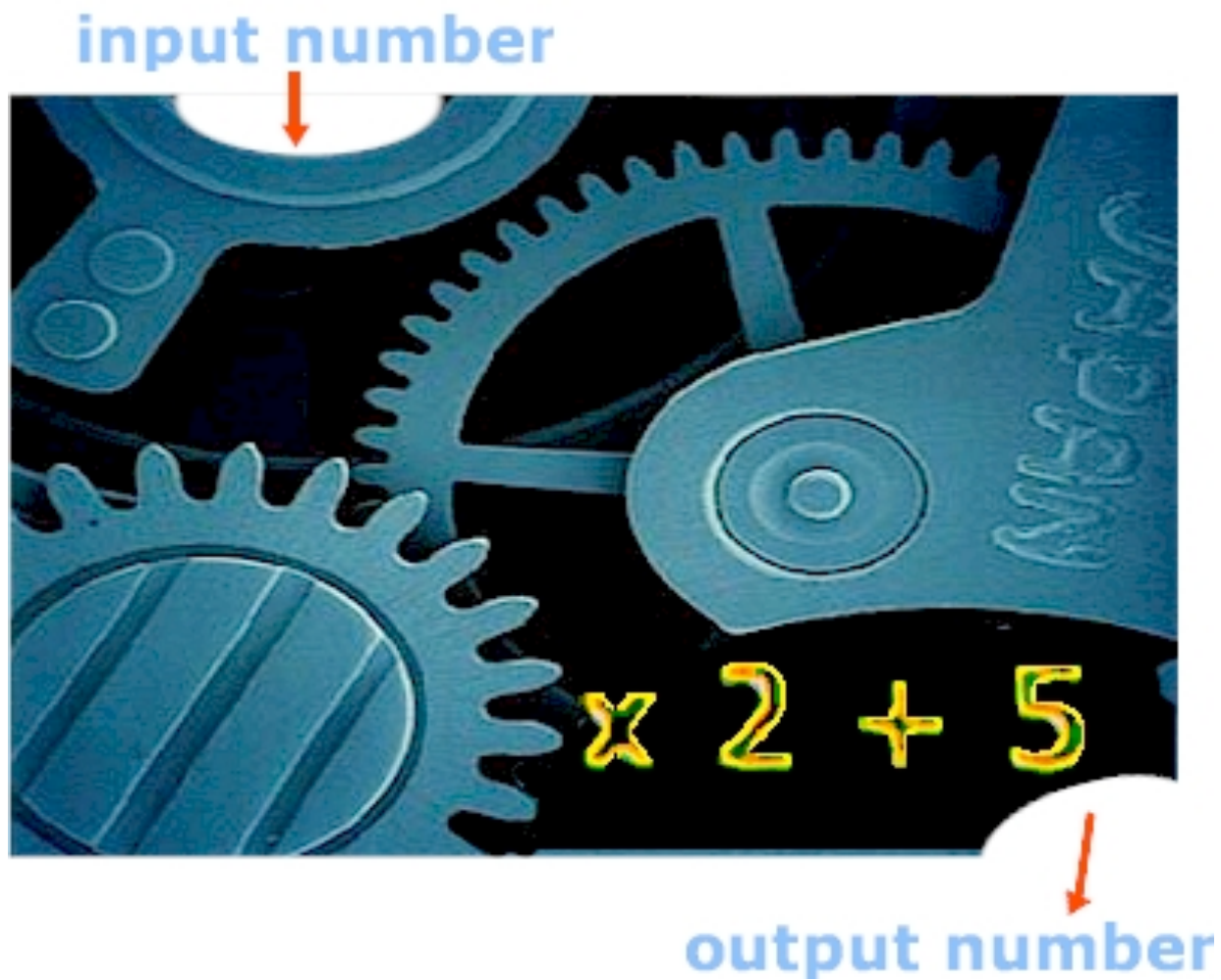# 2.1 Functions

A Foundation for Programming

any program you might want to write

objects

functions and modules ← build bigger programs and reuse code
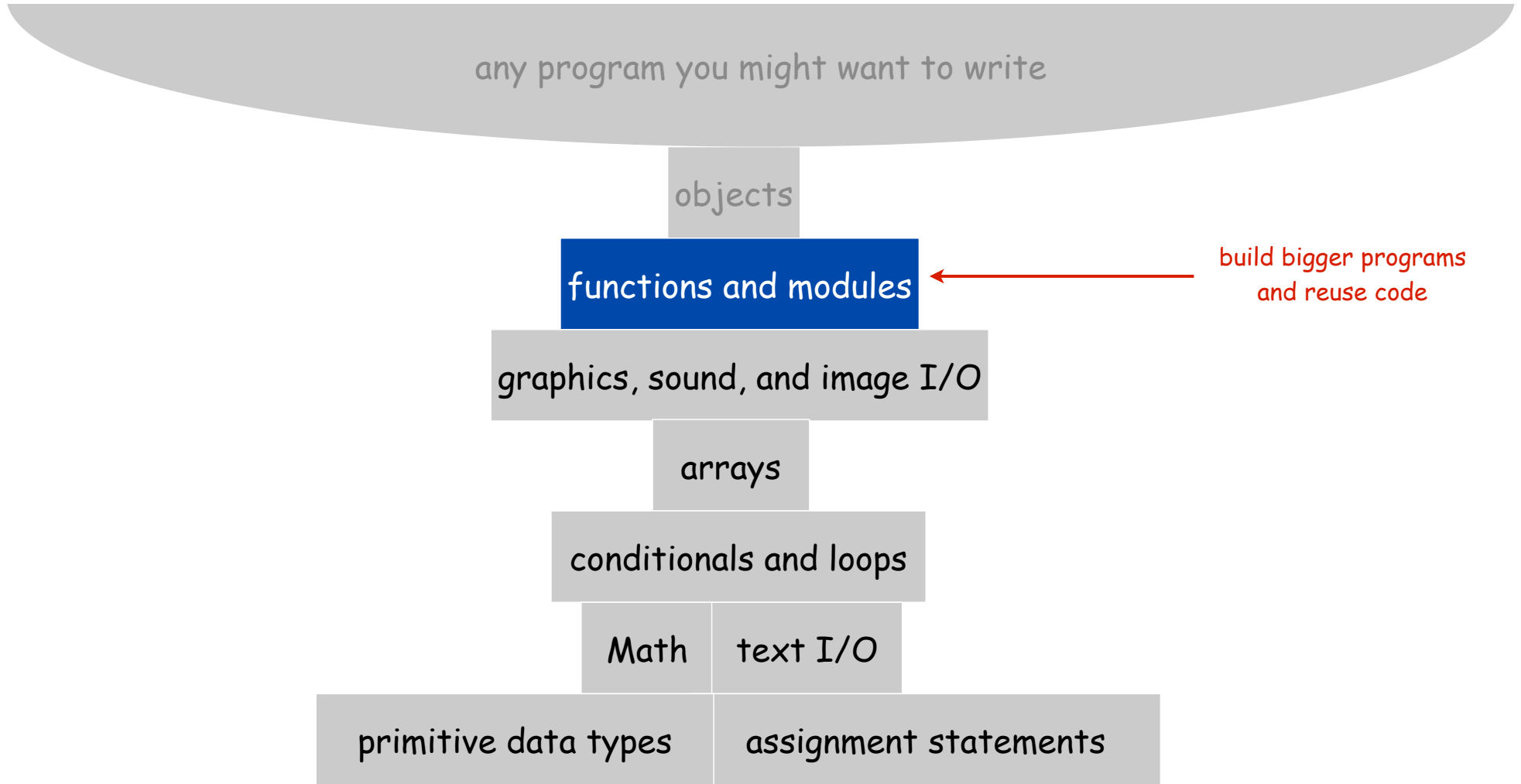
graphics, sound, and image I/O

arrays

conditionals and loops

Math | text I/O

primitive data types | assignment statements

Functions (Static Methods)

Java function.
- Takes zero or more input arguments.
- Returns zero or one output value.
- May cause side effects (e.g., output to standard draw). ← more general than mathematical functions
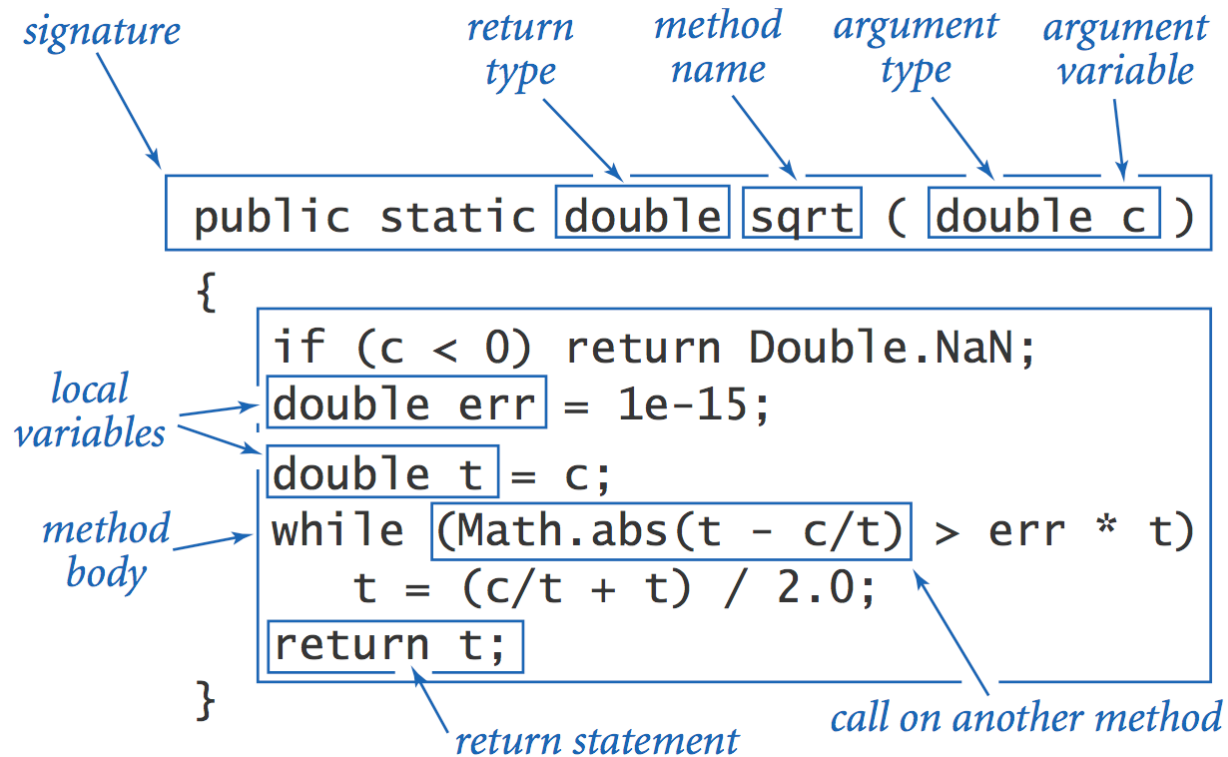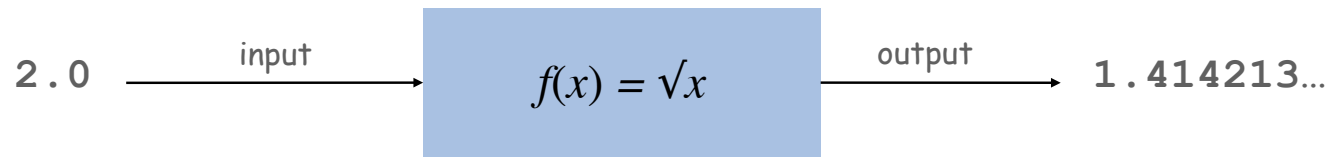
Applications.
- Scientists use mathematical functions to calculate formulas.
- Programmers use functions to build modular programs.
- You use functions for both.

Examples.
- Built-in functions: `Math.random()`, `Math.abs()`, `Integer.parseInt()`.
- Our I/O libraries: `StdIn.readInt()`, `StdDraw.line()`, `StdAudio.play()`.
- User-defined functions: `main()`.

Anatomy of a Java Function

Java functions. Easy to write your own.

$$2.0 \quad \xrightarrow{\text{input}} \quad \boxed{f(x) = \sqrt{x}} \quad \xrightarrow{\text{output}} \quad 1.414213\ldots$$

signature   *return type*   *method name*   *argument type*   *argument variable*

```
public static double sqrt ( double c )
{
    if (c < 0) return Double.NaN;
    double err = 1e-15;
    double t = c;
    while (Math.abs(t - c/t) > err * t)
        t = (c/t + t) / 2.0;
    return t;
}
```

local variables

method body

return statement

call on another method

Mumbojumbo Demystification, Part 2

```java
public class Gambler {
   public static void main(String[] args) {
      int stake  = Integer.parseInt(args[0]);
      int goal   = Integer.parseInt(args[1]);
      int trials = Integer.parseInt(args[2]);
      .  .  .
      .  .  .

}
```

Flow of Control

Key point. Functions provide a new way to control the flow of execution.

```java
public class Newton
{
    public static double sqrt(double c)
    {
        double epsilon = 1e-15;
        if (c < 0) return Double.NaN;
        double t = c;
        while (Math.abs(t - c/t) > epsilon * t)
            t = (c/t + t) / 2.0;
        return t;
    }

    public static void main(String[] args)
    {
        double[] a = new double[args.length];
        for (int i = 0; i < args.length; i++)
            a[i] = Double.parseDouble(args[i]);
        for (int i = 0; i < a.length; i++)
        {
            double x = sqrt(a[i]);
            StdOut.println(x);
        }
    }
}
```

Flow of Control

Key point. Functions provide a new way to control the flow of execution.

Summary of what happens when a function is called:
- Control transfers to the function code.
- Argument variables are assigned the values given in the call.
- Function code is executed.
- Return value is assigned in place of the function name in the calling code.
- Control transfers back to the calling code.

Note. This technique (standard in Java) is known as "pass by value".

other languages may use different methods

Scope

Scope (of a name). The code that can refer to that name.

Def. A variable's scope is code following the declaration in its block.

```
public class Newton
{
    public static double sqrt(double c)
    {
        double epsilon = 1e-15;                       ←——— scope of c
        if (c < 0) return Double.NaN;                 ←——— scope of epsilon
        double t = c;
        while (Math.abs(t - c/t) > epsilon * t)       ←——— scope of t
            t = (c/t + t) / 2.0;
        return t;
    }

    public static void main(String[] args)
    {
        double[] a = new double[args.length];
        for (int i = 0; i < args.length; i++)         ←——— scope of a
            a[i] = Double.parseDouble(args[i]);
        for (int i = 0; i < a.length; i++)
            System.out.println(sqrt(a[i]));
    }
}
```
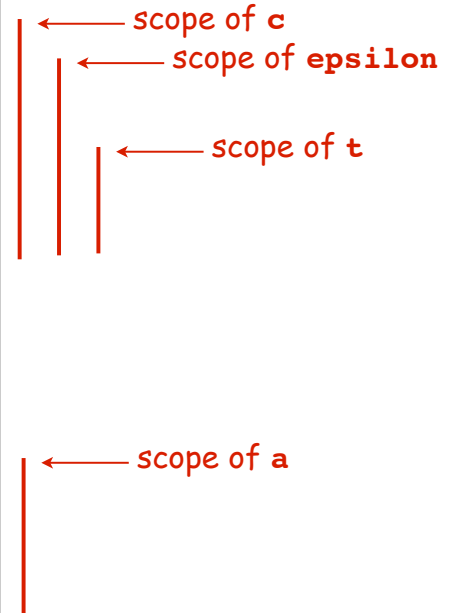
two different variables
with the same name i
each with two lines of scope

Best practice: declare variables so as to limit their scope.

Function Call Trace

```java
public class Newton
{
   public static double sqrt(double c)
   {
      double epsilon = 1e-15;
      if (c < 0) return Double.NaN;
      double t = c;
      while (Math.abs(t - c/t) > epsilon * t)
         t = (c/t + t) / 2.0;
      return t;
   }

   public static void main(String[] args)
   {
      double[] a = new double[args.length];
      for (int i = 0; i < args.length; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < a.length; i++)
         System.out.println(sqrt(a[i]));
   }
}
```

Functions Challenge 1

What happens when you compile and run the following code?

```java
public class Cubes1
{
    public static int cube(int i)
    {
        int j = i * i * i;
        return j;
    }
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        for (int i = 1; i <= N; i++)
            StdOut.println(i + " " + cube(i));
    }
}
```

Functions Challenge 2

What happens when you compile and run the following code?

```java
public class Cubes2
{
    public static int cube(int i)
    {
        int i = i * i * i;
        return i;
    }
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        for (int i = 1; i <= N; i++)
            StdOut.println(i + " " + cube(i));
    }
}
```

Functions Challenge 3

What happens when you compile and run the following code?

```java
public class Cubes3
{
   public static int cube(int i)
   {
      i = i * i * i;
   }
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

Functions Challenge 4

What happens when you compile and run the following code?

```java
public class Cubes4
{
    public static int cube(int i)
    {
        i = i * i * i;
        return i;
    }
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        for (int i = 1; i <= N; i++)
            StdOut.println(i + " " + cube(i));
    }
}
```

Functions Challenge 5

What happens when you compile and run the following code?

```java
public class Cubes5
{
    public static int cube(int i)
    {
        return i * i * i;
    }
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        for (int i = 1; i <= N; i++)
            StdOut.println(i + " " + cube(i));
    }
}
```
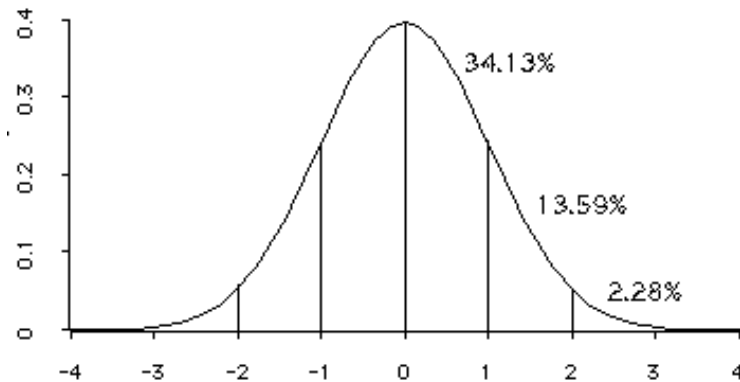
Example: Gaussian Distribution
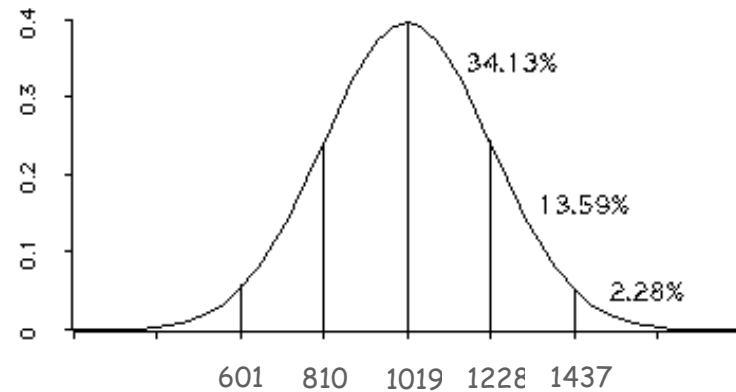
Gaussian Distribution

Standard Gaussian distribution.

• "Bell curve."

• Basis of most statistical analysis in social and physical sciences.

Ex. 2000 SAT scores follow a Gaussian distribution with
mean μ = 1019, stddev σ = 209.

$$\phi(x) = \tfrac{1}{\sqrt{2\pi}} \, e^{-x^2/2}$$

$$\phi(x, \mu, \sigma) = \tfrac{1}{\sigma\sqrt{2\pi}} \, e^{-(x-\mu)^2/2\sigma^2}$$

$$= \phi\!\left(\tfrac{x-\mu}{\sigma}\right)/\sigma$$

Java Function for $\phi(x)$

Mathematical functions. Use built-in functions when possible; build your own when not available.

```java
public class Gaussian
{

    public static double phi(double x)
    {
        return Math.exp(-x*x / 2) / Math.sqrt(2 * Math.PI);
    }

    public static double phi(double x, double mu, double sigma)
    {
        return phi((x - mu) / sigma) / sigma;
    }

}
```

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$$

$$\phi(x, \mu, \sigma) = \phi\left(\frac{x-\mu}{\sigma}\right) / \sigma$$

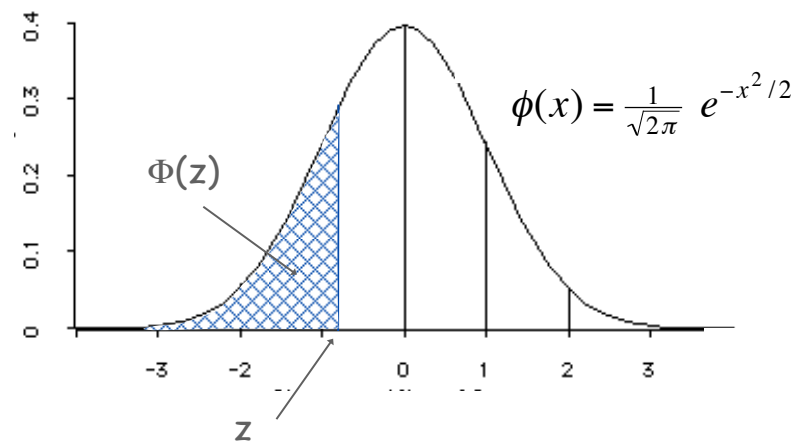**Overloading.** Functions with different signatures are different.

Multiple arguments. Functions can take any number of arguments.

Calling other functions. Functions can call other functions.

library or
user-defined

Gaussian Cumulative Distribution Function

Goal. Compute Gaussian cdf $\Phi(z)$.

Challenge. No "closed form" expression and not in Java library.

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$$

$\Phi(z)$

$$\Phi(z) = \int_{-\infty}^{z} \phi(x)\,dx$$

Taylor series

$$= \frac{1}{2} + \phi(z)\left(z + \frac{z^3}{3} + \frac{z^5}{3\cdot 5} + \frac{z^7}{3\cdot 5\cdot 7} + \cdots\right)$$

Bottom line. 1,000 years of mathematical formulas at your fingertips.

Java function for Φ(z)

```java
public class Gaussian
{

    public static double phi(double x)
        // as before

    public static double Phi(double z)
    {
        if (z < -8.0) return 0.0;
        if (z >  8.0) return 1.0;
        double sum = 0.0, term = z;
        for (int i = 3; sum + term != sum; i += 2) {
            sum  = sum + term;
            term = term * z * z / i;
        }
        return 0.5 + sum * phi(z);
    }

    public static double Phi(double z, double mu, double sigma)

    {

        return Phi((z - mu) / sigma);

    }

}
```

accurate with absolute error
less than $8 * 10^{-16}$

$\Phi(z, \mu, \sigma) = \int_{-\infty}^{z} \phi(z, \mu, \sigma) = \Phi((z - \mu) / \sigma)$

SAT Scores

Q. NCAA requires at least 820 for Division I athletes.
What fraction of test takers in 2000 did not qualify?

A. $\Phi(820, \mu, \sigma) \approx 0.17051$. [approximately 17%]


```
double fraction = Gaussian.Phi(820, 1019, 209);
```

Gaussian Distribution

Q. Why relevant in mathematics?
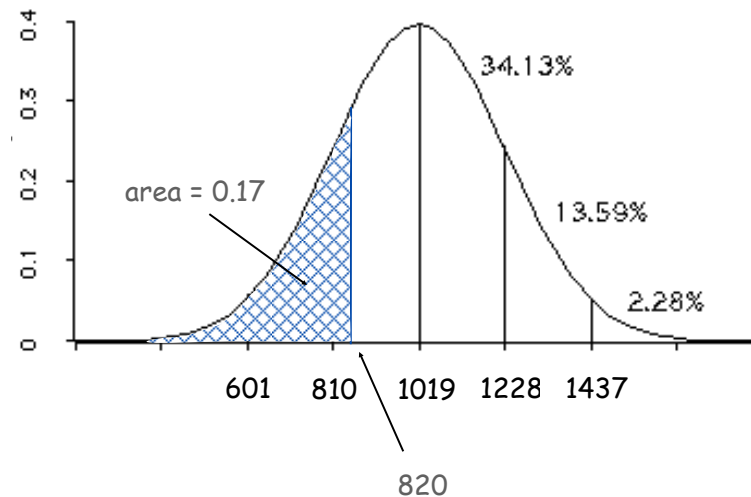
A. Central limit theorem: under very general conditions, average of
    a set of variables tends to the Gaussian distribution.

Q. Why relevant in the sciences?

A. Models a wide range of natural phenomena and random processes.

 • Weights of humans, heights of trees in a forest.

 • SAT scores, investment returns.

Caveat.

> Everybody believes in the exponential law of errors: the experimenters,
> because they think it can be proved by mathematics; and the
> mathematicians, because they believe it has been established by
> observation.   - M. Lippman in a letter to H. Poincaré

Libraries

Library. A module whose methods are primarily intended for use by many other programs.

Client. Program that calls library method(s).

API. Contract between client and implementation.

Implementation. Program that implements the methods in an API.

client

```
Gaussian.Phi(1019)
```

calls methods

API

```
public class Gaussian

    double phi(double x)     φ(x)
    double Phi(double z)     Φ(z)
```

*defines signatures
and describes methods*

implementation

```
public class Gaussian

    public static double phi(double x)

    public static double Phi(double z)
```

*Java code that
implements methods*
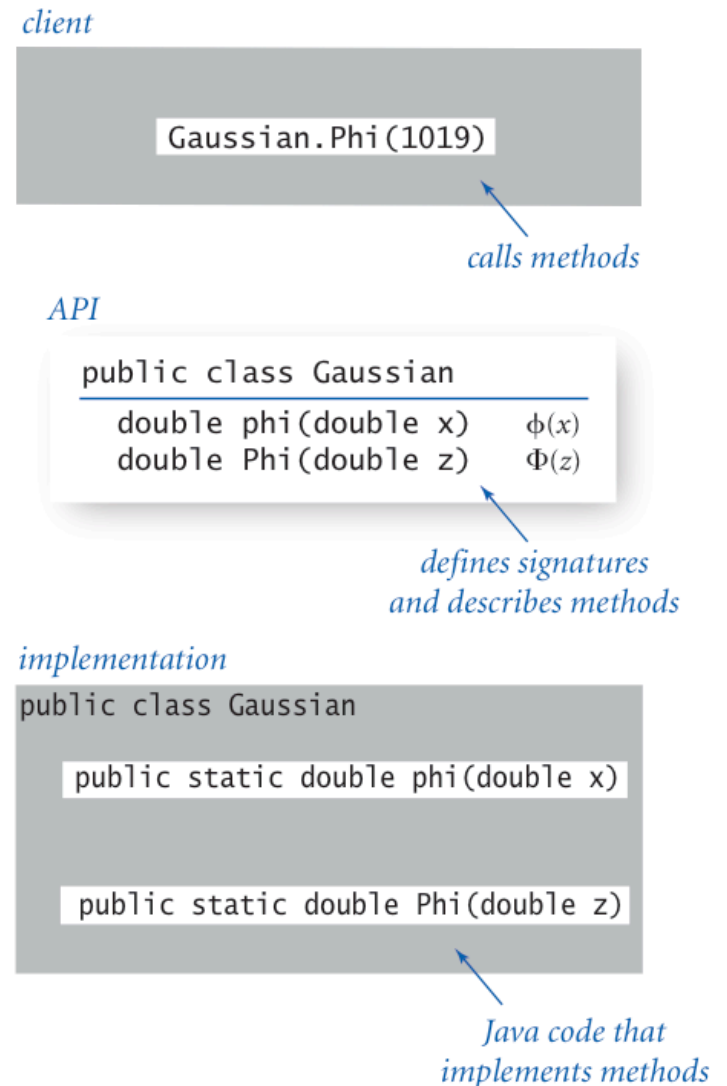
Libraries

Why use libraries?

- Makes code easier to understand.

- Makes code easier to debug.

- Makes code easier to maintain and improve.

- Makes code easier to reuse.

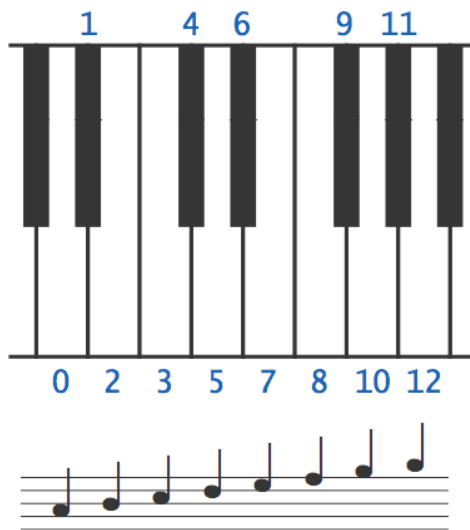# Digital Audio

Crash Course in Sound

Sound. Perception of the vibration of molecules in our eardrums.

Concert A. Sine wave, scaled to oscillated at 440Hz.

Other notes. 12 notes on chromatic scale, divided logarithmically.

note	i	frequency
A	0	440.00
A♯ or B♭	1	466.16
B	2	493.88
C	3	523.25
C♯ or D♭	4	554.37
D	5	587.33
D♯ or E♭	6	622.25
E	7	659.26
F	8	698.46
F♯ or G♭	9	739.99
G	10	783.99
G♯ or A♭	11	830.61
A	12	880.00

$440 \times 2^{i/12}$
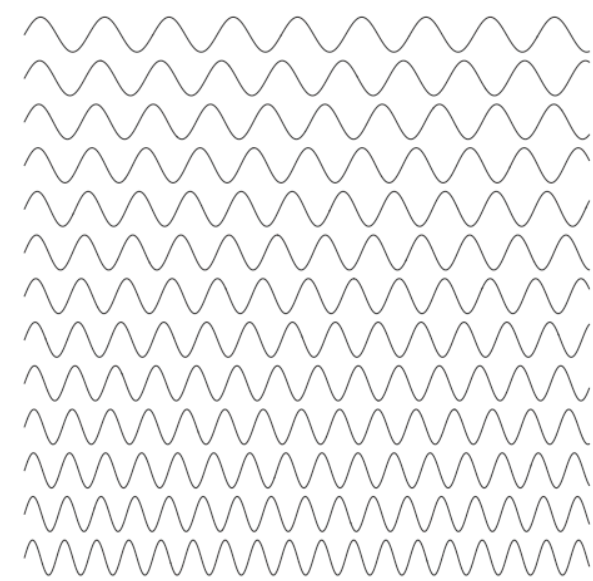
Notes, numbers, and waves

Digital Audio

Sampling. Represent curve by sampling it at regular intervals.

5,512 samples/second, 137 samples

11,025 samples/second, 275 samples

22,050 samples/second, 551 samples

44,100 samples/second, 1,102 samples

audio CD
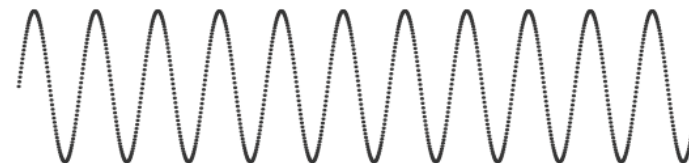
$$y(i) = \sin\left(\frac{2\pi \cdot i \cdot 440}{44{,}100}\right)$$

Warmup: Musical Tone

Musical tone. Create a music tone of a given frequency and duration.

```java
public class Tone
{
   public static void main(String[] args)
   {
      int sps = 44100;
      double hz       = Double.parseDouble(args[0]);
      double duration = Double.parseDouble(args[1]);
      int N = (int) (sps * duration);
      double[] a = new double[N+1];
      for (int i = 0; i <= N; i++)
         a[i] = Math.sin(2 * Math.PI * i * hz / sps);
      StdAudio.play(a);
   }
}
```

```
% java Tone 440 1.5
[ concert A for 1.5 seconds]
```

$$y(i) \; = \; \sin\left(\frac{2\pi \cdot i \cdot hz}{44{,}100}\right)$$

Play That Tune

Goal. Play pitches and durations from standard input on standard audio.

```java
public class PlayThatTune
{
   public static void main(String[] args)
   {
      int sps = 44100;
      while (!StdIn.isEmpty())
      {
            int pitch = StdIn.readInt();
            double duration = StdIn.readDouble();
            double hz = 440 * Math.pow(2, pitch / 12.0);
            int N = (int) (sps * duration);
            double[] a = new double[N+1];
            for (int i = 0; i <= N; i++)
                a[i] = Math.sin(2 * Math.PI * i * hz / sps);
            StdAudio.play(a);
      }
   }
}
```

```
% more elise.txt
7 .125
6 .125
7 .125
6 .125
7 .125
2 .125
5 .125
3 .125
0 .25
...
```

```
% java PlayThatTune < elise.txt
```

Musical Tone Function

Musical tone. Create a music tone of a given frequency and duration.

```java
public static double[] tone(double hz, double seconds)
{
    int SAMPLE_RATE = 44100;
    int N = (int) (seconds * SAMPLE_RATE);
    double[] a = new double[N+1];
    for (int i = 0; i <= N; i++)
        a[i] = Math.sin(2 * Math.PI * i * hz / SAMPLE_RATE);
    return a;
}
```

$$y(i) \; = \; \sin\left(\frac{2\pi \cdot i \cdot hz}{44{,}100}\right)$$

Remark. Can use arrays as function return value and/or argument.

Digital Audio in Java

Standard audio. Library for playing digital audio.

```
public class StdAudio
       void  play(String file)                    play the given .wav file
       void  play(double[] a)                      play the given sound wave
       void  play(double x)                        play sample for 1/44100 second
       void  save(String file, double[] a)   save to a .wav file
   double[]  read(String file)                     read from a .wav file
```
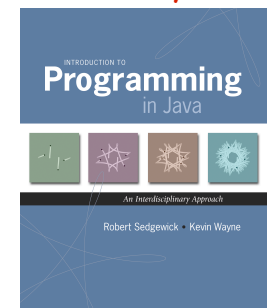
library developed
for this course
(also broadly useful)

Concert A. Play concert A for 1.5 seconds using StdAudio.

```
double[] a = tone(440, 1.5);

StdAudio.play(a);
```

Remark. Java arrays passed "by reference" (no copy made).

INTRODUCTION TO
Programming
in Java

An Interdisciplinary Approach

Robert Sedgewick · Kevin Wayne

Harmonics

Concert A with harmonics. Obtain richer sound by adding tones one octave above and below concert A.

880 Hz   220 Hz   440 Hz
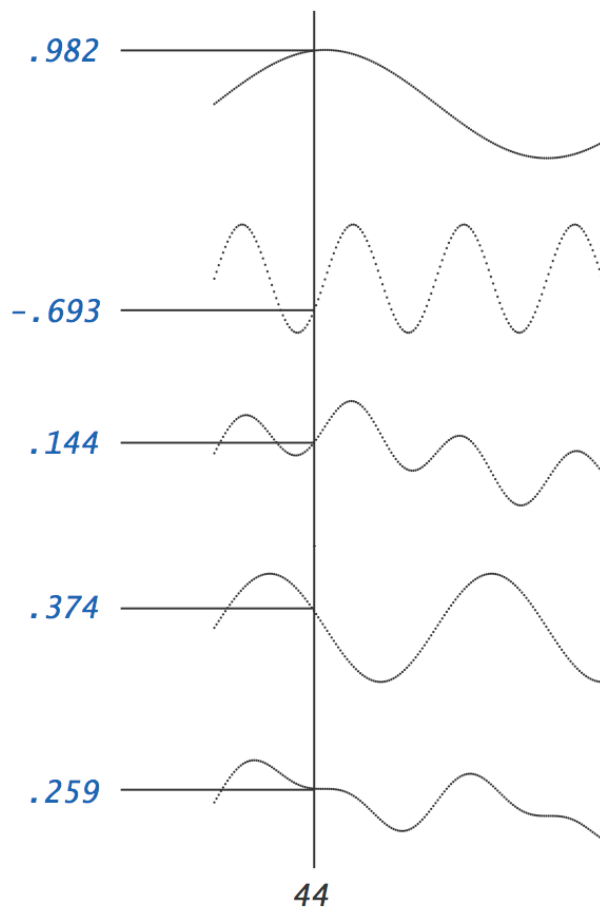

```
lo = tone(220, .0041);
lo[44] = .982
```

```
hi = tone(880, .0041);
hi[44] = -.693
```

```
h = sum(hi, lo, .5, .5);
h[44] = .5*lo[44]+.5*hi[44];
     = .5*.982 - .5*.693 = .144
```

```
A = tone(440, .0041);
A[44] = .374
```

```
sum(A, h, .5, .5);
A[44] + h[44] = .5*.144 + .5*.374
            = .259
```

.982

-.693

.144

.374

.259

44

Harmonics

```java
public class PlayThatTune                          // improved version with Harmonics
{
   // Return weighted sum of two arrays.
   public static double[] sum(double[] a, double[] b, double awt, double bwt) {
      double[] c = new double[a.length];
      for (int i = 0; i < a.length; i++)
         c[i] = a[i]*awt + b[i]*bwt;
      return c;
   }


   // Return a note of given pitch and duration.
   public static double[] note(int pitch, double duration) {
      double hz = 440.0 * Math.pow(2, pitch / 12.0);
      double[] a  = tone(1.0 * hz, duration);
      double[] hi = tone(2.0 * hz, duration);
      double[] lo = tone(0.5 * hz, duration);
      double[] h  = sum(hi, lo, .5, .5);
      return sum(a, h, .5, .5);
   }


   public static double[] tone(double hz, double t)
      // see previous slide


   public static void main(String[] args)
      // see next slide
}
```

Harmonics

Play that tune. Read in pitches and durations from standard input, and play using standard audio.

```java
public static void main(String[] args)
{
    while (!StdIn.isEmpty())
    {
        int pitch = StdIn.readInt();
        double duration = StdIn.readDouble();
        double[] a = note(pitch, duration);
        StdAudio.play(a);
    }
}
```

```
% more elise.txt        % java PlayThatTune < elise.txt
7 .125
6 .125
7 .125
6 .125
7 .125
2 .125
5 .125
3 .125
0 .25
```

```java
public class PlayThatTune
{
    public static double[] sum(double[] a, double[] b,
                               double awt, double bwt)
    {
        double[] c = new double[a.length];
        for (int i = 0; i < a.length; i++)
            c[i] = a[i]*awt + b[i]*bwt;
        return c;
    }

    public static double[] tone(double hz, double t)
    {
        int sps = 44100;
        int N = (int) (sps * t);
        double[] a = new double[N+1];
        for (int i = 0; i <= N; i++)
            a[i] = Math.sin(2 * Math.PI * i * hz / sps);
        return a;
    }

    public static double[] note(int pitch, double t)
    {
        double hz = 440.0 * Math.pow(2, pitch / 12.0);
        double[] a  = tone(hz, t);

        double[] hi = tone(2*hz, t);

        double[] lo = tone(hz/2, t);

        double[] h  = sum(hi, lo, .5, .5);

        return sum(a, h, .5, .5);

    }

    public static void main(String[] args)
    {
        while (!StdIn.isEmpty())
        {
            int pitch = StdIn.readInt();
            double duration = StdIn.readDouble();
            double[] a = note(pitch, duration);

            StdAudio.play(a);
        }
    }
}
```