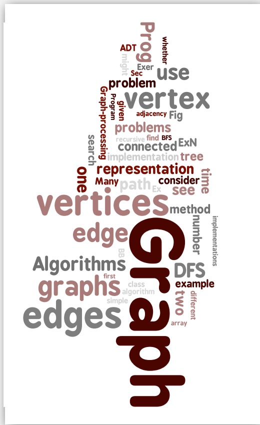
# 4.1 Undirected Graphs




- graph API
- depth-first search
- breadth-first search
- connected components
- challenges

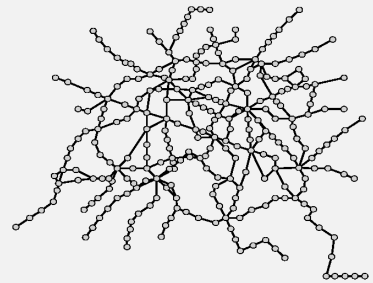


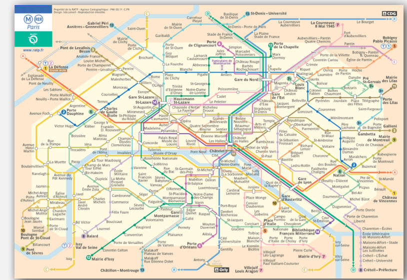
## Undirected graphs

Graph. Set of vertices connected pairwise by edges.

Why study graph algorithms?

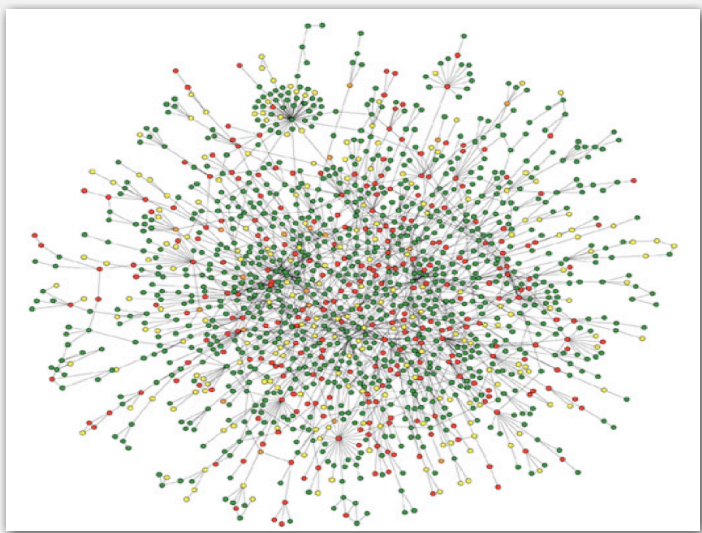
- Interesting and broadly useful abstraction.
- Challenging branch of computer science and discrete math.
- Hundreds of graph algorithms known.
- Thousands of practical applications.







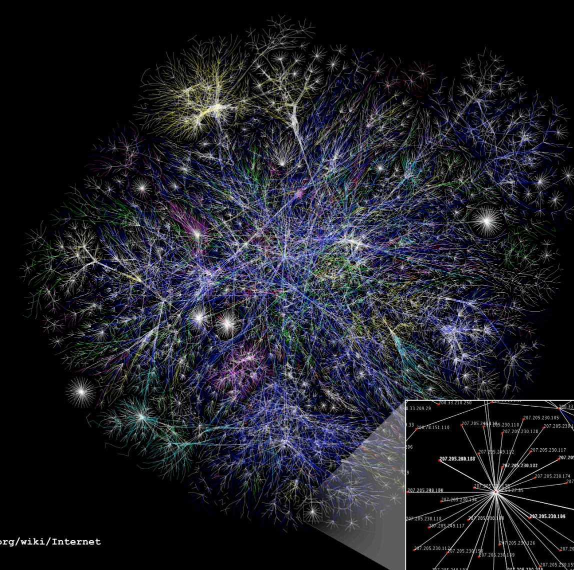
## Protein-protein interaction network



Reference: Jeong et al, Nature Review | Genetics

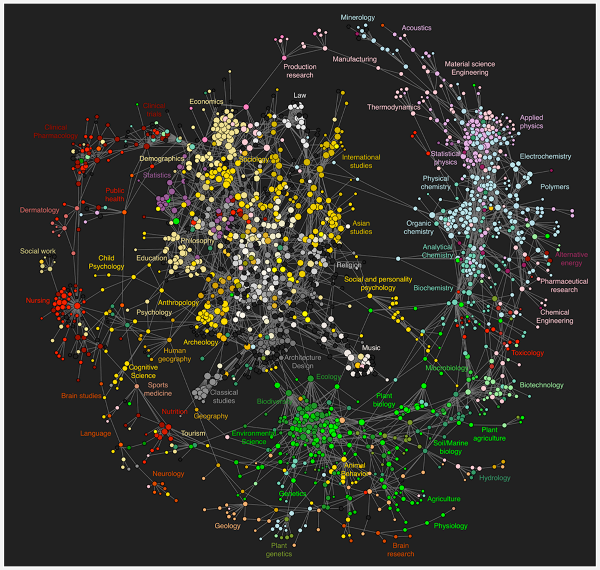


## The Internet as mapped by the Opte Project



http://en.wikipedia.org/wiki/Internet

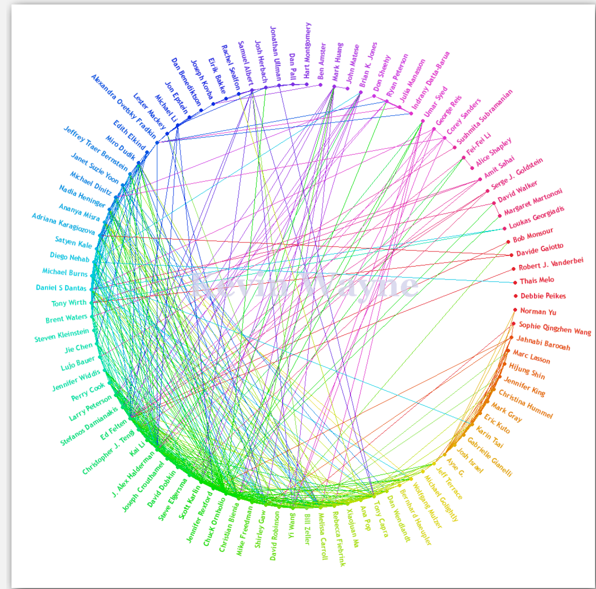
## Map of science clickstreams




http://www.plosone.org/article/info:doi/10.1371/journal.pone.0004803



## Kevin's facebook friends (Princeton network)



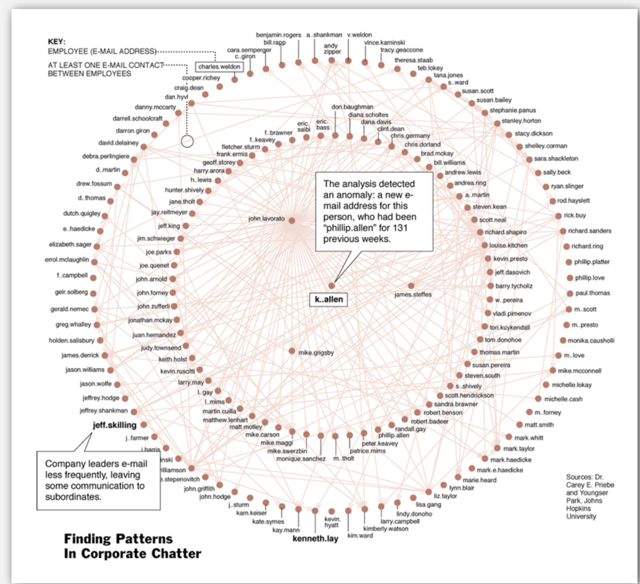


## 10 million Facebook friends

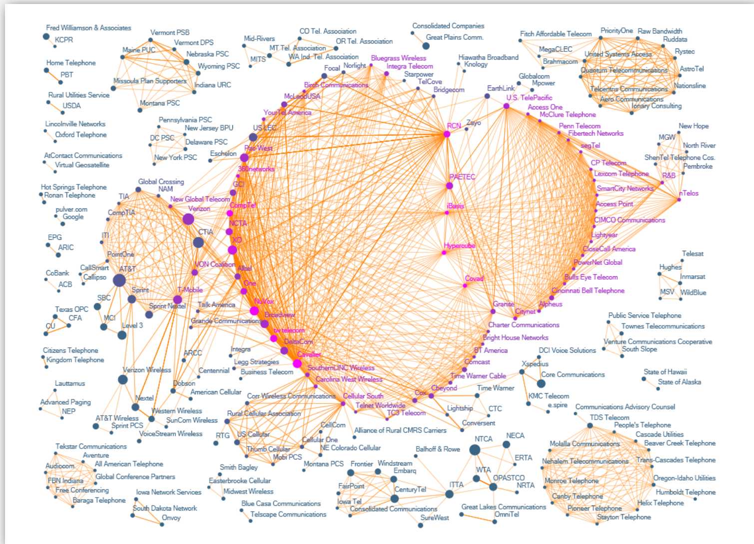

"Visualizing Friendships" by Paul Butler



## One week of Enron emails



## The evolution of FCC lobbying coalitions



"The Evolution of FCC Lobbying Coalitions" by Pierre de Vries in JoSS Visualization Symposium 2010



## Graph applications

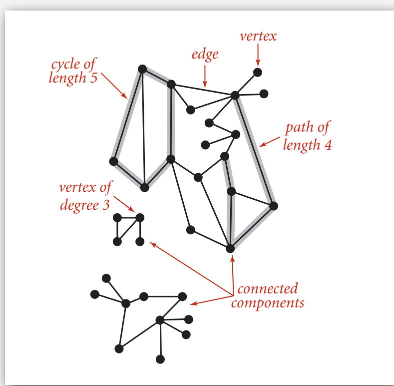
| graph | vertex | edge |
|---|---|---|
| communication | telephone, computer | fiber optic cable |
| circuit | gate, register, processor | wire |
| mechanical | joint | rod, beam, spring |
| financial | stock, currency | transactions |
| transportation | street intersection, airport | highway, airway route |
| internet | class C network | connection |
| game | board position | legal move |
| social relationship | person, actor | friendship, movie cast |
| neural network | neuron | synapse |
| protein network | protein | protein-protein interaction |
| chemical compound | molecule | bond |



## Graph terminology

Path. Sequence of vertices connected by edges.
Cycle. Path whose first and last vertices are the same.

Two vertices are connected if there is a path between them.






## Some graph-processing problems

Path. Is there a path between s and t?
Shortest path. What is the shortest path between s and t?

Cycle. Is there a cycle in the graph?
Euler tour. Is there a cycle that uses each edge exactly once?
Hamilton tour. Is there a cycle that uses each vertex exactly once?

Connectivity. Is there a way to connect all of the vertices?
MST. What is the best way to connect all of the vertices?
Biconnectivity. Is there a vertex whose removal disconnects the graph?

Planarity. Can you draw the graph in the plane with no crossing edges?
Graph isomorphism. Do two adjacency lists represent the same graph?

Challenge. Which of these problems are easy? difficult? intractable?

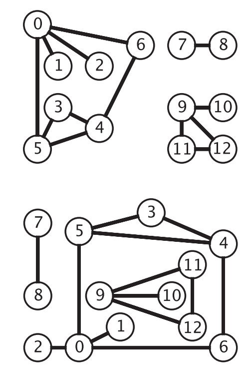
- **graph API**
- depth-first search
- breadth-first search
- connected components
- challenges



## Graph representation

Graph drawing. Provides intuition about the structure of the graph.

Caveat. Intuition can be misleading.



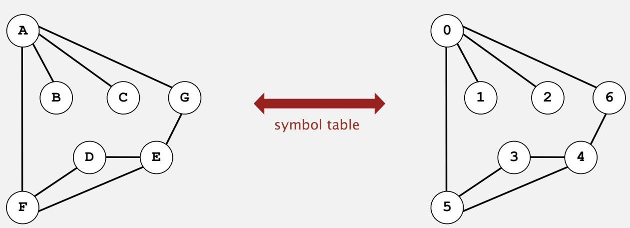

Two drawings of the same graph



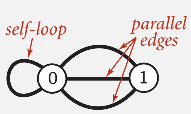
## Graph representation

Vertex representation.

- This lecture: use integers between `0` and `V-1`.
- Applications: convert between names and integers with symbol table.




Anomalies.






## Graph API

| | public class Graph | |
|---|---|---|
| | `Graph(int V)` | *create an empty graph with V vertices* |
| | `Graph(In in)` | *create a graph from input stream* |
| `void` | `addEdge(int v, int w)` | *add an edge v-w* |
| `Iterable<Integer>` | `adj(int v)` | *vertices adjacent to v* |
| `int` | `V()` | *number of vertices* |
| `int` | `E()` | *number of edges* |
| `String` | `toString()` | *string representation* |

```
In in = new In(args[0]);
Graph G = new Graph(in);

for (int v = 0; v < G.V(); v++)
   for (int w : G.adj(v))
      StdOut.println(v + "-" + w);
```

read graph from input stream

print out each edge (twice)

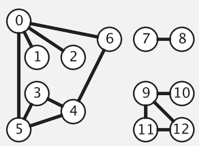
## Graph API: sample client

Graph input format.

**tinyG.txt**

V → 13
13 ← E

```
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```




```
% java Test tinyG.txt
0-6
0-2
0-1
0-5
1-0
2-0
3-5
3-4
…
12-11
12-9
```

```
In in = new In(args[0]);
Graph G = new Graph(in);

for (int v = 0; v < G.V(); v++)
   for (int w : G.adj(w))
      StdOut.println(v + "-" + w);
```

read graph from input stream

print out each edge (twice)



## Typical graph-processing code

*compute the degree of v*

```
public static int degree(Graph G, int v)
{
   int degree = 0;
   for (int w : G.adj(v)) degree++;
   return degree;
}
```

*compute maximum degree*

```
public static int maxDegree(Graph G)
{
   int max = 0;
   for (int v = 0; v < G.V(); v++)
      if (degree(G, v) > max)
         max = degree(G, v);
   return max;
}
```

*compute average degree*

```
public static int avgDegree(Graph G)
{
   return 2 * G.E() / G.V();
}
```

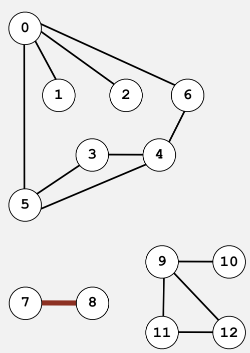
*count self-loops*

```
public static int numberOfSelfLoops(Graph G)
{
   int count = 0;
   for (int v = 0; v < G.V(); v++)
      for (int w : G.adj(v))
         if (v == w) count++;
   return count/2;
}
```



## Set-of-edges graph representation

Maintain a list of the edges (linked list or array).



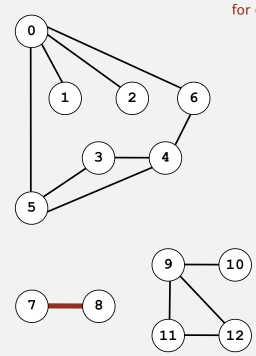

| | |
|---|---|
| 0 | 1 |
| 0 | 2 |
| 0 | 5 |
| 0 | 6 |
| 3 | 4 |
| 3 | 5 |
| 4 | 5 |
| 4 | 6 |
| 7 | 8 |
| 9 | 10 |
| 9 | 11 |
| 9 | 12 |
| 11 | 12 |



## Adjacency-matrix graph representation

Maintain a two-dimensional $V$-by-$V$ boolean array;

for each edge $v$-$w$ in graph: `adj[v][w] = adj[w][v] = true.`

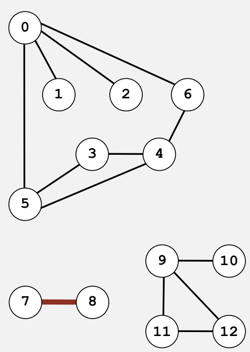


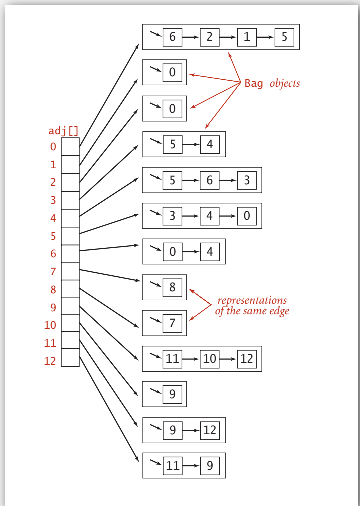

two entries for each edge

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

## Adjacency-list graph representation

Maintain vertex-indexed array of lists.
(use `Bag` abstraction)








## Adjacency-list graph representation: Java implementation

```
public class Graph
{
   private final int V;
   private Bag<Integer>[] adj;

   public Graph(int V)
   {
      this.V = V;
      adj = (Bag<Integer>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
   }

   public void addEdge(int v, int w)
   {
      adj[v].add(w);
      adj[w].add(v);
   }

   public Iterable<Integer> adj(int v)
   {  return adj[v];  }
}
```

adjacency lists ( use `Bag` data type )

create empty graph with `V` vertices

add edge `v-w` (parallel edges allowed)

iterator for vertices adjacent to `v`



## Graph representations

In practice. Use adjacency-lists representation.
- Algorithms based on iterating over vertices adjacent to v.
- Real-world graphs tend to be "sparse."

huge number of vertices, small average vertex degree

| representation | space | add edge | edge between v and w? | iterate over vertices adjacent to v? |
|---|---|---|---|---|
| list of edges | E | 1 | E | E |
| adjacency matrix | $V^2$ | 1 * | 1 | V |
| adjacency lists | E + V | 1 | degree(v) | degree(v) |

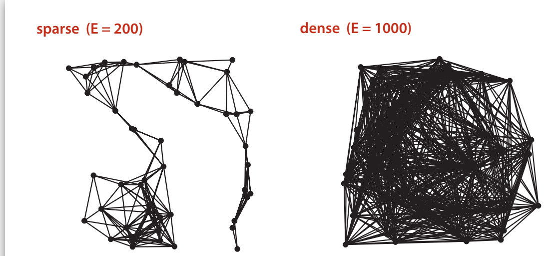
* disallows parallel edges



## Graph representations

In practice. Use adjacency-lists representation.
- Algorithms based on iterating over vertices adjacent to v.
- Real-world graphs tend to be "sparse."

huge number of vertices, small average vertex degree




Two graphs (V = 50)

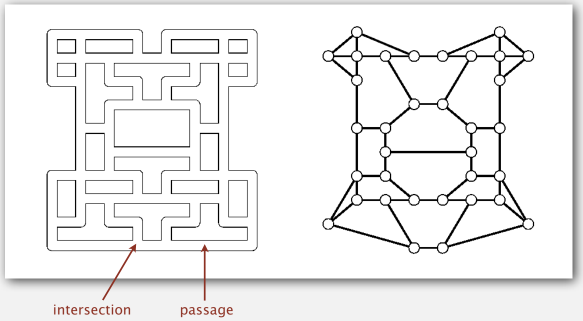
- graph API
- **depth-first search**
- breadth-first search
- connected components
- challenges



## Maze exploration

Maze graphs.

- Vertex = intersection.
- Edge = passage.



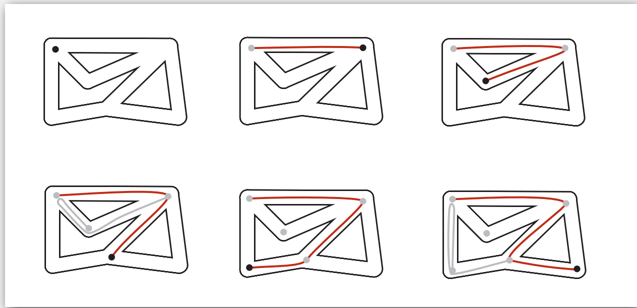

Goal. Explore every intersection in the maze.



## Trémaux maze exploration

Algorithm.

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options.



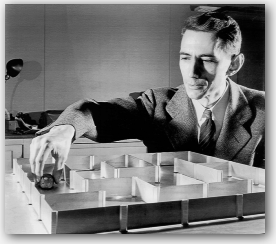


## Trémaux maze exploration

Algorithm.

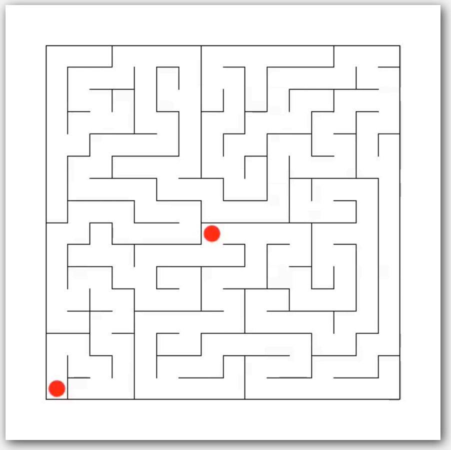
- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options.

First use? Theseus entered labyrinth to kill the monstrous Minotaur; Ariadne held ball of string.



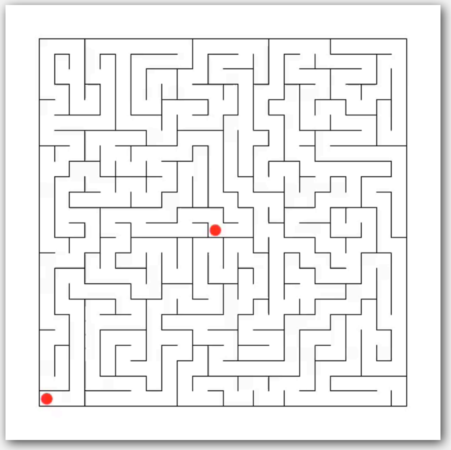
Claude Shannon (with Theseus mouse)

## Maze exploration





## Maze exploration





## Depth-first search

Goal. Systematically search through a graph.

Idea. Mimic maze exploration.

**DFS (to visit a vertex v)**

**Mark v as visited.**
**Recursively visit all unmarked vertices w adjacent to v.**

Typical applications. [ahead]

- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

## Design pattern for graph processing

Design pattern. Decouple graph data type from graph processing.

| public class Search | | |
|---|---|---|
| | Search(Graph G, int s) | *find vertices connected to s* |
| boolean | marked(int v) | *is vertex v connected to s?* |
| int | count() | *how many vertices connected to s?* |

Typical client program.

- Create a `Graph`.
- Pass the `Graph` to a graph-processing routine, e.g., `Search`.
- Query the graph-processing routine for information.

```
Search search = new Search(G, s);
for (int v = 0; v < G.V(); v++)
   if (search.marked(v))
      StdOut.println(v);
```

print all vertices connected to s

## Depth-first search (warmup)

**Goal.** Find all vertices connected to s.

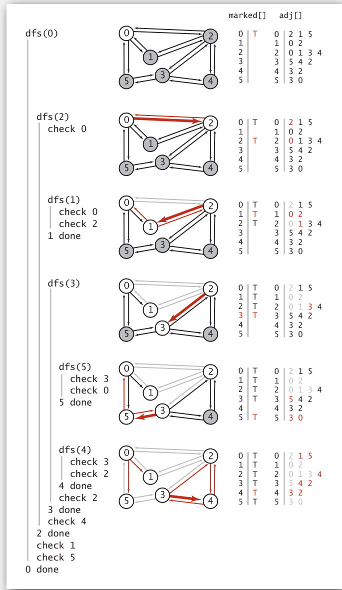
**Idea.** Mimic maze exploration.

**Algorithm.**

- Use recursion (ball of string).
- Mark each visited vertex.
- Return (retrace steps) when no unvisited options.

**Data structure.**

- `boolean[] marked` to mark visited vertices.




## Depth-first search (warmup)

```java
public class DepthFirstSearch
{
   private boolean[] marked;                          // true if connected to s

   public DepthFirstSearch(Graph G, int s)
   {
      marked = new boolean[G.V()];
      dfs(G, s);                                      // constructor marks vertices connected to s
   }

   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))                          // recursive DFS does the work
         if (!marked[w])
            dfs(G, w);
   }

   public boolean marked(int v)                       // client can ask whether vertex v is connected to s
   {  return marked[v];  }
}
```

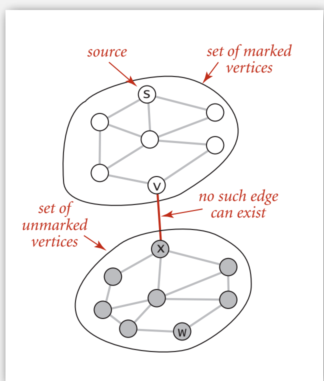


## Depth-first search properties

**Proposition.** DFS marks all vertices connected to $s$ in time proportional to the sum of their degrees.

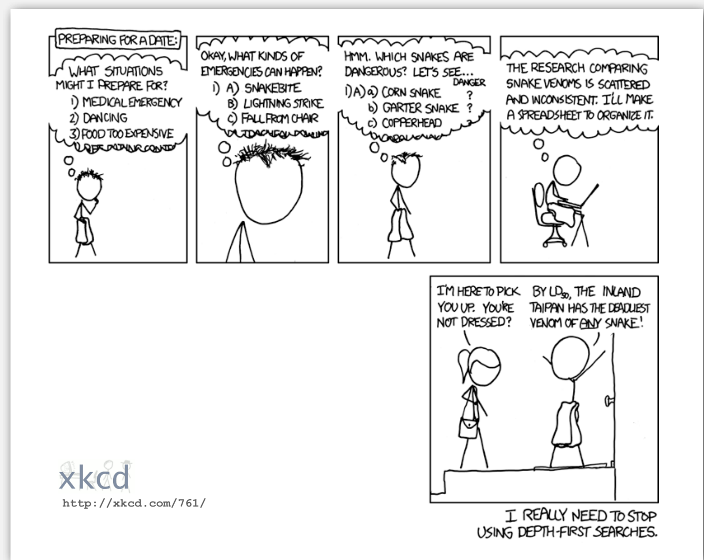
**Pf.**

- Correctness:
  - if $w$ marked, then $w$ connected to $s$ (why?)
  - if $w$ connected to $s$, then $w$ marked (if $w$ unmarked, then consider last edge on a path from $s$ to $w$ that goes from a marked vertex to an unmarked one)
- Running time: each vertex connected to $s$ is visited once.






## Depth-first search application: preparing for a date



## Depth-first search application: flood fill

Challenge. Flood fill (Photoshop magic wand).
Assumptions. Picture has millions to billions of pixels.

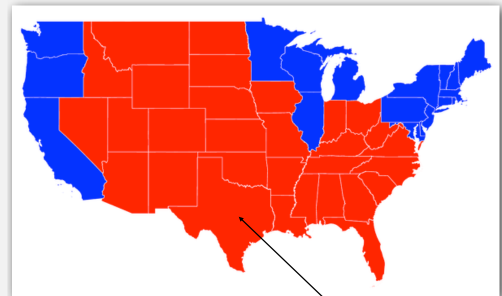
Q. How difficult?

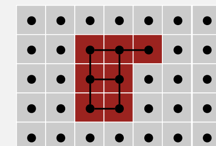


## Depth-first search application: flood fill

Change color of entire blob of neighboring red pixels to blue.

Build a grid graph.
- Vertex: pixel.
- Edge: between two adjacent red pixels.
- Blob: all pixels connected to given pixel.





## Depth-first search application: flood fill

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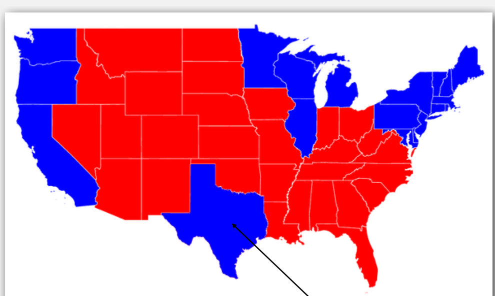
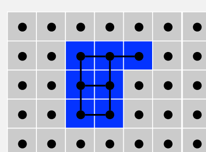
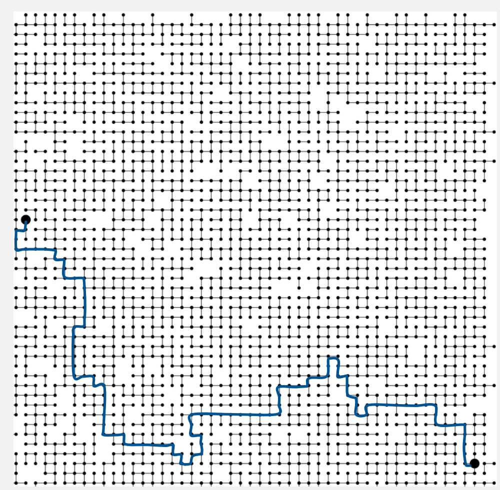




## Paths in graphs

Goal. Does there exist a path from $s$ to $t$?



## Paths in graphs: union-find vs. DFS

Goal. Does there exist a path from $s$ to $t$ ?

| method | preprocessing time | query time | space |
|---|---|---|---|
| union-find | V + E log* V | log* V † | V |
| DFS | E + V | 1 | E + V |

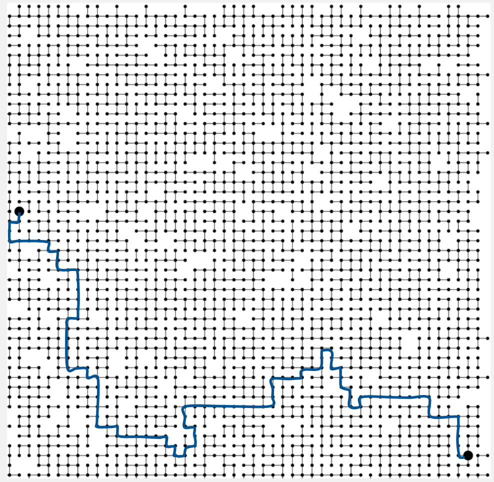
Union-find. Can intermix queries and edge insertions.

Depth-first search. Constant time per query.



## Pathfinding in graphs

Goal. Does there exist a path from $s$ to $t$ ? If yes, find any such path.





## Pathfinding in graphs

Goal. Does there exist a path from $s$ to $t$ ? If yes, find any such path.

| | | |
|---|---|---|
| public class Paths | | |
| | Paths(Graph G, int s) | *find paths in G from source s* |
| boolean | hasPathTo(int v) | *is there a path from s to v?* |
| Iterable<Integer> | pathTo(int v) | *path from s to v; null if no such path* |

Union-find. Not much help.

Depth-first search. After linear-time preprocessing, can recover path itself in time proportional to its length.

easy modification (stay tuned)



## Depth-first search (pathfinding)

Goal. Find paths to all vertices connected to a given source $s$.

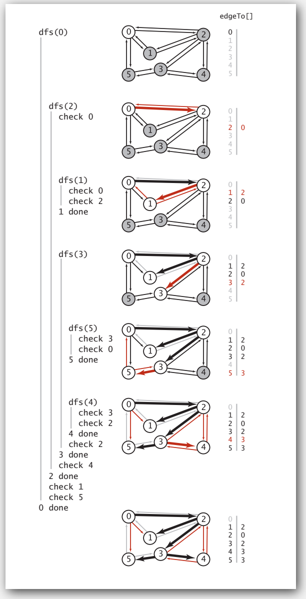
Idea. Mimic maze exploration.

Algorithm.

- Use recursion (ball of string).
- Mark each visited vertex by keeping
- track of edge taken to visit it.
- Return (retrace steps) when no unvisited options.

Data structures.

- `boolean[] marked` to mark visited vertices.
- `int[] edgeTo` to keep tree of paths.
- `(edgeTo[w] == v)` means that edge `v-w` was taken to visit `w` the first time



## Depth-first search (pathfinding)

```
public class DepthFirstPaths
{
   private boolean[] marked;
   private int[] edgeTo;
   private final int s;

   public DepthFirstPaths(Graph G, int s)
   {
      marked = new boolean[G.V()];
      edgeTo = new int[G.V()];
      this.s = s;
      dfs(G, s);
   }
   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w])
         {
            edgeTo[w] = v;
            dfs(G, w);
         }
   }

   public boolean hasPathTo(int v)
   public Iterable<Integer> pathTo(int v)
}
```

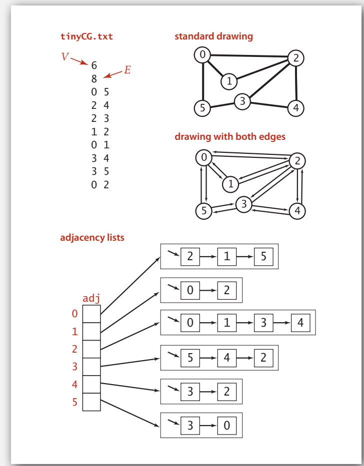
parent-link representation of DFS tree

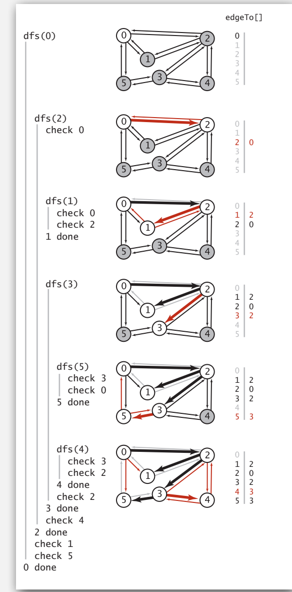
set parent link

ahead



## Depth-first search (pathfinding trace)

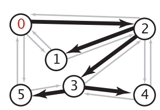


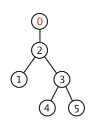



## Depth-first search (pathfinding iterator)

`edgeTo[]` is a parent-link representation of a tree rooted at `s`.



| edgeTo[] | |
|---|---|
| 0 | |
| 1 | 2 |
| 2 | 0 |
| 3 | 2 |
| 4 | 3 |
| 5 | 3 |



| x | path |
|---|---|
| 5 | 5 |
| 3 | 3 5 |
| 2 | 2 3 5 |
| 0 | 0 2 3 5 |

```
public boolean hasPathTo(int v)
{  return marked[v];  }

public Iterable<Integer> pathTo(int v)
{
   if (!hasPathTo(v)) return null;
   Stack<Integer> path = new Stack<Integer>();
   for (int x = v; x != s; x = edgeTo[x])
      path.push(x);
   path.push(s);
   return path;
}
```



## Depth-first search summary

Enables direct solution of simple graph problems.

- ✓ Does there exists a path between $s$ and $t$?
- ✓ Find path between $s$ and $t$.
- Connected components (stay tuned).
- Euler tour (see book).
- Cycle detection (see book).
- Bipartiteness checking (see book).

Basis for solving more difficult graph problems.

- Biconnected components (beyond scope).
- Planarity testing (beyond scope).

- graph API
- depth-first search
- **breadth-first search**
- connected components
- challenges



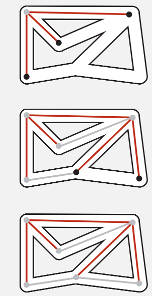
## Breadth-first search

Depth-first search. Put unvisited vertices on a stack.
Breadth-first search. Put unvisited vertices on a queue.

Shortest path. Find path from $s$ to $t$ that uses fewest number of edges.

**BFS (from source vertex s)**

**Put s onto a FIFO queue, and mark s as visited.**
**Repeat until the queue is empty:**
**- remove the least recently added vertex v**
**- add each of v's unvisited neighbors to the queue, and mark them as visited.**

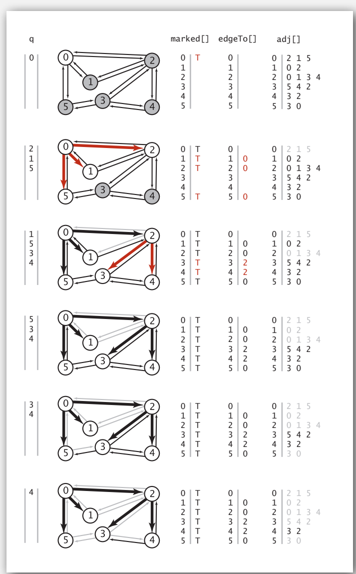


Intuition. BFS examines vertices in increasing distance from $s$.



## Breadth-first search (pathfinding)

```
private void bfs(Graph G, int s)
{
   Queue<Integer> q = new Queue<Integer>();
   q.enqueue(s);
   marked[s] = true;
   while (!q.isEmpty())
   {
      int v = q.dequeue();
      for (int w : G.adj(v))
         if (!marked[w])
         {
            q.enqueue(w);
            marked[w] = true;
            edgeTo[w] = v;
         }
   }
}
```



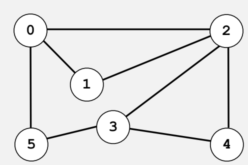



## Breadth-first search properties

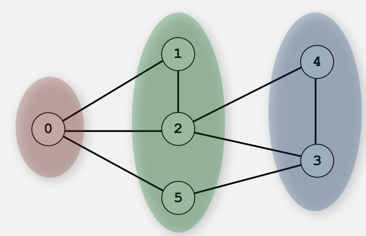
Proposition. BFS computes shortest path (number of edges) from $s$ in a connected graph in time proportional to $E + V$.

Pf.

- Correctness: queue always consists of zero or more vertices of distance $k$ from $s$, followed by zero or more vertices of distance $k + 1$.
- Running time: each vertex connected to $s$ is visited once.



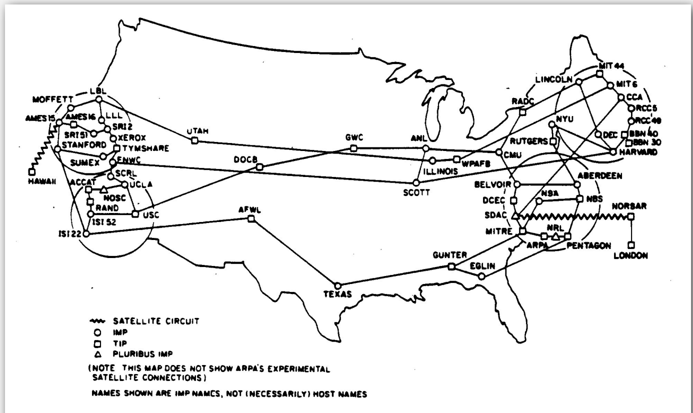

standard drawing




dist = 0 dist = 1 dist = 2

## Breadth-first search application: routing

Fewest number of hops in a communication network.

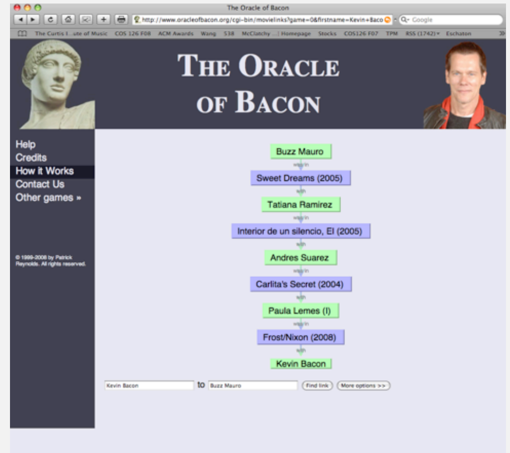



ARPANET, July 1977



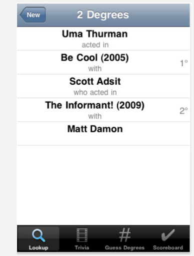
## Breadth-first search application: Kevin Bacon numbers

Kevin Bacon numbers.




http://oracleofbacon.org

Endless Games board game

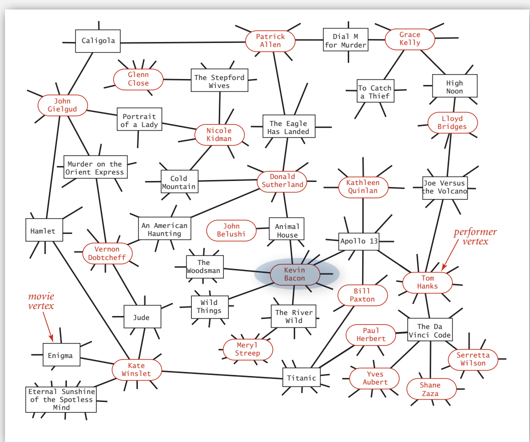



SixDegrees iPhone App



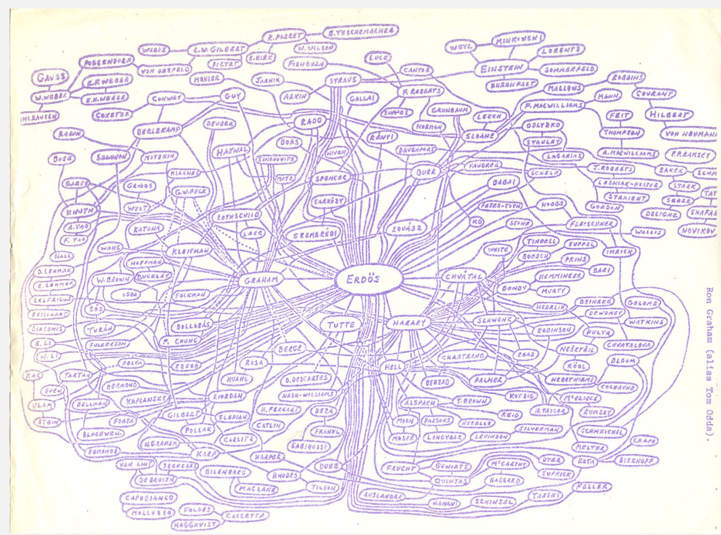
## Kevin Bacon graph

- Include a vertex for each performer and for each movie.
- Connect a movie to all performers that appear in that movie.
- Compute shortest path from s = Kevin Bacon.






## Breadth-first search application: Erdös numbers



hand-drawing of part of the Erdös graph by Ron Graham

- graph API
- depth-first search
- breadth-first search
- **connected components**
- challenges



## Connectivity queries

Def. Vertices $v$ and $w$ are connected if there is a path between them.

Goal. Preprocess graph to answer queries: is $v$ connected to $w$ ? in constant time.

| public class CC | | |
|---|---|---|
| | `CC(Graph G)` | *find connected components in G* |
| `boolean` | `connected(int v, int w)` | *are v and w connected?* |
| `int` | `count()` | *number of connected components* |
| `int` | `id(int v)` | *component identifier for v* |

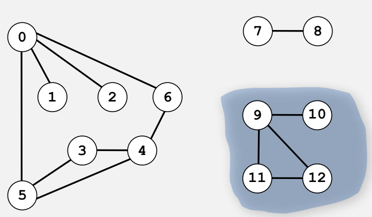
Union-Find? Not quite.
Depth-first search. Yes. [next few slides]



## Connected components

The relation "is connected to" is an equivalence relation:
- Reflexive: $v$ is connected to $v$.
- Symmetric: if $v$ is connected to $w$, then $w$ is connected to $v$.
- Transitive: if $v$ connected to $w$ and $w$ connected to $x$, then $v$ connected to $x$.

Def. A connected component is a maximal set of connected vertices.



3 connected components

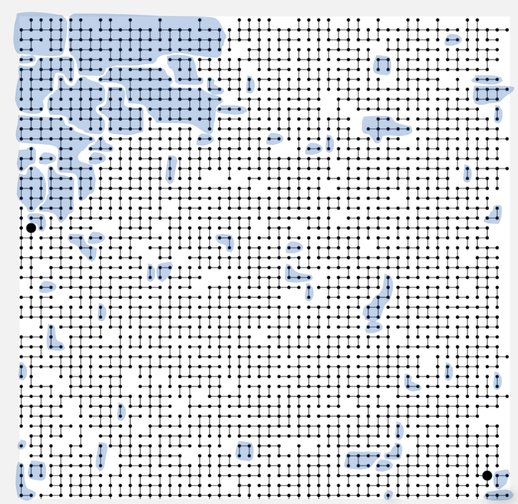
| v | id[v] |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |
| 5 | 0 |
| 6 | 0 |
| 7 | 1 |
| 8 | 1 |
| 9 | 2 |
| 10 | 2 |
| 11 | 2 |
| 12 | 2 |

Remark. Given connected components, can answer queries in constant time.



## Connected components

Def. A connected component is a maximal set of connected vertices.


63 connected components

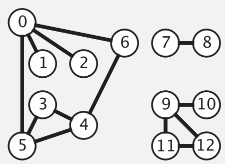
## Connected components

**Goal.** Partition vertices into connected components.

**Connected components**

Initialize all vertices v as unmarked.

For each unmarked vertex v, run DFS to identify all vertices discovered as part of the same component.




tinyG.txt

```
V → 13
    13 ← E
    0 5
    4 3
    0 1
    9 12
    6 4
    5 4
    0 2
    11 12
    9 10
    0 6
    7 8
    9 11
    5 3
```



## Finding connected components with DFS

```
public class CC
{
   private boolean marked[];
   private int[] id;
   private int count;

   public CC(Graph G)
   {
      marked = new boolean[G.V()];
      id = new int[G.V()];
      for (int v = 0; v < G.V(); v++)
      {
         if (!marked[v])
         {
            dfs(G, v);
            count++;
         }
      }
   }

   public int count()
   public int id(int v)
   private void dfs(Graph G, int v)

}
```

id[v] = id of component containing v

number of connected components

run DFS from one vertex in each component

see next slide



## Finding connected components with DFS (continued)

```
public int count()
{  return count;  }

public int id(int v)
{  return id[v];  }

private void dfs(Graph G, int v)
{
   marked[v] = true;
   id[v] = count;
   for (int w : G.adj(v))
      if (!marked[w])
         dfs(G, w);
}
```

number of connected components

id of component containing v

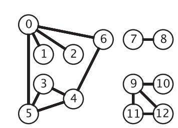
all vertices discovered in same call of dfs have same id



## Finding connected components with DFS (trace)

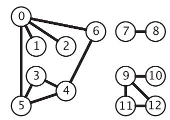
```
                    count  marked[]                     id[]
                           0 1 2 3 4 5 6 7 8 9 10 11 12  0 1 2 3 4 5 6 7 8 9 10 11 12

dfs(0)                0    T                             0
| dfs(6)              0    T           T                 0           0
| | check 0
| | dfs(4)            0    T       T   T                 0       0   0
| | | dfs(5)          0    T       T T T                 0       0 0 0
| | | | dfs(3)        0    T     T T T T                 0     0 0 0 0
| | | | | check 5
| | | | | check 4
| | | | 3 done
| | | | check 4
| | | | check 0
| | | 5 done
| | | check 6
| | | check 3
| | 4 done
| 6 done
| dfs(2)              0    T   T T T T T                 0   0 0 0 0 0
| | check 0
| 2 done
| dfs(1)              0    T T T T T T T                 0 0 0 0 0 0 0
| | check 0
| 1 done
| check 5
0 done
```



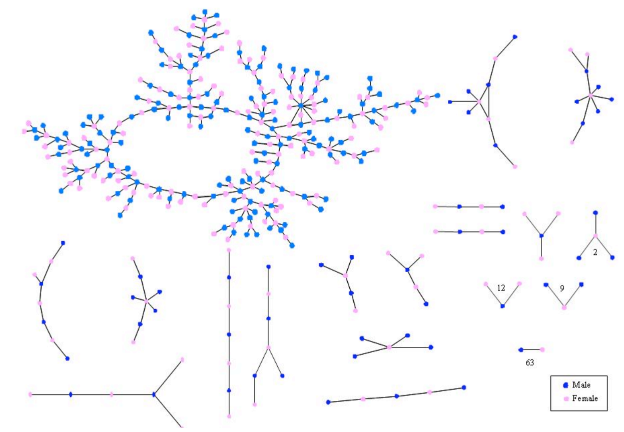
## Finding connected components with DFS (trace)

```
                 count    marked[]                     id[]
                          0 1 2 3 4 5 6 7 8 9 10 11 12  0 1 2 3 4 5 6 7 8 9 10 11 12
0 done
dfs(7)           1        T T T T T T T T              0 0 0 0 0 0 0 1
| dfs(8)         1        T T T T T T T T T            0 0 0 0 0 0 0 1 1
| | check 7
| 8 done
7 done
dfs(9)           2        T T T T T T T T T T          0 0 0 0 0 0 0 1 1 2
| dfs(11)        2        T T T T T T T T T T    T     0 0 0 0 0 0 0 1 1 2    2
| | check 9
| | dfs(12)      2        T T T T T T T T T T    T T   0 0 0 0 0 0 0 1 1 2    2 2
| | | check 11
| | | check 9
| | 12 done
| 11 done
| dfs(10)        2        T T T T T T T T T T T  T T   0 0 0 0 0 0 0 1 1 2 2  2 2
| | check 9
| 10 done
| check 12
9 done
```






## Connected components application: study spread of STDs




Relationship graph at "Jefferson High"

Peter Bearman, James Moody, and Katherine Stovel. Chains of affection: The structure of adolescent romantic and sexual networks. American Journal of Sociology, 110(1): 44–99, 2004.

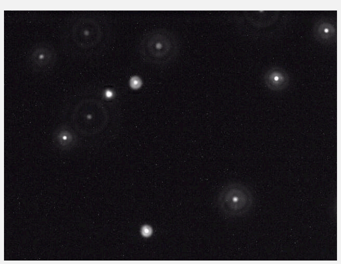


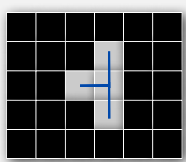
## Connected components application: particle detection

Particle detection. Given grayscale image of particles, identify "blobs."

- Vertex: pixel.
- Edge: between two adjacent pixels with grayscale value ≥ 70.
- Blob: connected component of 20-30 pixels.

black = 0
white = 255

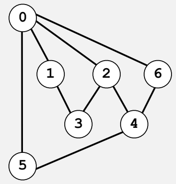




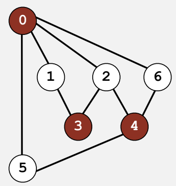
Particle tracking. Track moving particles over time.

## Graph-processing challenge 1

Problem. Is a graph bipartite?




0-1
0-2
0-5
0-6
1-3
2-3
2-4
4-5
4-6



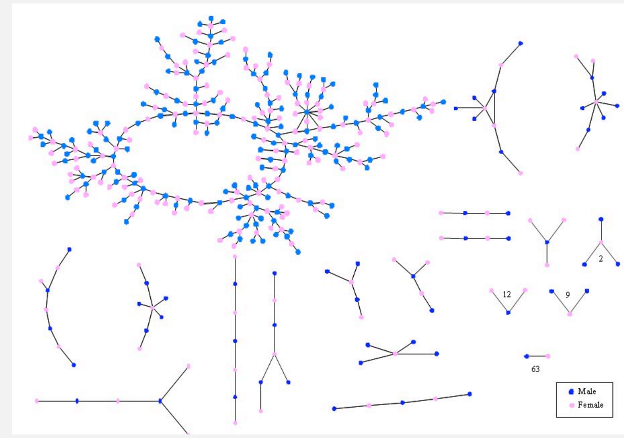

0-1
0-2
0-5
0-6
1-3
2-3
2-4
4-5
4-6

How difficult?

- Any COS 126 student could do it.
- Need to be a typical diligent COS 226 student.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.



## Bipartiteness application



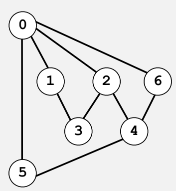

Relationship graph at "Jefferson High"

Peter Bearman, James Moody, and Katherine Stovel. Chains of affection: The structure of adolescent romantic and sexual networks. American Journal of Sociology, 110(1): 44–99, 2004.

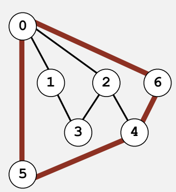


## Graph-processing challenge 2

Problem. Find a cycle.




0-1
0-2
0-5
0-6
1-3
2-3
2-4
4-5
4-6




How difficult?

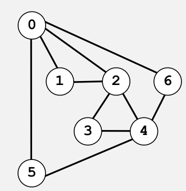
- Any COS 126 student could do it.
- Need to be a typical diligent COS 226 student.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.



## Graph-processing challenge 3

Problem. Find a cycle that uses every edge.

Assumption. Need to use each edge exactly once.




0-1-2-3-4-2-0-6-4-5-0

0-1
0-2
0-5
0-6
1-2
2-3
2-4
3-4
4-5
4-6

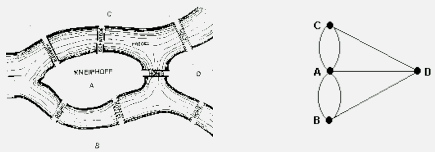
How difficult?

- Any COS 126 student could do it.
- Need to be a typical diligent COS 226 student.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

## Bridges of Königsberg

The Seven Bridges of Königsberg. [Leonhard Euler 1736]

> *" … in Königsberg in Prussia, there is an island A, called the Kneiphof; the river which surrounds it is divided into two branches … and these branches are crossed by seven bridges. Concerning these bridges, it was asked whether anyone could arrange a route in such a way that he could cross each bridge once and only once."*

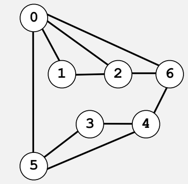


Euler tour. Is there a (general) cycle that uses each edge exactly once?
Answer. Yes iff connected and all vertices have even degree.
To find path. DFS-based algorithm (see textbook).



## Graph-processing challenge 4

Problem. Find a cycle that visits every vertex.
Assumption. Need to visit each vertex exactly once.



0-1
0-2
0-5
0-6
1-2
2-6
3-4
3-5
4-5
4-6

0-5-3-4-6-2-1-0

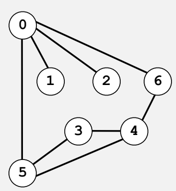
How difficult?

- Any COS 126 student could do it.
- Need to be a typical diligent COS 226 student.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

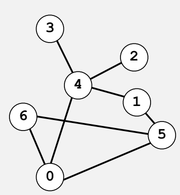


## Graph-processing challenge 5

Problem. Are two graphs identical except for vertex names?



0-1
0-2
0-5
0-6
3-4
3-5
4-5
4-6



0-4
0-5
0-6
1-4
1-5
2-4
3-4
5-6

0↔4, 1↔3, 2↔2, 3↔6, 4↔5, 5↔0, 6↔1

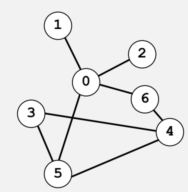
How difficult?

- Any COS 126 student could do it.
- Need to be a typical diligent COS 226 student.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

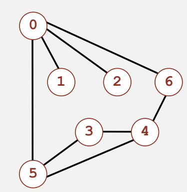


## Graph-processing challenge 6

Problem. Lay out a graph in the plane without crossing edges?



0-1
0-2
0-5
0-6
3-4
3-5
4-5
4-6



How difficult?

- Any COS 126 student could do it.
- Need to be a typical diligent COS 226 student.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.