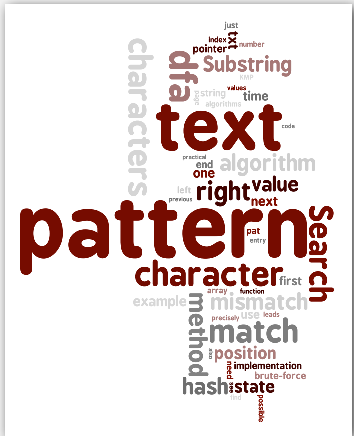
5.3 Substring Search



- brute force
- Knuth-Morris-Pratt
- Boyer-Moore
- Rabin-Karp

Substring search

Goal. Find pattern of length M in a text of length N.

typically N >> M

pattern → N E E D L E

text → I N A H A Y S T A C K N E E D L E I N A

match

Computer forensics. Search memory or disk for signatures, e.g., all URLs or RSA keys that the user has entered.

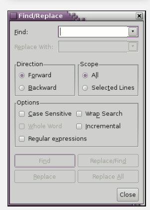
http://citp.princeton.edu/memory

Applications

- Parsers.
- Spam filters.
- Digital libraries.
- Screen scrapers.
- Word processors.
- Web search engines.
- Electronic surveillance.
- Natural language processing.
- Computational molecular biology.
- FBIs Digital Collection System 3000.
- Feature detection in digitized images.
- ...

SpamAssassin

LexisNexis
It's how you know



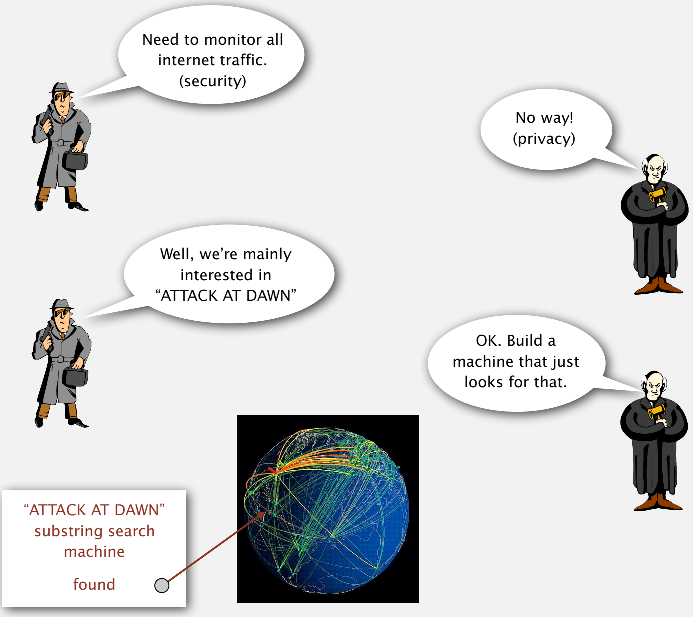
DCS 3000

Application: spam filtering

Identify patterns indicative of spam.

- `PROFITS`
- `L0SE WE1GHT`
- `herbal Viagra`
- `There is no catch.`
- `LOW MORTGAGE RATES`
- `This is a one-time mailing.`
- `This message is sent in compliance with spam regulations.`

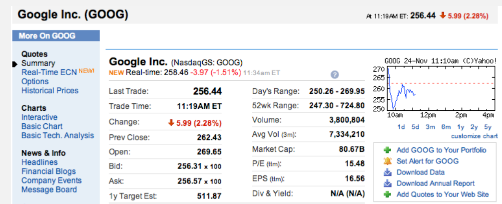
Application: electronic surveillance



Application: screen scraping

Goal. Extract relevant data from web page.

Ex. Find string delimited by `<b>` and `</b>` after first occurrence of pattern `Last Trade:`.



http://finance.yahoo.com/q?s=goog

```
...
<tr>
<td class= "yfnc_tablehead1"
width= "48%">
Last Trade:
</td>
<td class= "yfnc_tabledata1">
<big><b>452.92</b></big>
</td></tr>
<td class= "yfnc_tablehead1"
width= "48%">
Trade Time:
</td>
<td class= "yfnc_tabledata1">
...
```


Screen scraping: Java implementation

Java library. The `indexOf()` method in Java's string library returns the index of the first occurrence of a given string, starting at a given offset.

```
public class StockQuote
{
   public static void main(String[] args)
   {
      String name = "http://finance.yahoo.com/q?s=";
      In in = new In(name + args[0]);
      String text = in.readAll();
      int start    = text.indexOf("Last Trade:", 0);
      int from     = text.indexOf("<b>",  start);
      int to       = text.indexOf("</b>", from);
      String price = text.substring(from + 3, to);
      StdOut.println(price);
   }
}
```

```
% java StockQuote goog
564.35

% java StockQuote msft
26.04
```


- brute force
- Knuth-Morris-Pratt
- Boyer-Moore
- Rabin-Karp

Brute-force substring search

Check for pattern starting at each text position.

i	j	i+j	0	1	2	3	4	5	6	7	8	9	10
		txt →	A	B	A	C	A	D	A	B	R	A	C
0	2	2	A	B	R	A ← pat							
1	0	1		A	B	R	A						
2	1	3			A	B	R	A					
3	0	3				A	B	R	A				
4	1	5					A	B	R	A			
5	0	5						A	B	R	A		
6	4	10							A	B	R	A	

entries in red are mismatches

entries in gray are for reference only

entries in black match the text

return i *when* j *is* M

match

Brute-force substring search: Java implementation

Check for pattern starting at each text position.

i = 4, j = 3	0	1	2	3	4	5	6	7	8	9	10
	A	B	A	C	A	D	A	B	R	A	C
					A	D	A	C	R		

```
public static int search(String pat, String txt)
{
   int M = pat.length();
   int N = txt.length();
   for (int i = 0; i <= N - M; i++)
   {
      int j;
      for (j = 0; j < M; j++)
         if (txt.charAt(i+j) != pat.charAt(j))
            break;
      if (j == M) return i;   ← index in text where pattern starts
   }
   return N;   ← not found
}
```


Brute-force substring search: worst case

Brute-force algorithm can be slow if text and pattern are repetitive.

i	j	i+j	0	1	2	3	4	5	6	7	8	9
		txt →	A	A	A	A	A	A	A	A	A	B
0	4	4	A	A	A	A	B ← pat					
1	4	5		A	A	A	A	B				
2	4	6			A	A	A	A	B			
3	4	7				A	A	A	A	B		
4	4	8					A	A	A	A	B	
5	5	10						A	A	A	A	B

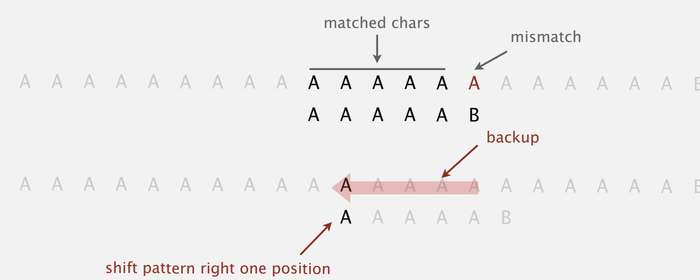
Worst case. $\sim M N$ char compares.

Backup

In typical applications, we want to avoid backup in text stream.

- Treat input as stream of data.
- Abstract model: standard input.

Brute-force algorithm needs backup for every mismatch.



Approach 1. Maintain buffer of size M (build backup into standard input).

Approach 2. Stay tuned.

Brute-force substring search: alternate implementation

Same sequence of char compares as previous implementation.

- `i` points to end of sequence of already-matched chars in text.
- `j` stores number of already-matched chars (end of sequence in pattern).

i = 6, j = 3

0	1	2	3	4	5	6	7	8	9	10
A	B	A	C	A	D	A	B	R	A	C
				A	D	A	C	R		

```
public static int search(String pat, String txt)
{
   int i, N = txt.length();
   int j, M = pat.length();
   for (i = 0, j = 0; i < N && j < M; i++)
   {
      if (txt.charAt(i) == pat.charAt(j)) j++;
      else { i -= j; j = 0;  }
   }
   if (j == M) return i - M;
   else            return N;
}
```

backup

Algorithmic challenges in substring search

Brute-force is often not good enough.

Theoretical challenge. Linear-time guarantee. ← fundamental algorithmic problem

Practical challenge. Avoid backup in text stream. ← often no room or time to save text

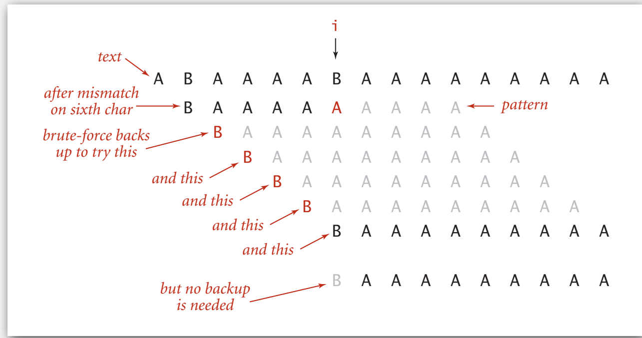
```
Now is the time for all people to come to the aid of their party. Now is the time for all good people to
come to the aid of their party. Now is the time for many good people to come to the aid of their party.
Now is the time for all good people to come to the aid of their party. Now is the time for a lot of good
people to come to the aid of their party. Now is the time for all of the good people to come to the aid of
their party. Now is the time for all good people to come to the aid of their party. Now is the time for
each good person to come to the aid of their party. Now is the time for all good people to come to the aid
of their party. Now is the time for all good Republicans to come to the aid of their party. Now is the
time for all good people to come to the aid of their party. Now is the time for many or all good people to
come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now
is the time for all good Democrats to come to the aid of their party. Now is the time for all people to
come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now
is the time for many good people to come to the aid of their party. Now is the time for all good people to
come to the aid of their party. Now is the time for a lot of good people to come to the aid of their
party. Now is the time for all of the good people to come to the aid of their party. Now is the time for
all good people to come to the aid of their attack at dawn party. Now is the time for each person to come
to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is
the time for all good Republicans to come to the aid of their party. Now is the time for all good people
to come to the aid of their party. Now is the time for many or all good people to come to the aid of their
party. Now is the time for all good people to come to the aid of their party. Now is the time for all good
Democrats to come to the aid of their party.
```


Knuth-Morris-Pratt substring search

Intuition. Suppose we are searching in text for pattern **BAAAAAAAAA**.

- Suppose we match 5 chars in pattern, with mismatch on 6^{th} char.
- We know previous 6 chars in text are **BAAAAB**.
- Don't need to back up text pointer!

assuming { A, B } alphabet



Knuth-Moris-Pratt algorithm. Clever method to always avoid backup. (!)

Deterministic finite state automaton (DFA)

DFA is abstract string-searching machine.

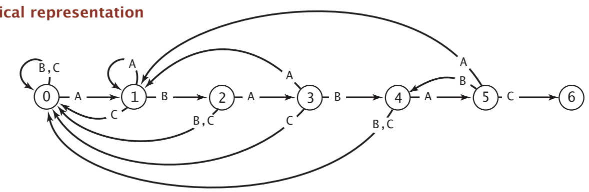
- Finite number of states (including start and halt).
- Exactly one transition for each char in alphabet.
- Accept if sequence of transitions leads to halt state.

internal representation

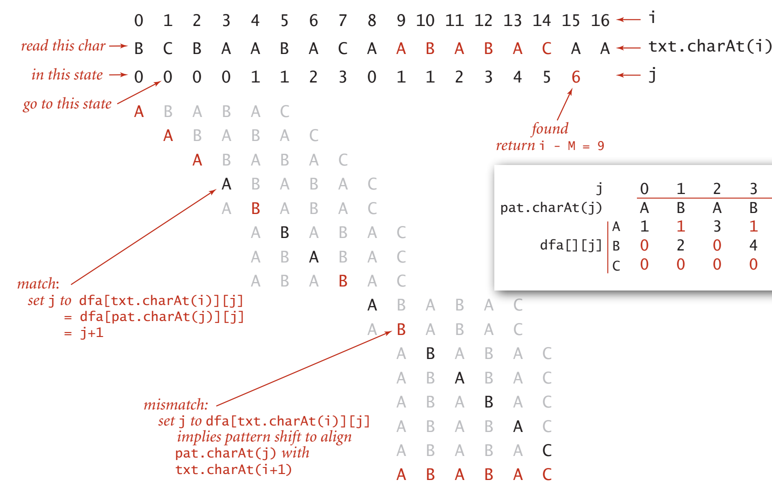
j		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

If in state `j` reading char `c`:
  if `j` is 6 halt and accept
  else move to state `dfa[c][j]`

graphical representation



KMP substring search: trace



j		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

Trace of KMP substring search (DFA simulation) for A B A B A C

Interpretation of Knuth-Morris-Pratt DFA

Q. What is interpretation of DFA state after reading in `txt[i]`?

A. State = number of characters in pattern that have been matched.
(length of longest prefix of `pat[]` that is a suffix of `txt[0..i]`)

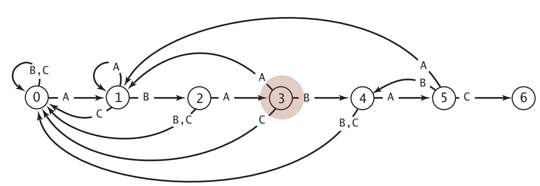
Ex. DFA is in state `3` after reading in character `txt[6]`.

	0	1	2	3	4	5	6	7	8
txt →	B	C	B	A	A	B	A	C	A

suffix of text[0..6]

	0	1	2	3	4	5
pat →	A	B	A	B	A	C

prefix of pat[]



KMP search: Java implementation

Key differences from brute-force implementation.

- Text pointer `i` never decrements.
- Need to precompute `dfa[][]` from pattern.

```
public int search(String txt)
{
   int i, j, N = txt.length();
   for (i = 0, j = 0; i < N && j < M; i++)
      j = dfa[txt.charAt(i)][j];
   if (j == M) return i - M;
   else        return N;
}
```

← no backup

Running time.

- Simulate DFA on text: at most N character accesses.
- Build DFA: how to do efficiently? [warning: tricky algorithm ahead]

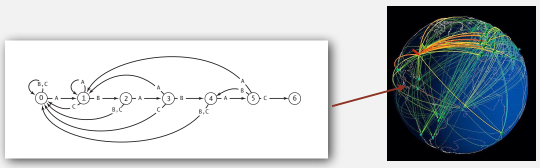
KMP search: Java implementation

Key differences from brute-force implementation.

- Text pointer `i` never decrements.
- Need to precompute `dfa[][]` from pattern.
- Could use input stream.

```
public int search(In in)
{
   int i, j;
   for (i = 0, j = 0; !in.isEmpty() && j < M; i++)
      j = dfa[in.readChar()][j];
   if (j == M) return i - M;
   else        return NOT_FOUND;
}
```

no backup



Building DFA from pattern: easy case

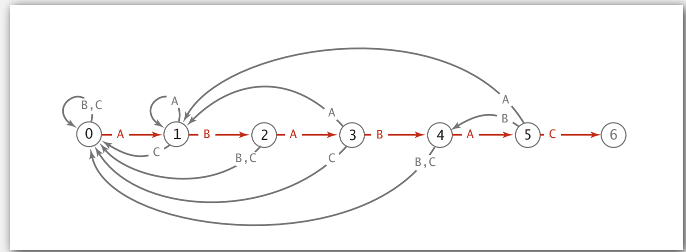
Match transition. If in state `j` and next char `c == pat.charAt(j)`, then go to state `j+1`.

first j characters of pattern have already been matched

next char matches

now first j+1 characters of pattern have been matched

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C



Knuth-Morris-Pratt construction

Include one state for each character in pattern (plus accept state).

j		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A						
	B						
	C						

Constructing the DFA for KMP substring search for A B A B A C

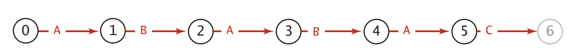
Knuth-Morris-Pratt construction

Match transition. For each state `j`, `dfa[pat.charAt(j)][j] = j+1`.

first j characters of pattern have already been matched

now first j+1 characters of pattern have been matched

j		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1		3		5	
	B		2		4		
	C						6



Constructing the DFA for KMP substring search for A B A B A C

Building DFA from pattern: tricky case

Mismatch transition.

Suppose DFA is in state `j` and next char is `c != pat.charAt(j)`

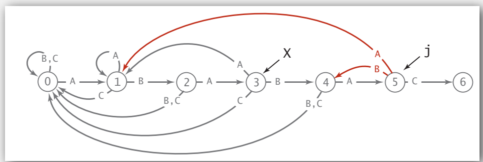
```
. . . A B A B A C . . .   ← match
      A B A B A A         ← mismatch case 1
      A B A B A B         ← mismatch case 2
```

Brute-force solution: Back up `j-1` chars and restart.

Key idea 1. The last `j` characters of input are known to be `pat[1..j-1]`, so use the DFA itself to find what state (X) it would be in if restarted after backup

Key idea 2. Maintain the value of X while constructing DFA.

Ex.



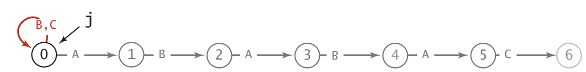

```
pat[1..j-1] = B A B A

X = 3
dfa['A'][5] = dfa['A'][X] = 1
dfa['B'][5] = dfa['B'][X] = 4
dfa['C'][5] = 6

new X = dfa['C'][X] = 0
```


Knuth-Morris-Pratt construction

		j	0	1	2	3	4	5
pat.charAt(j)			A	B	A	B	A	C
dfa[][j]	A		1		3		5	
	B		0	2		4		
	C		0					6



Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`, `dfa[c][j] = dfa[c][X]`; then update `X = dfa[pat.charAt(j)][X]`.

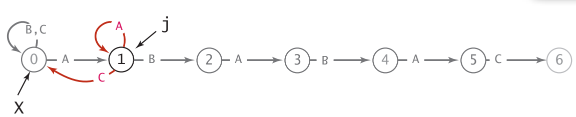
X = simulation of empty string

		j	0	1	2	3	4	5
pat.charAt(j)			A	B	A	B	A	C
dfa[][j]	A		1	1	3		5	
	B		0	2		4		
	C		0	0				6

```
j = 1
pat[1..j-1] = ''

X = 0
dfa['A'][1] = dfa['A'][X] = 1
dfa['B'][1] = 2
dfa['C'][1] = dfa['C'][X] = 0

new X = dfa['B'][X] = 0
```



Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`, `dfa[c][j] = dfa[c][X]`; then update `X = dfa[pat.charAt(j)][X]`.

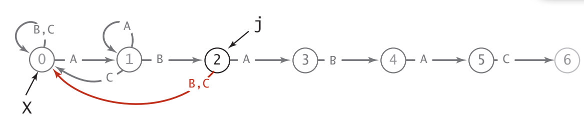
X = simulation of B

		j	0	1	2	3	4	5
pat.charAt(j)			A	B	A	B	A	C
dfa[][j]	A		1	1	3		5	
	B		0	2	0	4		
	C		0	0	0			6

```
j = 2
pat[1..j-1] = 'B'

X = 0
dfa['A'][2] = 3
dfa['B'][2] = dfa['B'][X] = 0
dfa['C'][2] = dfa['C'][X] = 0

new X = dfa['A'][X] = 1
```



Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`, `dfa[c][j] = dfa[c][X]`; then update `X = dfa[pat.charAt(j)][X]`.

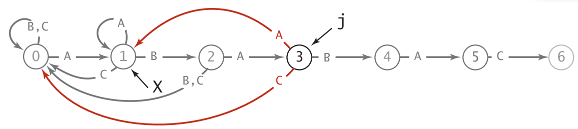
X = simulation of BA

	j	0	1	2	3	4	5
	pat.charAt(j)	A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	
	B	0	2	0	4		
	C	0	0	0	0		6

```
j = 3
pat[1..j-1] = 'BA'

X = 1
dfa['A'][3] = dfa['A'][X] = 1
dfa['B'][3] = 4
dfa['C'][3] = dfa['C'][X] = 0

new X = dfa['B'][X] = 2
```



Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`, `dfa[c][j] = dfa[c][X]`; then update `X = dfa[pat.charAt(j)][X]`.

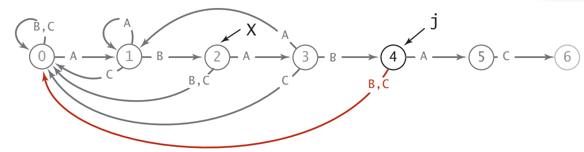
X = simulation of BAB

	j	0	1	2	3	4	5
	pat.charAt(j)	A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	
	B	0	2	0	4	0	
	C	0	0	0	0	0	6

```
j = 4
pat[1..j-1] = 'BAB'

X = 2
dfa['A'][4] = 5
dfa['B'][4] = dfa['B'][X] = 0
dfa['C'][4] = dfa['C'][X] = 0

new X = dfa['B'][X] = 3
```



Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`, `dfa[c][j] = dfa[c][X]`; then update `X = dfa[pat.charAt(j)][X]`.

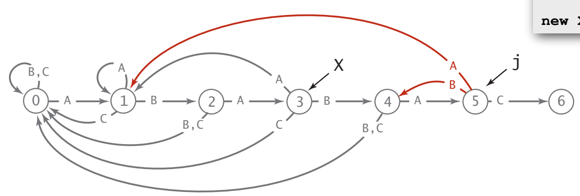
X = simulation of BABA

	j	0	1	2	3	4	5
	pat.charAt(j)	A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

```
j = 5
pat[1..j-1] = 'BABA'

X = 3
dfa['A'][5] = dfa['A'][X] = 1
dfa['B'][5] = dfa['B'][X] = 4
dfa['C'][5] = 6

new X = dfa['C'][X] = 0
```



Constructing the DFA for KMP substring search: Java implementation

For each state `j`:

- Copy `dfa[][X]` to `dfa[][j]` for mismatch case.
- Set `dfa[pat.charAt(j)][j]` to `j+1` for match case.
- Update `X`.

```
public KMP(String pat)
{
   this.pat = pat;
   M = pat.length();
   dfa = new int[R][M];
   dfa[pat.charAt(0)][0] = 1;
   for (int X = 0, j = 1; j < M; j++)
   {
      for (int c = 0; c < R; c++)
         dfa[c][j] = dfa[c][X];          // copy mismatch cases
      dfa[pat.charAt(j)][j] = j+1;       // set match case
      X = dfa[pat.charAt(j)][X];         // update restart state
   }
}
```

Running time. M character accesses (but space proportional to $R\,M$).

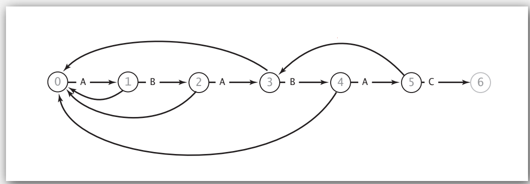
KMP substring search analysis

Proposition. KMP substring search accesses no more than $M + N$ chars to search for a pattern of length M in a text of length N.

Pf. Each pattern char accessed once when constructing the DFA; each text char accessed once (in the worst case) when simulating the DFA.

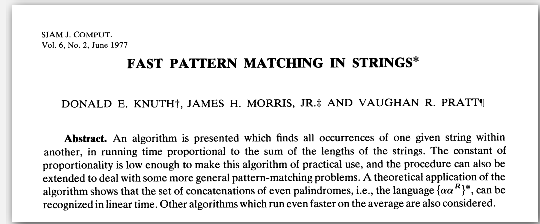
Proposition. KMP constructs `dfa[][]` in time and space proportional to $R M$.

Larger alphabets. Improved version of KMP constructs `nfa[]` in time and space proportional to M.



Knuth-Morris-Pratt: brief history

- Independently discovered by two theoreticians and a hacker.
  - Knuth: inspired by esoteric theorem, discovered linear-time algorithm
  - Pratt: made running time independent of alphabet size
  - Morris: built a text editor for the CDC 6400 computer
- Theory meets practice.



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FAST PATTERN MATCHING IN STRINGS*

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Abstract. An algorithm is presented which finds all occurrences of one given string within another, in running time proportional to the sum of the lengths of the strings. The constant of proportionality is low enough to make this algorithm of practical use, and the procedure can also be extended to deal with some more general pattern-matching problems. A theoretical application of the algorithm shows that the set of concatenations of even palindromes, i.e., the language $\{\alpha\alpha^R\}^*$, can be recognized in linear time. Other algorithms which run even faster on the average are also considered.

Don Knuth

Jim Morris

Vaughan Pratt

Knuth-Morris-Pratt application

A string s is a cyclic rotation of t if s and t have the same length and s is a suffix of t followed by a prefix of t.

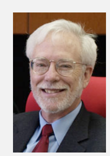
yes	yes	no
ROTATEDSTRING	ABABABBABBABA	ROTATEDSTRING
STRINGROTATED	BABBABBABAABA	GNIRTSDETATOR

Problem. Given two strings s and t, design a linear-time algorithm that determines if s is a cyclic rotation of t.

Solution.
- Check that s and t are the same length.
- Search for s in $t + t$ using KMP.

- brute force
- Knuth-Morris-Pratt
- **Boyer-Moore**
- Rabin-Karp

Robert Boyer



J. Strother Moore

Boyer-Moore: mismatched character heuristic

Intuition.

- Scan characters in pattern from right to left.
- Can skip M text chars when finding one not in the pattern.

i	j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	text →	F	I	N	D	I	N	A	H	A	Y	S	T	A	C	K	N	E	E	D	L	E	I	N	A
0	5	N	E	E	D	L	E ← *pattern*																		
5	5						N	E	E	D	L	E													
11	4												N	E	E	D	L	E							
15	0																N	E	E	D	L	E			

return i = 15

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

A. Compute `right[c]` = rightmost occurrence of character `c` in `pat`.

```
right = new int[R];
for (int c = 0; c < R; c++)
   right[c] = -1;
for (int j = 0; j < M; j++)
   right[pat.charAt(j)] = j;
```

c		N 0	E 1	E 2	D 3	L 4	E 5	right[c]
A	-1	-1	-1	-1	-1	-1	-1	-1
B	-1	-1	-1	-1	-1	-1	-1	-1
C	-1	-1	-1	-1	-1	-1	-1	-1
D	-1	-1	-1	-1	3	3	3	3
E	-1	-1	1	2	2	2	5	5
...								-1
L	-1	-1	-1	-1	-1	4	4	4
M	-1	-1	-1	-1	-1	-1	-1	-1
N	-1	0	0	0	0	0	0	0
...								-1

Boyer-Moore skip table computation

Boyer-Moore: mismatched character heuristic

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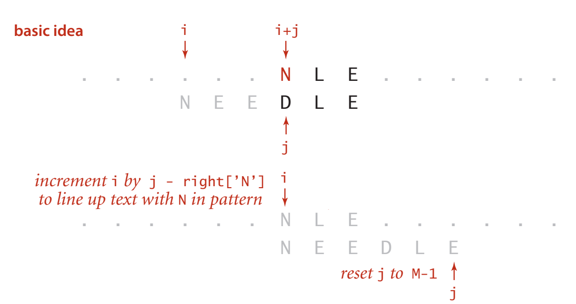
A. Compute `right[c]` = rightmost occurrence of character `c` in `pat`.



Mismatched character heuristic (mismatch in pattern)

Boyer-Moore: mismatched character heuristic

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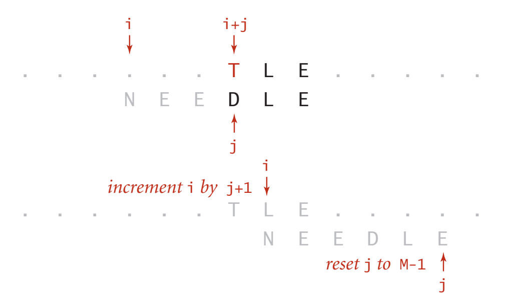

Mismatched character heuristic (mismatch not in pattern)

Character not in pattern? Set `right[c]` to -1.

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

A. Compute `right[c]` = rightmost occurrence of character `c` in `pat`.

```
                i           i+j
                ↓            ↓
  . . . . . . . E   L   E . . . . . . .
            N E E D   L   E
                      ↑
                      j
```

lining up text with rightmost E *would shift pattern left*

```
  . . . . . . . E L E . . . . . . .
        N E E D L E
```

so increment i *by* 1

```
                 i
                 ↓
  . . . . . . . E L E . . . . . . .
              N E E D L E
                        ↑
                        j
```

reset j *to* M-1

Heuristic no help? Increment `i` and reset `j` to `M-1`

Boyer-Moore: Java implementation

```java
public int search(String txt)
{
   int N = txt.length();
   int M = pat.length();
   int skip;
   for (int i = 0; i <= N-M; i += skip)
   {
      skip = 0;
      for (int j = M-1; j >= 0; j--)
      {
         if (pat.charAt(j) != txt.charAt(i+j))      // compute skip value
         {
            skip = Math.max(1, j - right[txt.charAt(i+j)]);
            break;
         }
      }
      if (skip == 0) return i;                     // match
   }
   return N;
}
```


Boyer-Moore: analysis

Property. Substring search with the Boyer-Moore mismatched character heuristic takes about ~ N / M character compares to search for a pattern of length M in a text of length N. sublinear

Worst-case. Can be as bad as ~ $M N$.

i	skip		0	1	2	3	4	5	6	7	8	9	
		txt →	B	B	B	B	B	B	B	B	B	B	
0	0		A	B	B	B	B	← pat					
1	1			A	B	B	B	B					
2	1				A	B	B	B	B				
3	1					A	B	B	B	B			
4	1						A	B	B	B	B		
5	1							A	B	B	B	B	

Boyer-Moore variant. Can improve worst case to ~ $3 N$ by adding a KMP-like rule to guard against repetitive patterns.

- brute force
- Knuth-Morris-Pratt
- Boyer-Moore
- **Rabin-Karp**

Michael Rabin, Turing Award '76
and Dick Karp, Turing Award '85

Rabin-Karp fingerprint search

Basic idea = modular hashing.

- Compute a hash of pattern characters 0 to $M - 1$.
- For each i, compute a hash of text characters i to $M + i - 1$.
- If pattern hash = text substring hash, check for a match.

pat.charAt(i)

i	0	1	2	3	4	
	2	6	5	3	5	% 997 = 613

txt.charAt(i)

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	3	1	4	1	5	9	2	6	5	3	5	8	9	7	9	3	
0	3	1	4	1	5												% 997 = 508
1		1	4	1	5	9											% 997 = 201
2			4	1	5	9	2										% 997 = 715
3				1	5	9	2	6									% 997 = 971
4					5	9	2	6	5								% 997 = 442
5						9	2	6	5	3							% 997 = 929
6 ← *return i = 6*							2	6	5	3	5						% 997 = 613 *match*

Efficiently computing the hash function

Modular hash function. Using the notation t_i for `txt.charAt(i)`, we wish to compute

$$x_i = t_i R^{M-1} + t_{i+1} R^{M-2} + \ldots + t_{i+M-1} R^0 \pmod{Q}$$

Intuition. M-digit, base-R integer, modulo Q.

Horner's method. Linear-time method to evaluate degree-M polynomial.

pat.charAt()

i	0	1	2	3	4	
	2	6	5	3	5	
0	2					% 997 = 2
1	2	6				% 997 = (2*10 + 6) % 997 = 26
2	2	6	5			% 997 = (26*10 + 5) % 997 = 265
3	2	6	5	3		% 997 = (265*10 + 3) % 997 = 659
4	2	6	5	3	5	% 997 = (659*10 + 5) % 997 = 613

Computing the hash value for the pattern with Horner's method

```
// Compute hash for M-digit key
private long hash(String key, int M)
{
   long h = 0;
   for (int j = 0; j < M; j++)
      h = (R * h + key.charAt(j)) % Q;
   return h;
}
```


Efficiently computing the hash function

Challenge. How to efficiently compute x_{i+1} given that we know x_i.

$$x_i = t_i R^{M-1} + t_{i+1} R^{M-2} + \ldots + t_{i+M-1} R^0$$

$$x_{i+1} = t_{i+1} R^{M-1} + t_{i+2} R^{M-2} + \ldots + t_{i+M} R^0$$

Key property. Can update hash function in constant time!

$$x_{i+1} = \underbrace{x_i R}_{\text{shift left}} - \underbrace{t_i R^M}_{\text{subtract leftmost digit}} + \underbrace{t_{i+M}}_{\text{add new rightmost digit}}$$

i	...	2	3	4	5	6	7	...	
current value	1	4	1	5	9	2	6	5	*text*
new value		4	1	5	9	2	6	5	

	4	1	5	9	2	*current value*
−	4	0	0	0	0	
		1	5	9	2	*subtract leading digit*
			*	1	0	*multiply by radix*
	1	5	9	2	0	
				+	6	*add new trailing digit*
	1	5	9	2	6	*new value*

Rabin-Karp substring search example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	3	1	4	1	5	9	2	6	5	3	5	8	9	7	9	3	
0	3																% 997 = 3
1	3	1															% 997 = (3*10 + 1) % 997 = 31
2	3	1	4														% 997 = (31*10 + 4) % 997 = 314
3	3	1	4	1													% 997 = (314*10 + 1) % 997 = 150
4	3	1	4	1	5												% 997 = (150*10 + 5) % 997 = 508
5		1	4	1	5	9											% 997 = ((508 + 3*(997 - 30))*10 + 9) % 997 = 201
6			4	1	5	9	2										% 997 = ((201 + 1*(997 - 30))*10 + 2) % 997 = 715
7				1	5	9	2	6									% 997 = ((715 + 4*(997 - 30))*10 + 6) % 997 = 971
8					5	9	2	6	5								% 997 = ((971 + 1*(997 - 30))*10 + 5) % 997 = 442
9						9	2	6	5	3							% 997 = ((442 + 5*(997 - 30))*10 + 3) % 997 = 929
10 ← *return i-M+1 = 6*							2	6	5	3	5						% 997 = ((929 + 9*(997 - 30))*10 + 5) % 997 = 613 *match*

Rabin-Karp: Java implementation

```
public class RabinKarp
{
   private long patHash;    // pattern hash value
   private int M;           // pattern length
   private long Q;          // modulus
   private int R;           // radix
   private long RM;         // R^(M-1) % Q

   public RabinKarp(String pat) {
      M = pat.length();
      R = 256;
      Q = longRandomPrime();

      RM = 1;
      for (int i = 1; i <= M-1; i++)
         RM = (R * RM) % Q;
      patHash = hash(pat, M);
   }

   private long hash(String key, int M)
   {  /* as before */  }

   public int search(String txt)
   {  /* see next slide */  }
}
```

a large prime (but not so large to cause long overflow)

precompute R^{M-1} (mod Q)

Rabin-Karp: Java implementation (continued)

Monte Carlo version. Return match if hash match.

```
public int search(String txt)
{
   int N = txt.length();
   int txtHash = hash(txt, M);
   if (patHash == txtHash) return 0;
   for (int i = M; i < N; i++)
   {
      txtHash = (txtHash + Q - RM*txt.charAt(i-M) % Q) % Q;
      txtHash = (txtHash*R + txt.charAt(i)) % Q;
      if (patHash == txtHash) return i - M + 1;
   }
   return N;
}
```

check for hash collision using rolling hash function

Las Vegas version. Check for substring match if hash match; continue search if false collision.

Rabin-Karp analysis

Theory. If Q is a sufficiently large random prime (about $M N^2$), then the probability of a false collision is about $1 / N$.

Practice. Choose Q to be a large prime (but not so large as to cause overflow). Under reasonable assumptions, probability of a collision is about $1 / Q$.

Monte Carlo version.

- Always runs in linear time.
- Extremely likely to return correct answer (but not always!).

Las Vegas version.

- Always returns correct answer.
- Extremely likely to run in linear time (but worst case is $M N$).

Rabin-Karp fingerprint search

Advantages.

- Extends to 2d patterns.
- Extends to finding multiple patterns.

Disadvantages.

- Arithmetic ops slower than char compares.
- Poor worst-case guarantee.
- Requires backup.

Q. How would you extend Rabin-Karp to efficiently search for any one of P possible patterns in a text of length N?

Substring search cost summary

Cost of searching for an M-character pattern in an N-character text.

algorithm	version	operation count: guarantee	operation count: typical	backup in input?	correct?	extra space
brute force	—	$M N$	$1.1\ N$	*yes*	*yes*	1
Knuth-Morris-Pratt	*full DFA (Algorithm 5.6)*	$2N$	$1.1\ N$	*no*	*yes*	MR
	mismatch transitions only	$3N$	$1.1\ N$	*no*	*yes*	M
Boyer-Moore	*full algorithm*	$3N$	N / M	*yes*	*yes*	R
	mismatched char heuristic only (Algorithm 5.7)	$M N$	N / M	*yes*	*yes*	R
Rabin-Karp†	*Monte Carlo (Algorithm 5.8)*	$7\ N$	$7\ N$	*no*	*yes†*	1
	Las Vegas	$7\ N$ †	$7\ N$	*yes*	*yes*	1

† probabilisitic guarantee, with uniform hash function