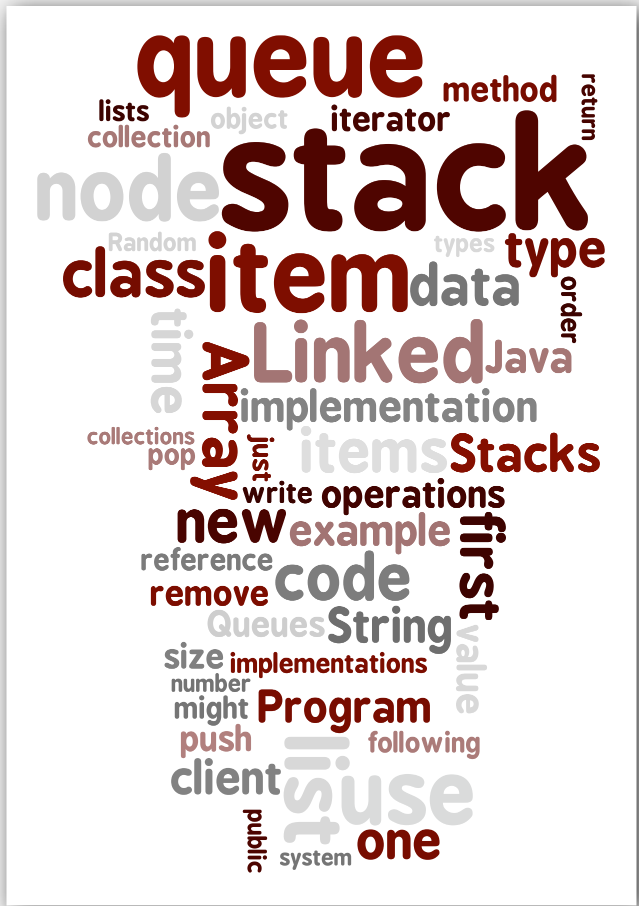
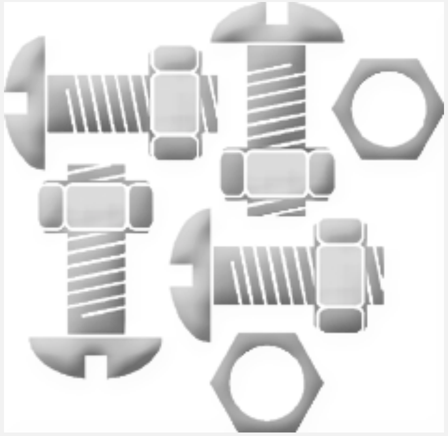
1.3 Stacks and Queues



- stacks
- dynamic resizing
- queues
- generics
- iterators
- applications

Fundamental data types.

- Values: sets of objects.
- Operations: insert, remove, test if empty.
- Intent is clear when we insert.
- Which item do we remove?



LIFO = "last in first out"

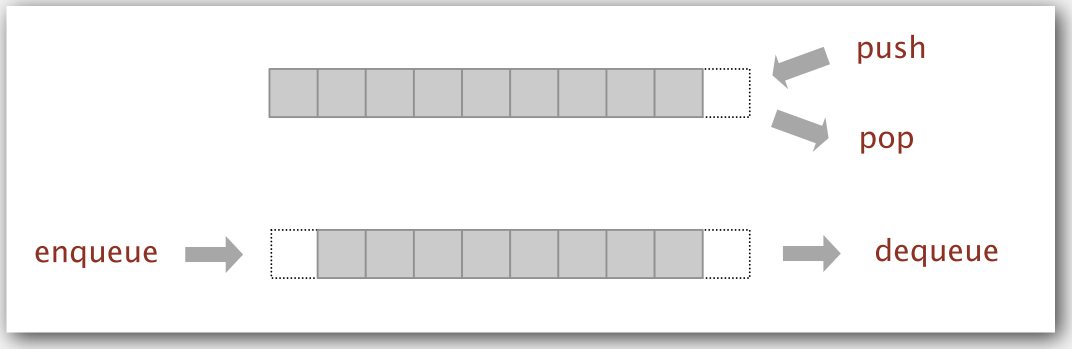
Stack. Remove the item most recently added.

Analogy. Cafeteria trays, Web surfing.

FIFO = "first in first out"

Queue. Remove the item least recently added.

Analogy. Registrar's line.



Client, implementation, interface

Separate interface and implementation.

Ex: stack, queue, priority queue, symbol table, union-find, ...

Benefits.

- Client can't know details of implementation $\Rightarrow$ client has many implementation from which to choose.
- Implementation can't know details of client needs $\Rightarrow$ many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

Client: program using operations defined in interface.
Implementation: actual code implementing operations.
Interface: description of data type, basic operations.

- stacks
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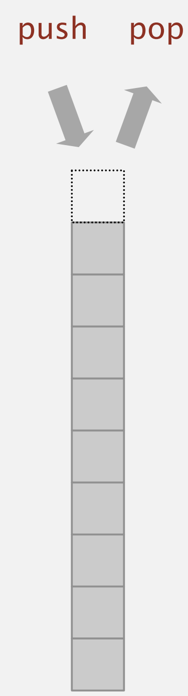
Stack API

Warmup. Stack of strings objects.

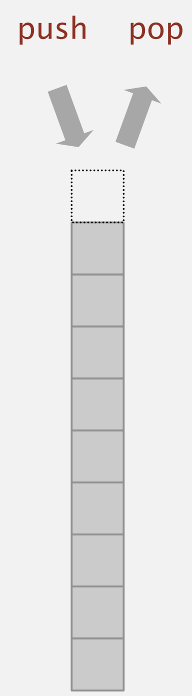
public class StackOfStrings		
	`StackOfStrings()`	*create an empty stack*
`void`	`push(String s)`	*insert a new item onto stack*
`String`	`pop()`	*remove and return the item most recently added*
`boolean`	`isEmpty()`	*is the stack empty?*
`int`	`size()`	*number of items on the stack*



Challenge. Reverse sequence of strings from standard input.

Stack test client

```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-")) StdOut.print(stack.pop());
      else                  stack.push(item);
   }
}
```




```
% more tobe.txt
to be or not to - be - - that - - - is

% java StackOfStrings < tobe.txt
to be not that or be
```

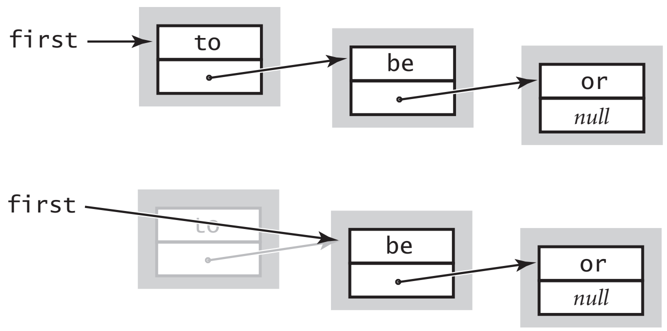
Stack pop: linked-list implementation

save item to return

```
String item = first.item;
```

save item to return

```
first = first.next;
```



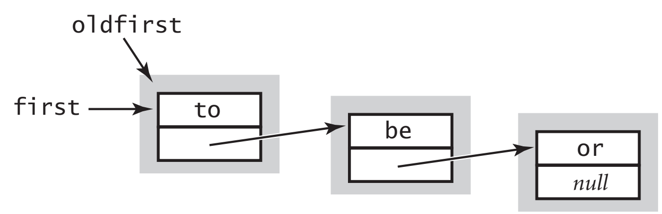
return saved item

```
return item;
```

Stack push: linked-list implementation

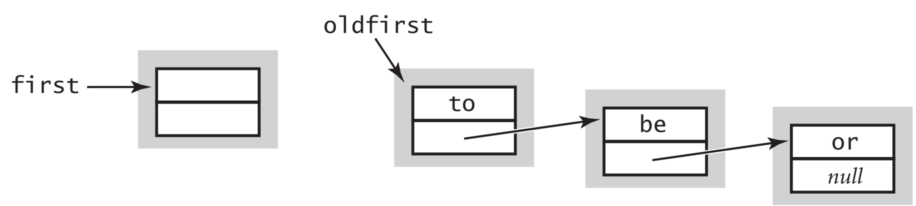
save a link to the list

```
Node oldfirst = first;
```



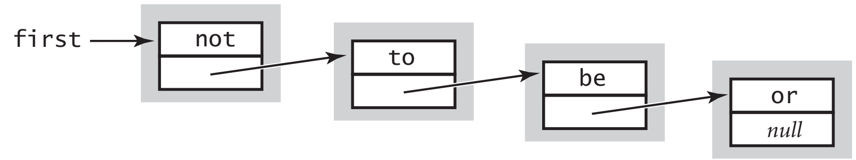
create a new node for the beginning

```
first = new Node();
```



set the instance variables in the new node

```
first.item = "not";
first.next = oldfirst;
```



Stack: linked-list implementation in Java

```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;   }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      if (isEmpty()) throw new RuntimeException();
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

inner class

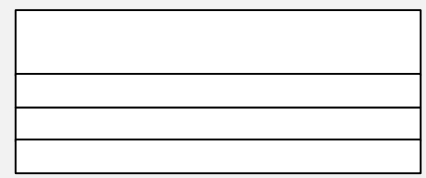
stack underflow

Stack: linked-list implementation performance

Proposition. Using a linked-list implementation of a stack, every operation takes constant time in the worst case.

Proposition. Uses ~ $40\ N$ bytes to represent a stack with N items.

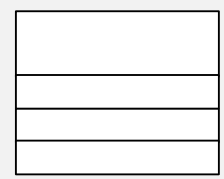
- assume 64-bit machine (8 bytes per reference)
- extra overhead for inner class




```
private class Node
{
   String item;
   Node next;
}
```

16 bytes (object overhead)
8 bytes (extra for inner class)
8 bytes (reference to String)
8 bytes (reference to Node)

40 bytes per stack item



Remark. Analysis includes memory for the stack (but not the strings themselves, which the client owns).

Lesson. "Swollen" pointers can use up memory on 64-bit machines!

Array implementation of a stack.

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.

`s[]`	to	be	or	not	to	be	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9
							`N`			`capacity = 10`

Defect. Stack overflows when `N` exceeds capacity. [stay tuned]

Stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

a cheat (stay tuned)

decrement N;
then use to index into array

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   return item;
}
```

this version avoids "loitering":
garbage collector reclaims memory
only if no outstanding references

- stacks
- **dynamic resizing**
- queues
- generics
- iterators
- applications

Stack: dynamic-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

First try.

- `push()`: increase size of `s[]` by 1.
- `pop()`: decrease size of `s[]` by 1.

Too expensive.

- Need to copy all item to a new array.
- Inserting first N items takes time proportional to $1 + 2 + \ldots + N \sim N^2 / 2$.

infeasible for large N

Challenge. Ensure that array resizing happens infrequently.

Stack: dynamic-array implementation

"repeated doubling"

Q. How to grow array?

A. If array is full, create a new array of twice the size, and copy items.

```
public StackOfStrings() {  s = new String[1];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

cost of array resizing is now
$2 + 4 + 8 + \ldots + N \sim 2N$

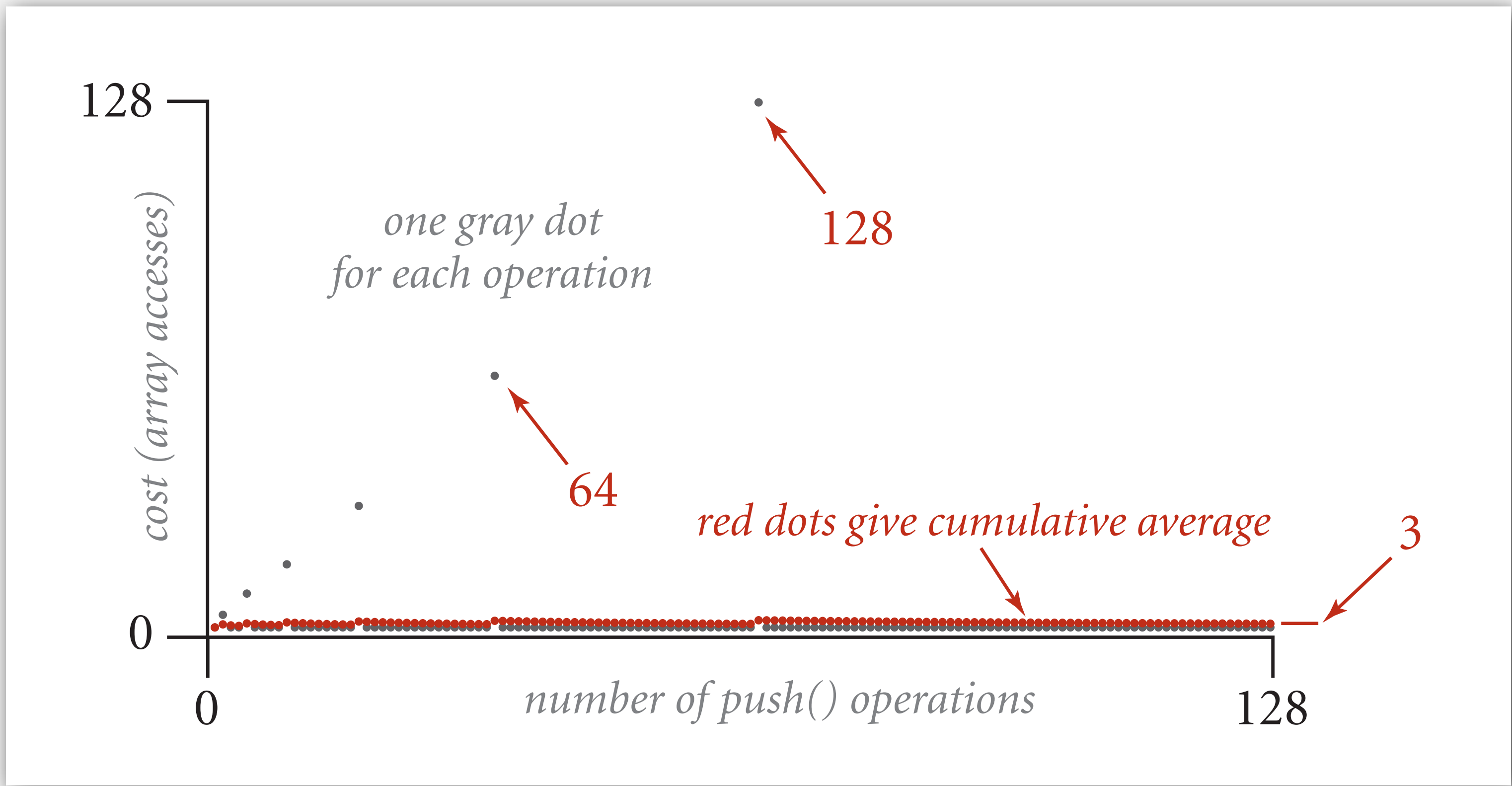
Consequence. Inserting first N items takes time proportional to N (not N^2).

Stack: amortized cost of adding to a stack

Cost of inserting first N items. $N + (2 + 4 + 8 + \ldots + N) \sim 3N.$

1 array accesses per push

k array accesses to double to size k



Q. How to shrink array?

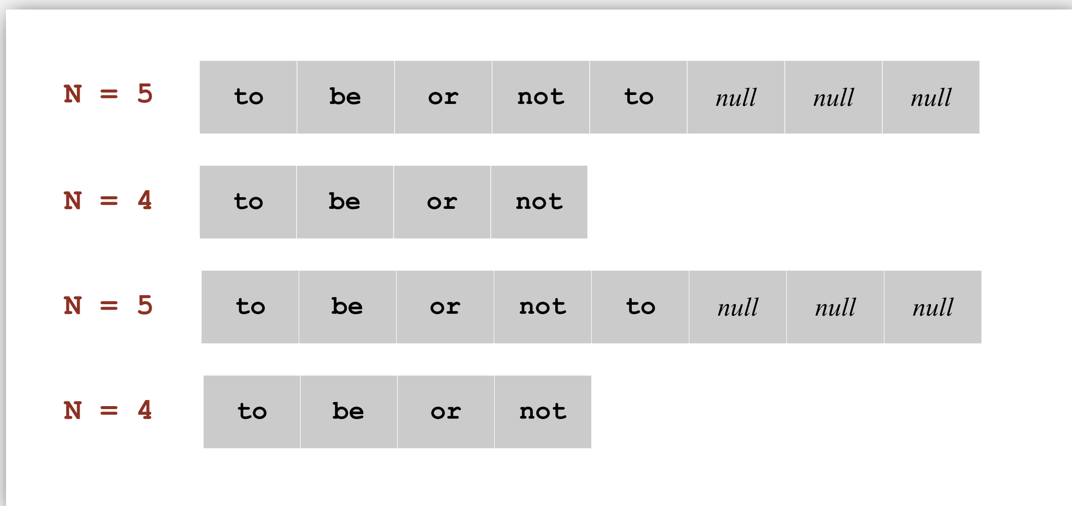
First try.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is one-half full.

"thrashing"

Too expensive.

- Consider push-pop-push-pop-… sequence when array is full.
- Takes time proportional to N per operation in worst case.



Stack: dynamic-array implementation

Q. How to shrink array?

Efficient solution.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length / 2);
   return item;
}
```

Invariant. Array is between 25% and 100% full.

Stack: dynamic-array implementation trace

StdIn	StdOut	N	a.length	a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]
		0	1	*null*							
to		1	1	to							
be		2	2	to	be						
or		3	4	to	be	or	*null*				
not		4	4	to	be	or	not				
to		5	8	to	be	or	not	to	*null*	*null*	*null*
-	to	4	8	to	be	or	not	*null*	*null*	*null*	*null*
be		5	8	to	be	or	not	be	*null*	*null*	*null*
-	be	4	8	to	be	or	not	*null*	*null*	*null*	*null*
-	not	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
that		4	8	to	be	or	that	*null*	*null*	*null*	*null*
-	that	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
-	or	2	4	to	be	*null*	*null*				
-	be	1	2	to	*null*						
is		2	2	to	is						

Stack dynamic-array implementation: performance

Amortized analysis. Average running time per operation over a worst-case sequence of operations.

Proposition. Starting from empty stack (with dynamic resizing), any sequence of M push and pop operations takes time proportional to M.

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and shrinking (applies to the worst-case N for push and pop)

running time for doubling stack with N items

Stack dynamic array implementation: memory usage

Proposition. Uses between $\sim 8\,N$ and $\sim 32\,N$ bytes to represent a stack with N items.

- $\sim 8\,N$ when full.
- $\sim 32\,N$ when one-quarter full.

```
public class DoublingStackOfStrings
{
   private String[] s;
   private int N = 0;
   …
}
```

8 bytes × array size (for `private String[] s;`)

4 bytes (for `private int N = 0;`)

Remark. Analysis includes memory for the stack (but not the strings themselves, which the client owns).

Stack implementations: dynamic array vs. linked List

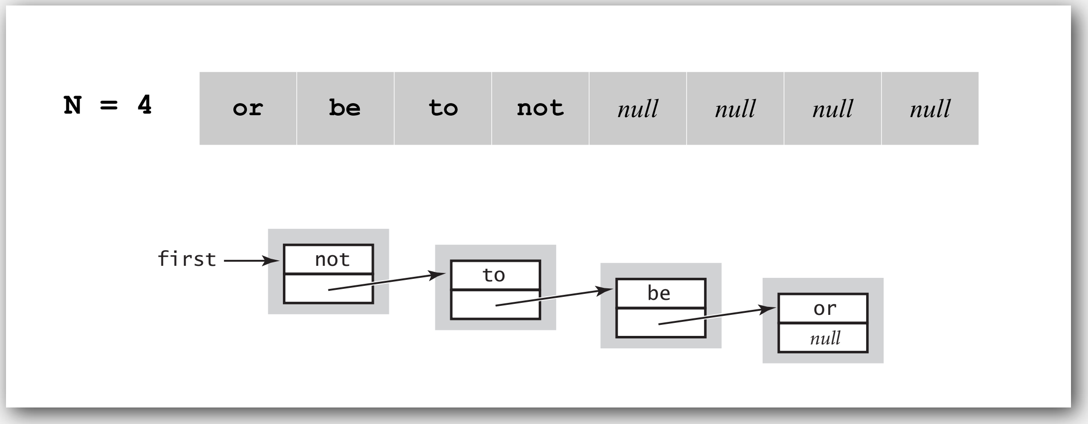
Tradeoffs. Can implement a stack with either dynamic array or linked list; client can use interchangeably. Which one is better?

Linked-list implementation.

- Every operation takes constant time in the worst case.
- Uses extra time and space to deal with the links.

Dynamic-array implementation.

- Every operation takes constant amortized time.
- Less wasted space.



- stacks
- dynamic resizing
- **queues**
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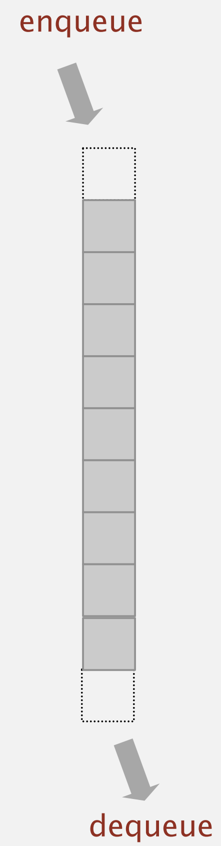
Queue API

public class QueueOfStrings		
	`QueueOfStrings()`	*create an empty queue*
`void`	`enqueue(String s)`	*insert a new item onto queue*
`String`	`dequeue()`	*remove and return the item least recently added*
`boolean`	`isEmpty()`	*is the queue empty?*
`int`	`size()`	*number of items on the queue*



Queue test client

```
public static void main(String[] args)
{
   QueueOfStrings q = new QueueOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-")) StdOut.print(q.dequeue());
      else                  q.enqueue(item);
   }
}
```

```
% more tobe.txt
to be or not to - be - - that - - - is

% java QueueOfStrings < tobe.txt
to be or not to be
```

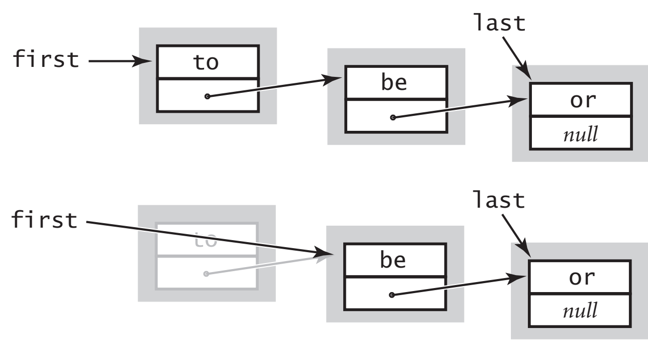
Queue dequeue: linked-list implementation

save item to return

```
String item = first.item;
```

save item to return

```
first = first.next;
```



return saved item

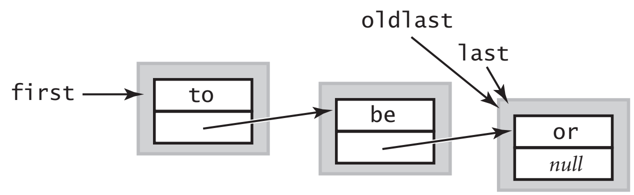
```
return item;
```

Remark. Identical code to linked-list stack `pop()`.

Queue enqueue: linked-list implementation

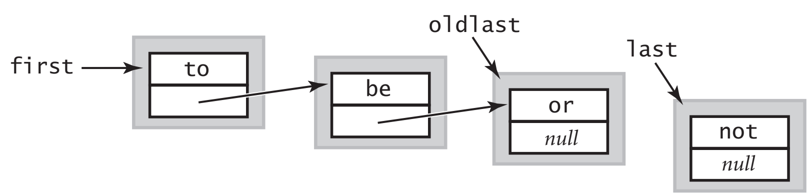
save a link to the last node

```
Node oldlast = last;
```



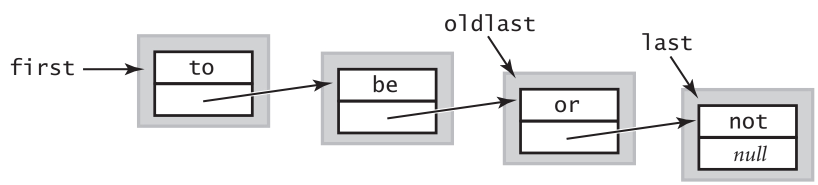
create a new node for the end

```
Node last = new Node();
last.item = "not";
last.next = null;
```



link the new node to the end of the list

```
oldlast.next = last;
```

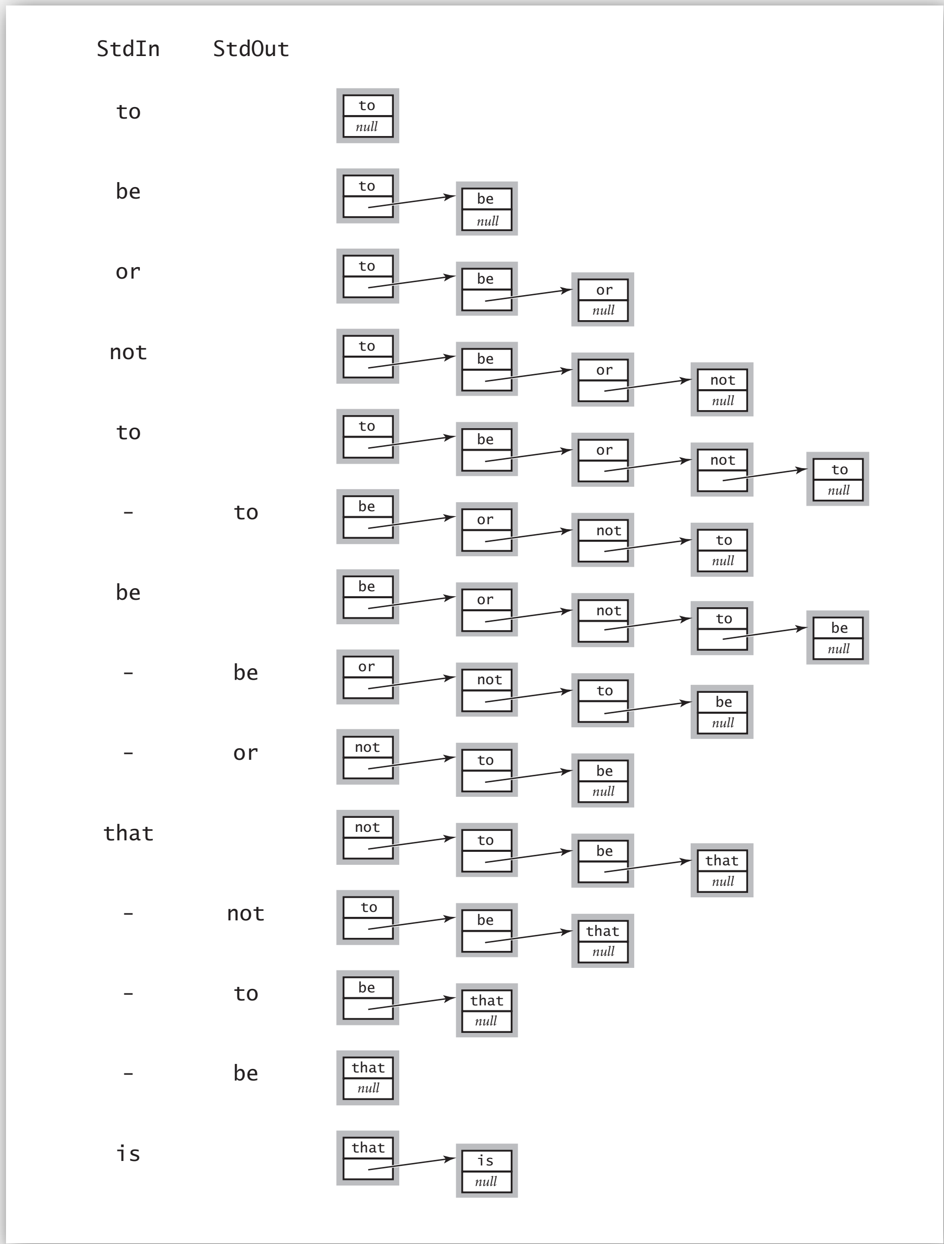


Queue: linked-list implementation in Java

```
public class QueueOfStrings
{
   private Node first, last;

   private class Node
   {  /* same as in StackOfStrings */   }

   public boolean isEmpty()
   {  return first == null;   }

   public void enqueue(String item)
   {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```

special cases for empty queue

Queue: linked-list trace



Queue: dynamic array implementation

Array implementation of a queue.

- Use array `q[]` to store items in queue.
- `enqueue()`: add new item at `q[tail]`.
- `dequeue()`: remove item from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.
- Add dynamic resizing.

- stacks
- dynamic resizing
- queues
- **generics**
- iterators
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Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

Attempt 1. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

@#$*! most reasonable approach until Java 1.5.

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

Attempt 2. Implement a stack with items of type `Object`.

- Casting is required in client.
- Casting is error-prone: run-time error if types mismatch.

```
StackOfObjects s = new StackOfObjects();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

Attempt 3. Java generics.

- Avoid casting in client.
- Discover type mismatch errors at compile-time instead of run-time.

type parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = s.pop();
```

compile-time error (`s.push(b);`)

Guiding principles. Welcome compile-time errors; avoid run-time errors.

Generic stack: linked-list implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

generic type name

Generic stack: array implementation

```
public class ArrayStackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it should be

```
public class ArrayStack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

```
public class ArrayStackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it is

```
public class ArrayStack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = (Item[]) new Object[capacity]; }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

the ugly cast

Generic data types: autoboxing

Q. What to do about primitive types?

Wrapper type.

- Each primitive type has a wrapper object type.
- Ex: `Integer` is wrapper type for `int`.

Autoboxing. Automatic cast between a primitive type and its wrapper.

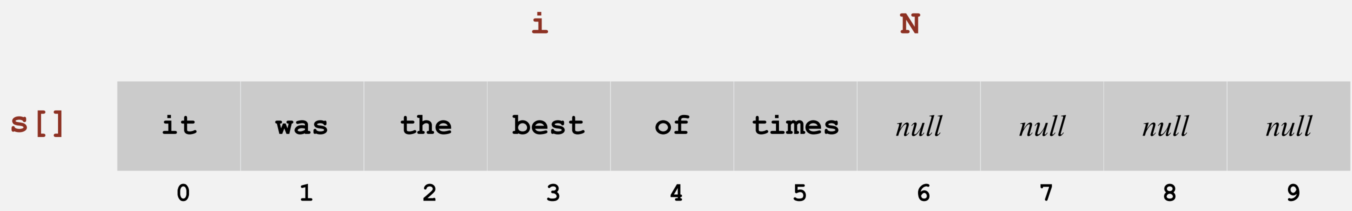
Syntactic sugar. Behind-the-scenes casting.

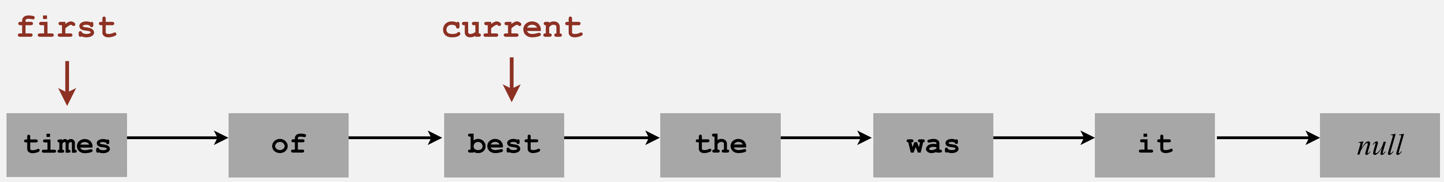
```
Stack<Integer> s = new Stack<Integer>();
s.push(17);          // s.push(new Integer(17));
int a = s.pop();     // int a = s.pop().intValue();
```

Bottom line. Client code can use generic stack for any type of data.

- stacks
- dynamic resizing
- queues
- generics
- **iterators**
- applications

Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.





Java solution. Make stack implement the `Iterable` interface.

Iterators

Q. What is an `Iterable` ?

A. Has a method that returns an `Iterator`.

Q. What is an `Iterator` ?

A. Has methods `hasNext()` and `next()`.

Q. Why make data structures `Iterable` ?

A. Java supports elegant client code.

Iterable interface

```
public interface Iterable<Item>
{
   Iterator<Item> iterator();
}
```

Iterator interface

```
public interface Iterator<Item>
{
   boolean hasNext();
   Item next();
   void remove();   ← optional; use at your own risk
}
```

"foreach" statement

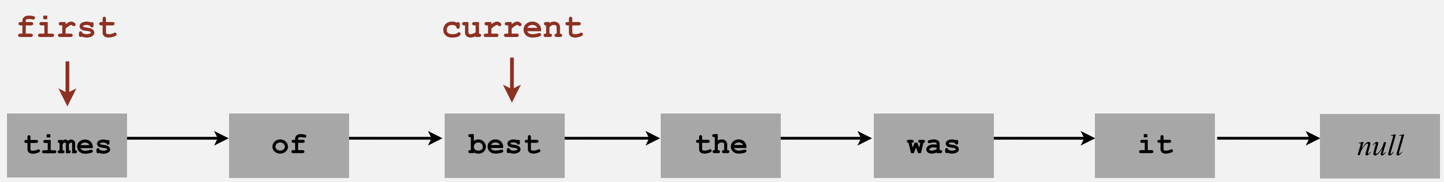
```
for (String s : stack)
   StdOut.println(s);
```

equivalent code

```
Iterator<String> i = stack.iterator();
while (i.hasNext())
{
   String s = i.next();
   StdOut.println(s);
}
```

Stack iterator: linked-list implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

    public Iterator<Item> iterator() { return new ListIterator(); }

    private class ListIterator implements Iterator<Item>
    {
        private Node current = first;

        public boolean hasNext() {  return current != null;  }
        public void remove()     {  /* not supported */      }
        public Item next()
        {
            Item item = current.item;
            current   = current.next;
            return item;
        }
    }
}
```



Stack iterator: array implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    …

    public Iterator<Item> iterator() { return new ArrayIterator(); }

    private class ArrayIterator implements Iterator<Item>
    {
        private int i = N;

        public boolean hasNext() {  return i > 0;           }
        public void remove()     {  /* not supported */   }
        public Item next()       {  return s[--i];         }
    }
}
```

				i			N			
s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

Iteration: concurrent modification

Q. What if client modifies the data structure while iterating?

A. A fail-fast iterator throws a `ConcurrentModificationException`.

concurrent modification

```
for (String s : stack)
   stack.push(s);
```

To detect:

- Count total number of `push()` and `pop()` operations in `Stack`.
- Save current count in `*Iterator` subclass upon creation.
- Check that two values are still equal when calling `next()` and `hasNext()`.

- stacks
- dynamic resizing
- queues
- generics
- iterators
- **applications**

Summary

Three ADTs for processing collections of objects:

- Stack
- Queue
- Bag

Generic implementations

- reusable code (don't need new implementation for new type)
- compile-time type checks

Iteration

- stack: LIFO order
- queue: FIFO order
- bag: arbitrary order

easy implementation:
stack without pop() or queue without get()

Bag

API

`public class Bag<Item> implements Iterable<Item>`		
	`Bag()`	*create an empty bag*
`void`	`add(Item x)`	*add an item*
`int`	`size()`	*number of items in bag*
`Iterable<Item>`	`iterator()`	*iterator for all items in bag*

typical client
(average the numbers on StdIn)

```
public static void main(String[] args)
{
   Bag<Double> numbers = new Bag<Double>();
   while (!StdIn.isEmpty())
      numbers.add(StdIn.readDouble());
   int N = numbers.size();
   double sum = 0.0;
   for (Double s : numbers) sum += s;
   double avg = sum/N;
   StdOut.println("Average: " + avg);
}
```

Sweet spot: Save for iteration where order doesn't matter.

Queue

API

	public class Queue<Item> implements Iterable<Item>	
	`Queue()`	*create an empty queue*
`void`	`enqueue(Item x)`	*add an item*
`Item`	`dequeue()`	*remove the least recently added item*
`int`	`size()`	*number of items in queue*
`Iterable<Item>`	`iterator()`	*iterator for all items in queue*

typical client
(put the ints in a file into an array)

```
public static int[] readInts(String name)
{
   In in = new In(name);
   Queue<Integer> q = new Queue<Integer>();
   while (!in.isEmpty())
      q.enqueue(in.readInt());
   int N = q.size();
   int[] a = new int[N];
   for (int i = 0; i < N; i++)
      a[i] = q.dequeue();
   return a;
}
```

key point: don't need to know file size

Sweet spot: Save for later use where order does matter.

Stack

API

	public class Stack<Item> implements Iterable<Item>	
	`Stack()`	*create an empty stack*
`void`	`push(Item x)`	*add an item*
`Item`	`pop()`	*remove the most recently added item*
`int`	`size()`	*number of items in queue*
`Iterable<Item>`	`iterator()`	*iterator for all items in queue*

sample client
(print the strings on StdIn in reverse order)

```
public class Reverse
{
   public static void main(String[] args)
   {
      Stack<String> stack = new Stack<String>();
      while (!StdIn.isEmpty())
         stack.push(StdIn.readString());
      for (String s : stack)
         StdOut.println(s);
   }
}
```

Sweet spot: Support recursive computation (stay tuned).

Java collections library

List interface. `java.util.List` is API for ordered collection of items.

`public interface List<Item> implements Iterable<Item>`		
	`List()`	*create an empty list*
`boolean`	`isEmpty()`	*is the list empty?*
`int`	`size()`	*number of items*
`void`	`add(Item item)`	*append item to the end*
`Item`	`get(int index)`	*return item at given index*
`Item`	`remove(int index)`	*return and delete item at given index*
`boolean`	`contains(Item item)`	*does the list contain the given item?*
`Iteartor<Item>`	`iterator()`	*iterator over all items in the list*
`...`		

Implementations. `java.util.ArrayList` uses dynamic array; `java.util.LinkedList` uses linked list.

`java.util.Stack`.

- Supports `push()`, `pop()`, `size()`, `isEmpty()`, and iteration.
- Also implements `java.util.List` interface from previous slide, including, `get()`, `remove()`, and `contains()`.
- Bloated and poorly-designed API ⇒ don't use.

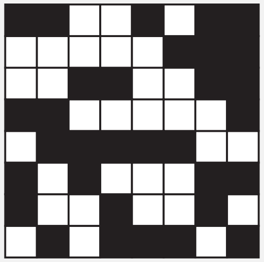
`java.util.Queue`. An interface, not an implementation of a queue.

Best practices. Use our implementations of `Stack`, `Queue`, and `Bag`.

War story (from COS 226)

Generate random open sites in an N-by-N percolation system.

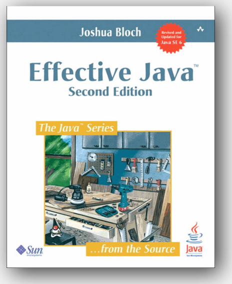
- Jenny: pick (i, j) at random; if already open, repeat.
  Takes ~ $c_1 N^2$ seconds.
- Kenny: create a `java.util.LinkedList` of N^2 closed sites.
  Pick an index at random and delete.
  Takes ~ $c_2 N^4$ seconds.



Kenny

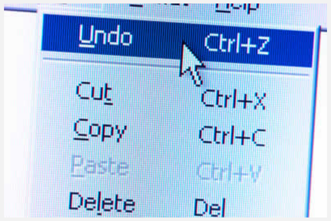
Lesson. Don't use a library until you understand its API!

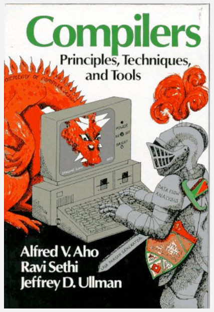
This course. Can't use a library until we've implemented it in class.



Stack applications

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.
- ...





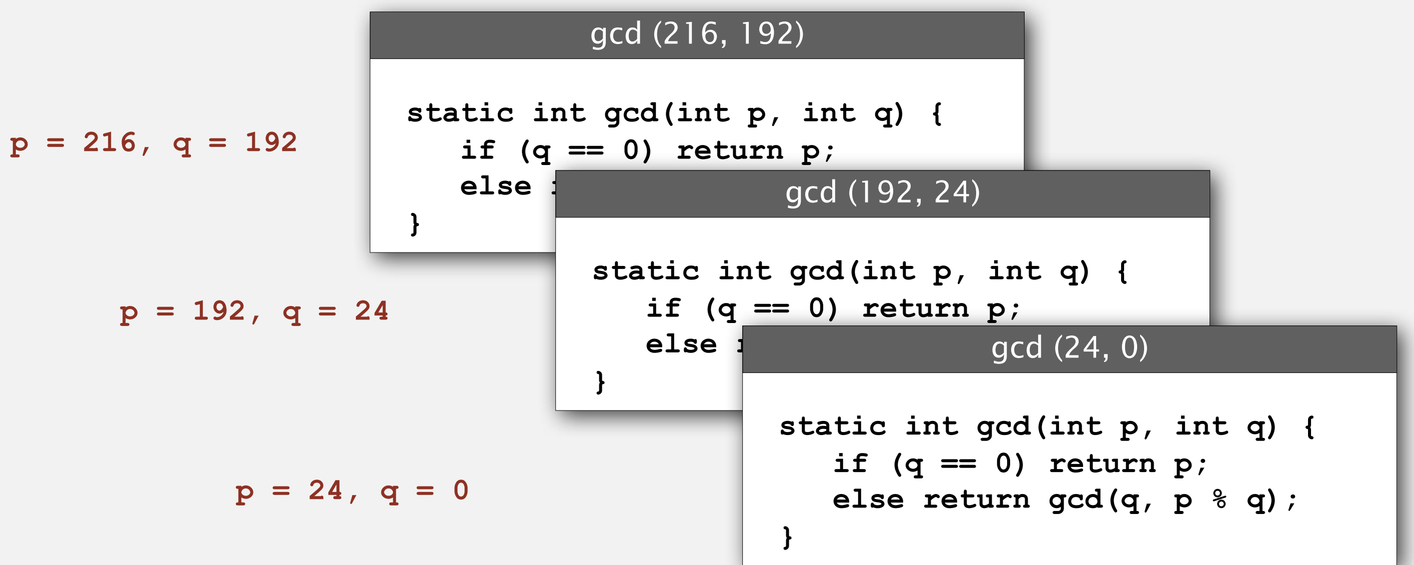
Function calls

How a compiler implements a function.

- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.

Note. Can always use an explicit stack to remove recursion.



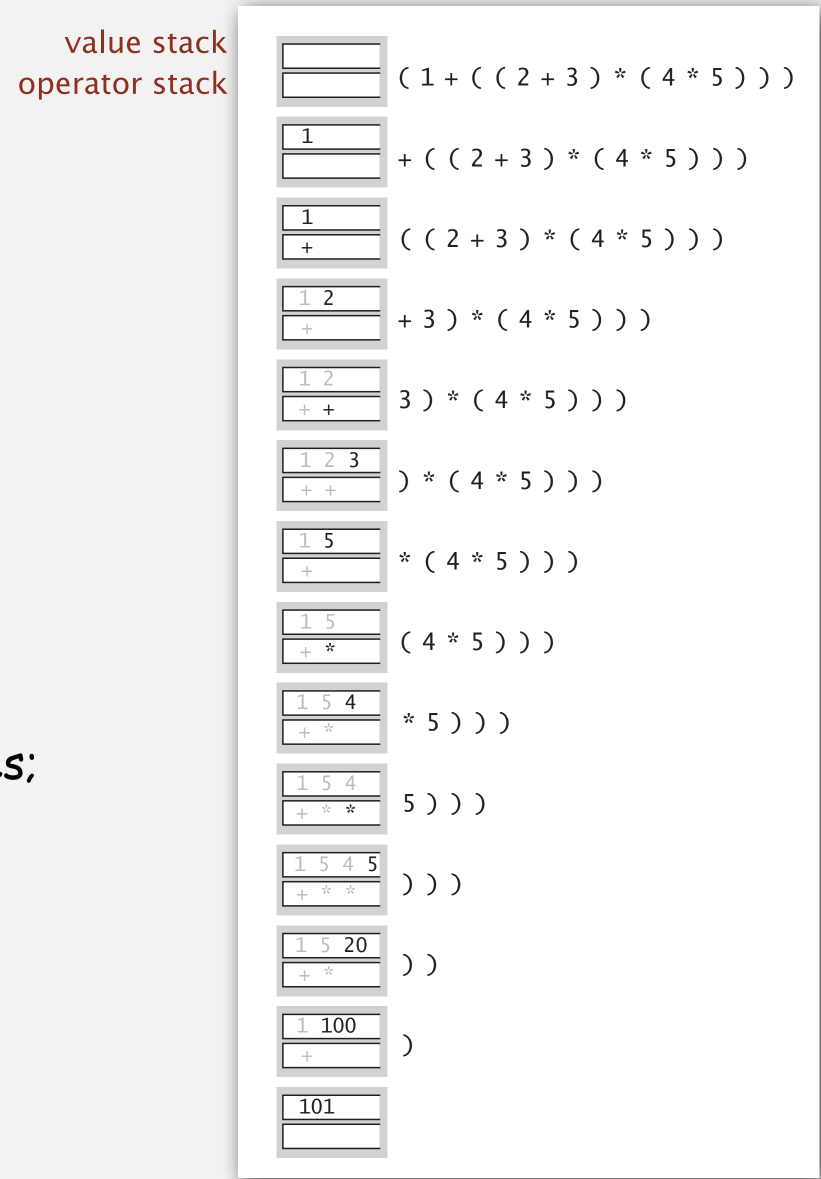
Goal. Evaluate infix expressions.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

operand operator

Two-stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.



Context. An interpreter!

Arithmetic expression evaluation

```
public class Evaluate
{
   public static void main(String[] args)
   {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")"))
         {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Q. Why correct?

A. When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity.

Stack-based programming languages

Observation 1. The 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

Observation 2. All of the parentheses are redundant!

```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, …

PostScript. [Warnock-Geschke 1980s]

- Postfix program code.
- Turtle graphics commands.
- Variables, types, text, loops, conditionals, functions, ...

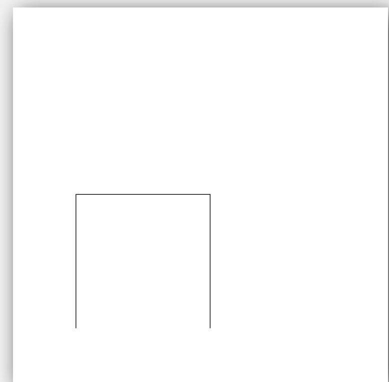
units are points (72 per inch)

```
%!
 100 100 moveto
 100 300 lineto
 300 300 lineto
 300 100 lineto
 stroke
```

define a path

draw the path

a PostScript program



its output

Simple virtual machine, but not a toy.

- Easy to specify published page.
- Easy to implement in printers.
- Revolutionized the publishing world.

PostScript

Page description language.

- Explicit stack.
- Full computational model
- Graphics engine.

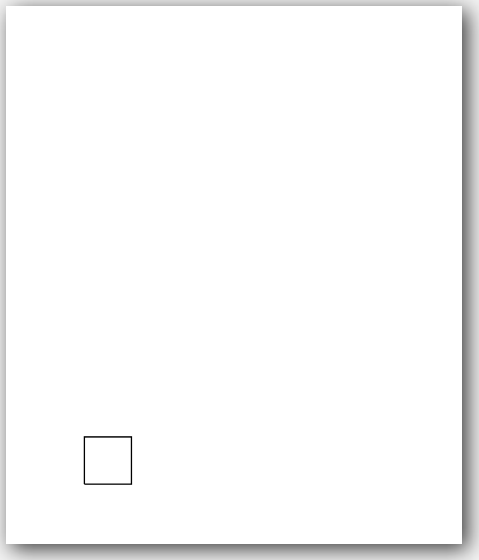
Basics.

- `%!`: "I am a PostScript program."
- Literal: "push me on the stack."
- Function calls take arguments from stack.
- Turtle graphics built in.

a PostScript program

```
%!
72 72 moveto
0 72 rlineto
72 0 rlineto
0 -72 rlineto
-72 0 rlineto
2 setlinewidth
stroke
```

its output



PostScript

Data types.

- Basic: integer, floating point, boolean, ...
- Graphics: font, path, curve, ...
- Full set of built-in operators.

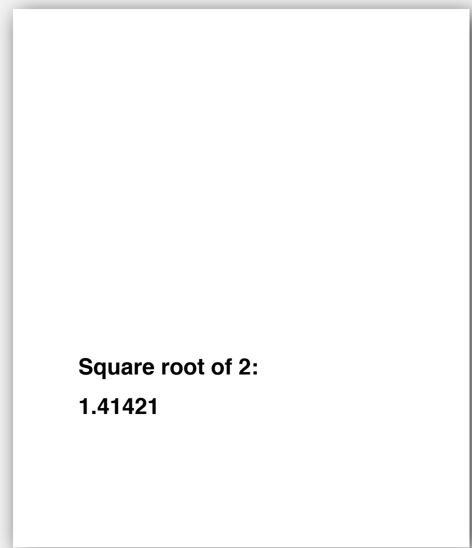
Text and strings.

- Full font support.
- **show** (display a string, using current font).
- **cvs** (convert anything to a string).

`System.out.print()`

`toString()`

```
%!
/Helvetica-Bold findfont 16 scalefont setfont
72 168 moveto
(Square root of 2:) show
72 144 moveto
2 sqrt 10 string cvs show
```



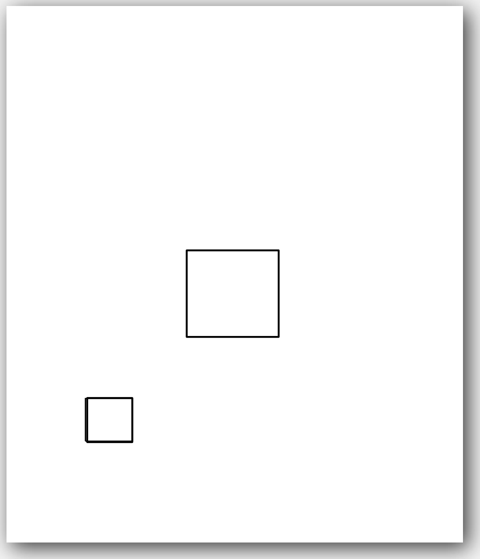
Variables (and functions).

- Identifiers start with `/`.
- `def` operator associates id with value.
- Braces.
- args on stack.

function definition →

function calls →

```
%!
/box
{
  /sz exch def
  0 sz rlineto
  sz 0 rlineto
  0 sz neg rlineto
  sz neg 0 rlineto
} def

72 144 moveto
72 box
288 288 moveto
144 box
2 setlinewidth
stroke
```



For loop.

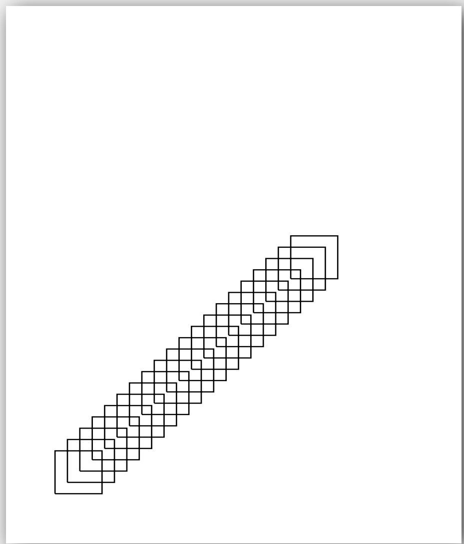
- "from, increment, to" on stack.
- Loop body in braces.
- `for` operator.

If-else conditional.

- Boolean on stack.
- Alternatives in braces.
- `if` operator.

... (hundreds of operators)

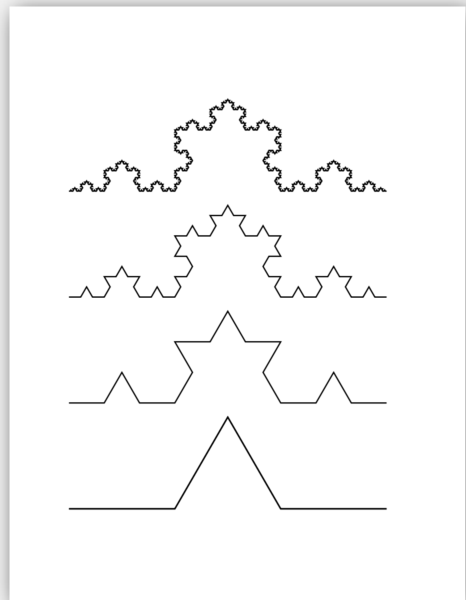
```
%!
\box
{
  ...
}

1 1 20
{ 19 mul dup 2 add moveto 72 box }
for
stroke
```

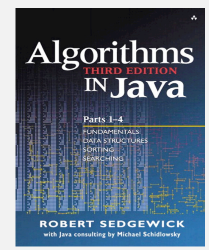


PostScript applications

Algorithms, 3^{rd} edition. Figures created directly in PostScript.

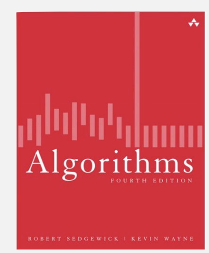
```
%!
72 72 translate

/kochR
  {
    2 copy ge { dup 0 rlineto }
      {
        3 div
        2 copy kochR 60 rotate
        2 copy kochR -120 rotate
        2 copy kochR 60 rotate
        2 copy kochR
      } ifelse
    pop pop
  } def

0   0 moveto   81 243 kochR
0  81 moveto   27 243 kochR
0 162 moveto    9 243 kochR
0 243 moveto    1 243 kochR
stroke
```





see page 218

Algorithms, 4^{th} edition. Figures created using enhanced version of `StdDraw` that saves to PostScript for vector graphics.



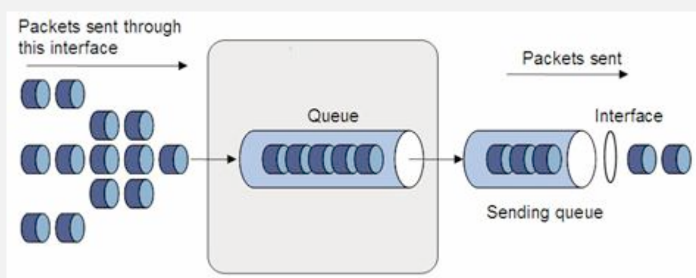
Queue applications

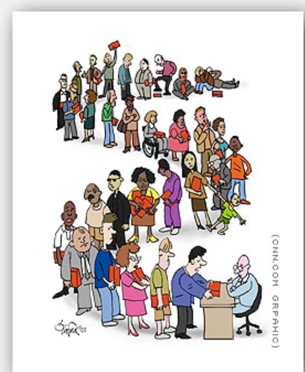
Familiar applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.





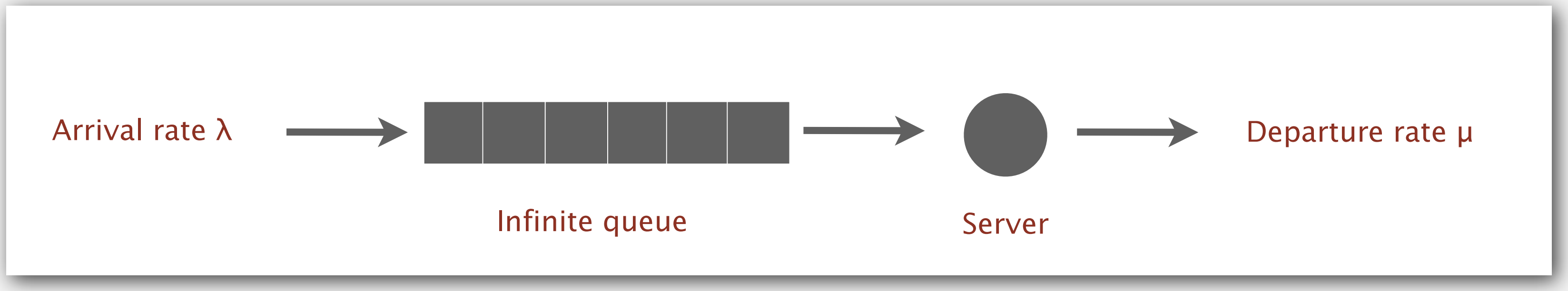
M/M/1 queuing model

M/M/1 queue.

- Customers arrive according to Poisson process at rate of λ per minute.
- Customers are serviced with rate of μ per minute.

interarrival time has exponential distribution $\Pr[X \leq x] = 1 - e^{-\lambda x}$

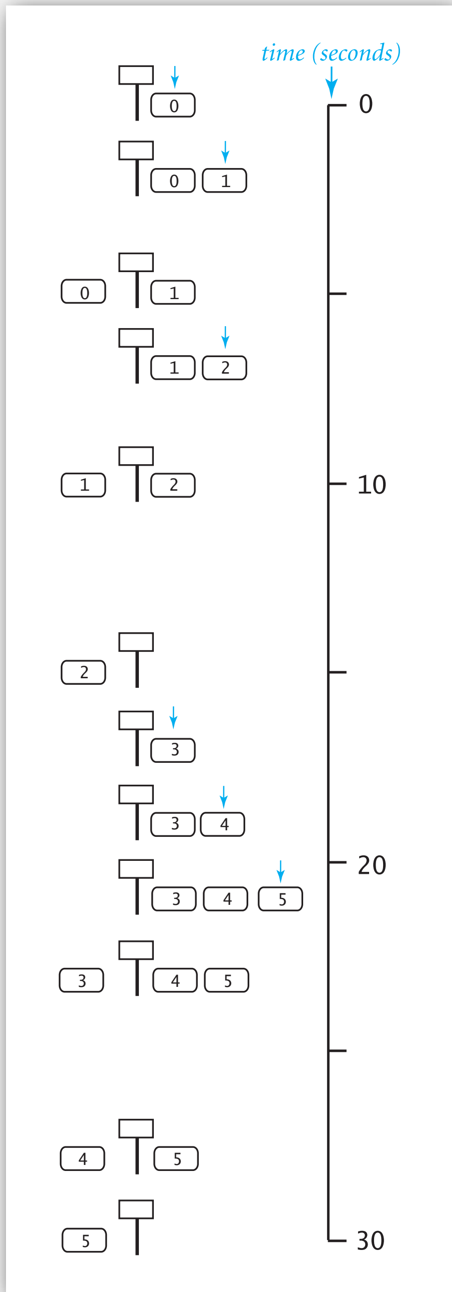
service time has exponential distribution $\Pr[X \leq x] = 1 - e^{-\mu x}$



Q. What is average wait time W of a customer in system?

Q. What is average number of customers L in system?

M/M/1 queuing model: example simulation



	arrival	departure	wait
0	0	5	5
1	2	10	8
2	7	15	8
3	17	23	6
4	19	28	9
5	21	30	9

M/M/1 queuing model: event-based simulation

```
public class MM1Queue
{
    public static void main(String[] args) {
        double lambda = Double.parseDouble(args[0]);   // arrival rate
        double mu     = Double.parseDouble(args[1]);   // service rate
        double nextArrival = StdRandom.exp(lambda);
        double nextService = nextArrival + StdRandom.exp(mu);

        Queue<Double> queue = new Queue<Double>();
        Histogram hist = new Histogram("M/M/1 Queue", 60);

        while (true)
        {
            while (nextArrival < nextService)
            {
                queue.enqueue(nextArrival);
                nextArrival += StdRandom.exp(lambda);
            }

            double arrival = queue.dequeue();
            double wait = nextService - arrival;
            hist.addDataPoint(Math.min(60,  (int)  (Math.round(wait))));
            if  (queue.isEmpty()) nextService = nextArrival + StdRandom.exp(mu);
            else                  nextService = nextService + StdRandom.exp(mu);
        }
    }
}
```

queue of arrival times

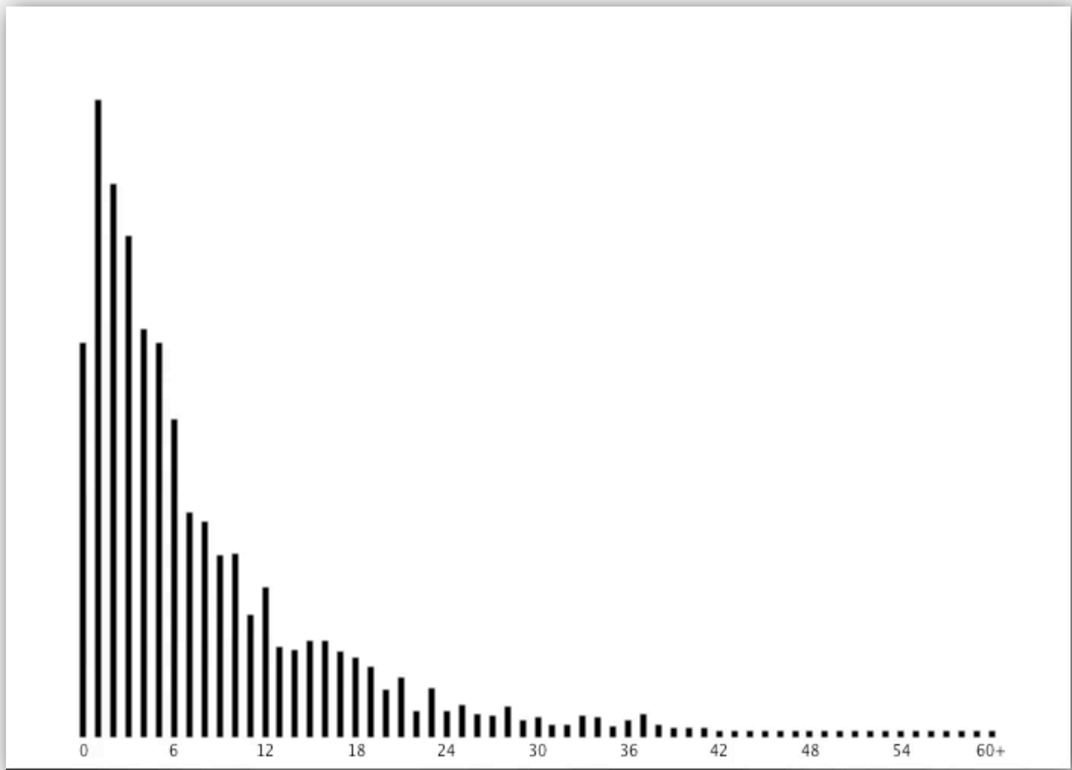
next event is an arrival

next event is a service completion

M/M/1 queuing model: experiments

Observation. If service rate μ is much larger than arrival rate λ, customers gets good service.

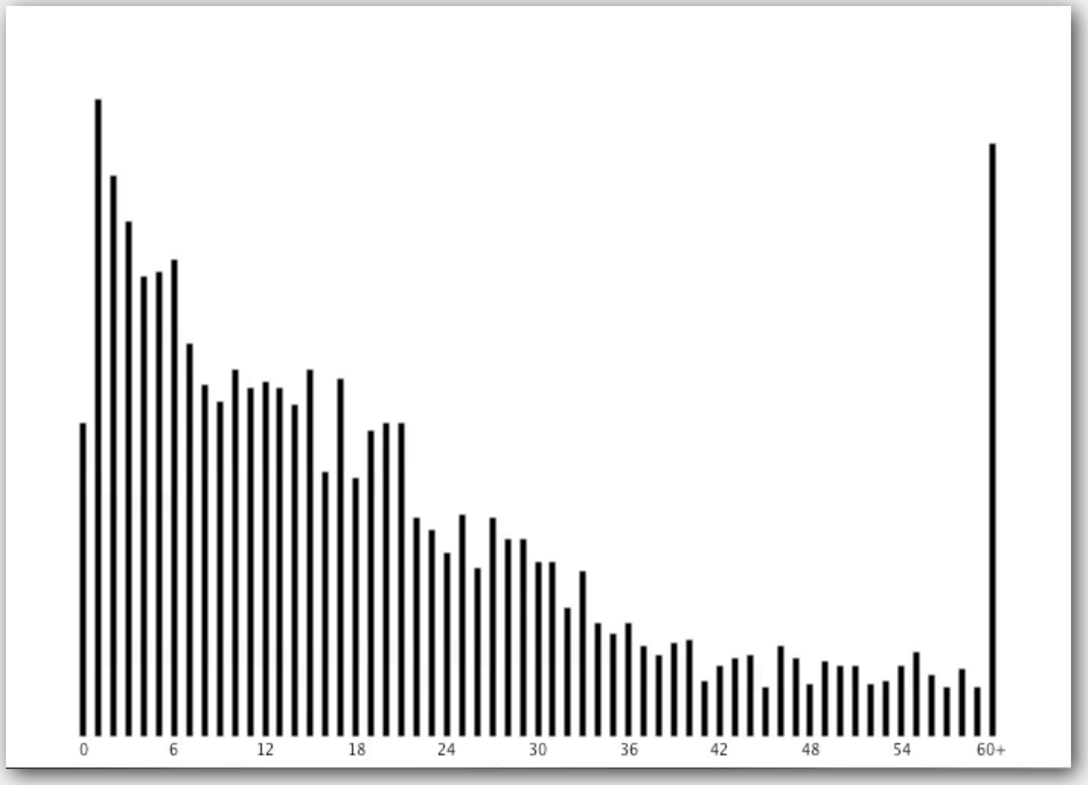
```
% java MM1Queue .2 .333
```



M/M/1 queuing model: experiments

Observation. As service rate μ approaches arrival rate λ, services goes to h***.

```
% java MM1Queue .2 .25
```



M/M/1 queuing model: experiments

Observation. As service rate μ approaches arrival rate λ, services goes to h***.

```
% java MM1Queue .2 .21
```

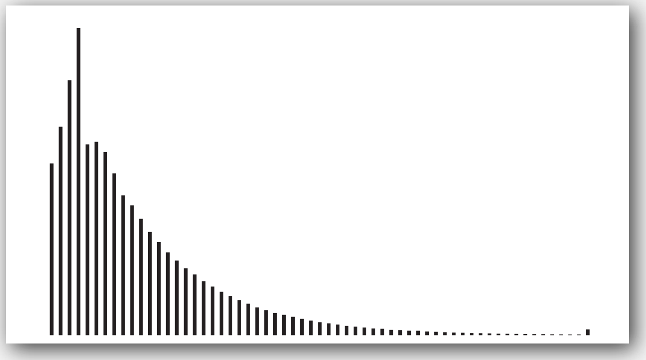
M/M/1 queuing model: analysis

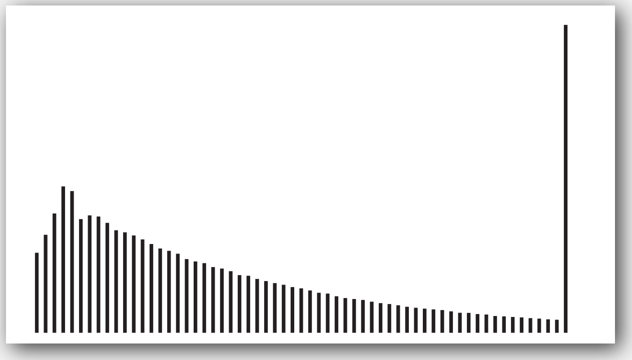
M/M/1 queue. Exact formulas known.

wait time W and queue length L approach infinity
as service rate approaches arrival rate

Little's Law

$$W = \frac{1}{\mu - \lambda}, \quad L = \lambda W$$





More complicated queueing models. Event-based simulation essential!

Queueing theory. See ORF 309.