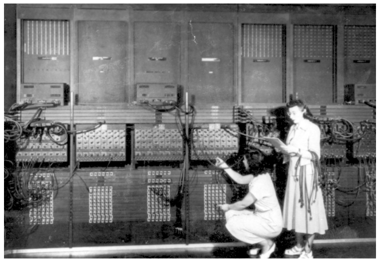
A Little History

Electronic Numerical Integrator and Calculator (ENIAC).

- First widely known general purpose electronic computer.
- Conditional jumps, programmable.
- Programming: change switches and cable connections.
- Data: enter numbers using punch cards.

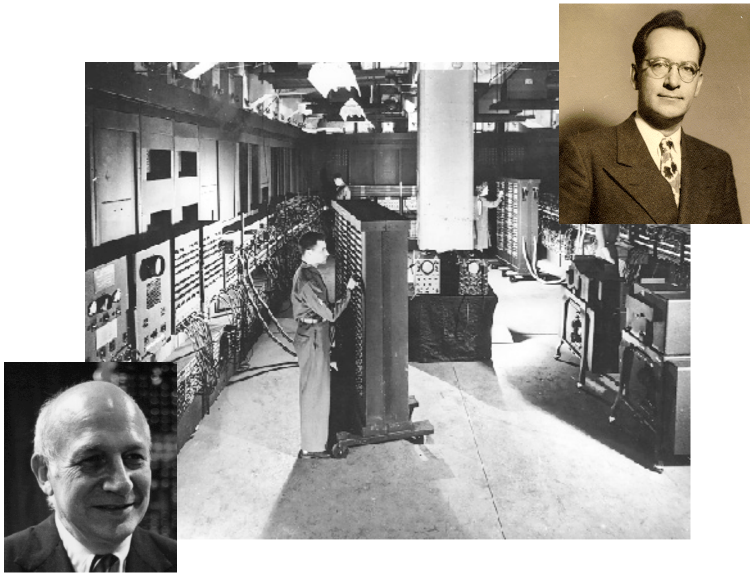
30 tons
30 x 50 x 8.5 ft
17,468 vacuum tubes
300 multiply/sec

John Mauchly (left) and J. Presper Eckert (right)
http://cs.swau.edu/~durkin/articles/history_computing.html



ENIAC, Ester Gerston (left), Gloria Gordon (right)
US Army photo: http://ftp.arl.mil/ftp/historic-computers

ENIAC



Basic Characteristics of TOY Machine

TOY is a general-purpose computer.
- Sufficient power to perform ANY computation.
- Limited only by amount of memory and time.

Stored-program computer. [von Neumann memo, 1944]
- Data and program encoded in binary.
- Data and program stored in SAME memory.
- Can change program without rewiring.

Outgrowth of Alan Turing's work. (stay tuned)

All modern computers are general-purpose computers and have same (von Neumann) architecture.

John von Neumann

Maurice Wilkes (left)
EDSAC (right)

Harvard vs. Princeton

Harvard architecture.
- Separate program and data memories.
- Can't load game from disk (data) and execute (program).
- Used in some microcontrollers.

Von Neumann architecture.
- Program and data stored in same memory.
- Used in almost all computers.

Q. What's the difference between Harvard and Princeton?
A. At Princeton, data and programs are the same.