COS 226

Algorithms and Data Structures
Princeton University
Spring 2011

Robert Sedgewick

Course Overview

- outline
- why study algorithms?
- usual suspects
- coursework
- resources

COS 226 course overview

What is COS 226?

- Intermediate-level survey course.
- Programming and problem solving with applications.
- Algorithm: method for solving a problem.
- Data structure: method to store information.

topic	data structures and algorithms
data types	stack, queue, union-find, priority queue
sorting	quicksort, mergesort, heapsort, radix sorts
searching	hash table, BST, red-black tree
graphs	BFS, DFS, Prim, Kruskal, Dijkstra
strings	KMP, regular expressions, TST, Huffman, LZW
advanced	B-tree, suffix arrays, maxflow, simplex

Why study algorithms?

Their impact is broad and far-reaching.

Internet. Web search, packet routing, distributed file sharing, ...

Biology. Human genome project, protein folding, ...

Computers. Circuit layout, file system, compilers, ...

Computer graphics. Movies, video games, virtual reality, ...

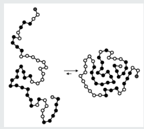
Security. Cell phones, e-commerce, voting machines, ...

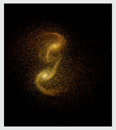
Multimedia. CD player, DVD, MP3, JPG, DivX, HDTV, ...

Transportation. Airline crew scheduling, map routing, ...

Physics. N-body simulation, particle collision simulation, ...

...

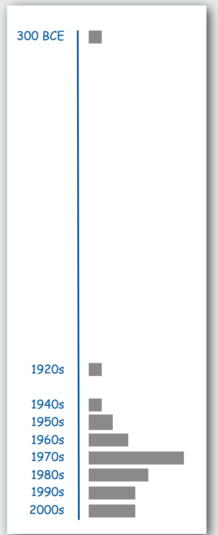





Why study algorithms?

Old roots, new opportunities.

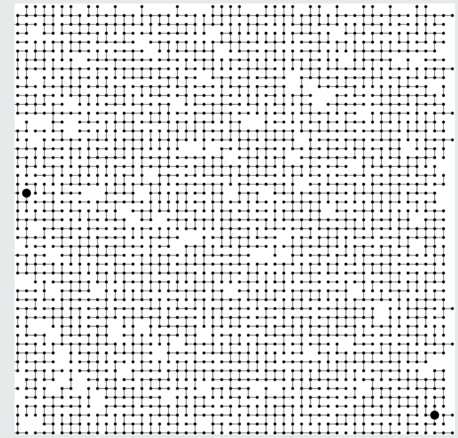
- Study of algorithms dates at least to Euclid.
- Some important algorithms were discovered by undergraduates!



Why study algorithms?

To solve problems that could not otherwise be addressed.

Ex. Network connectivity. [stay tuned]



Why study algorithms?

For intellectual stimulation.

> *"For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense, and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing."* — Francis Sullivan

> *"An algorithm must be seen to be believed."* — D. E. Knuth

Why study algorithms?

They may unlock the secrets of life and of the universe.

Computational models are replacing mathematical models in scientific inquiry.

$$E = mc^2$$
$$F = ma$$
$$F = \frac{G m_1 m_2}{r^2}$$
$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \Psi(r) = E \, \Psi(r)$$

20th century science
(formula based)

```
for (double t = 0.0; true; t = t + dt)
   for (int i = 0; i < N; i++)
   {
      bodies[i].resetForce();
      for (int j = 0; j < N; j++)
         if (i != j)
            bodies[i].addForce(bodies[j]);
   }
```

21st century science
(algorithm based)

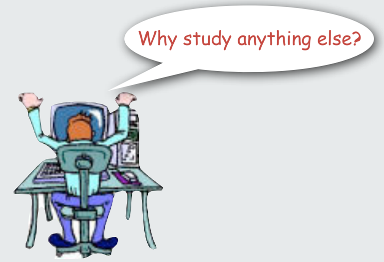
> *"Algorithms: a common language for nature, human, and computer."* — Avi Wigderson

Why study algorithms?

For fun and profit.

Why study algorithms?

- Their impact is broad and far-reaching.
- Old roots, new opportunities.
- To solve problems that could not otherwise be addressed.
- For intellectual stimulation.
- They may unlock the secrets of life and of the universe.
- For fun and profit.



The usual suspects

Lectures. Introduce new material.

Precepts. Discussion, problem-solving, background for programming assignment.
First precept meets this week

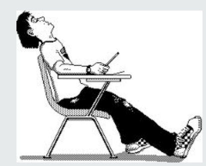
What	When	Where	Who	Office	Office Hours
L01	MW 11-12:20	Friend 101	Prof. Sedgewick	CS 319	W 1:30-2:30
P01	Th 12:30	CS 102	Maia Ginsburg (lead)	CS 205	*
P01A	Th 12:30	Friend 108	Josh Kroll	Sherrard 320	*
P01B	Th 12:30	Friend 109	Dave Walker	CS 211	*
P03	Th 3:30	Friend 109	Maia Ginsburg	CS 205	*
P02	F 11	CS 102	RobSchapire	CS 407	*
P02A	F 11	Friend 109	Aman Dhesi	CS 103B	*

* see web page for updates

UTAs in Friend 106/107 (check web page for hours).

All questions answered

90-minute lecture? Soporiphic!



Each lecture will include a 5-minute break for a brief AQA session.

- ask a question!
- email a question
- preferably not about course content

All students must meet RS at office hours

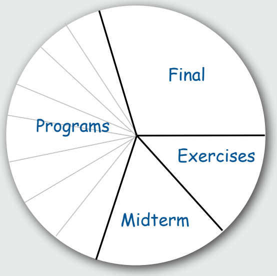
- W 1:30 to 2:30 (more after break)

Coursework and grading

8 programming assignments. 40%
- Electronic submission.
- Due 11pm, starting Tuesday 2/9.

Exercises. 15%
- Due at beginning of lecture, starting Monday 2/8.

Exams.
- Closed-book with cheatsheet.
- Midterm. 15%
- Final. 30%

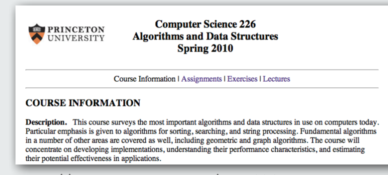


Staff discretion. To adjust borderline cases.

everyone needs to
- attend precept
- meet me in office hours

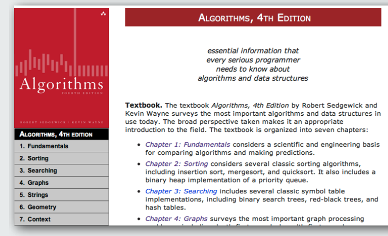
Resources (web)

Course content.
- Course info.
- Exercises.
- Lecture slides.
- Programming assignments.
- Submit assignments.



http://www.princeton.edu/~cos226

Booksite (under construction).
- Brief summary of content.
- Download code from lecture, book, exercises, examples
- Links to references



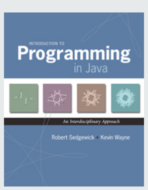
introcs.cs.princeton.edu
algs4.cs.princeton.edu

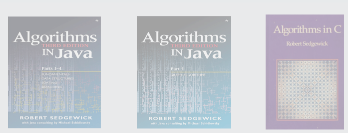
Resources (books)

Required readings.
- Algorithms 4th edition.
- to be published in early March

Preliminary edition (Fall 2010)
- on reserve in the library
- or, borrow a copy from a friend
- see the web for some Algs4 excerpts

Recommended Java reference.
- Introduction to Programming in Java.





What's ahead?

Lecture 1. Union find. ← today
Precept 1. Meets Thursday or Friday.
Lecture 2. Analysis of algorithms.

Exercise 1. Due in precept Thursday or Friday.
Assignment 1. Due via electronic submission at 11pm on Tuesday.

Right course? See Maia after lecture.
Placed out of COS 126? Review the following in Intro to Programming in Java
- Section 1.5: I/O and command-line interface.
- Section 4.1: Analysis of algorithms.
- Section 4.3: Stacks and queues.

Not registered? Go to any precept this week.
Change precept? Use SCORE. ← see Colleen Kenny-McGinley in CS 210 if the only precept you can attend is closed

1.5 Case Study

- dynamic connectivity
- quick find
- quick union
- improvements
- applications

Algorithms in Java, 4th Edition · Robert Sedgewick and Kevin Wayne · Copyright © 2009 · January 30, 2011 12:04:28 PM

Subtext of today's lecture (and this course)

Steps to developing a usable algorithm.

- Model the problem.
- Find an algorithm to solve it.
- Fast enough? Fits in memory?
- If not, figure out why.
- Find a way to address the problem.
- Iterate until satisfied.

The scientific method.

Mathematical analysis.

- dynamic connectivity
- quick find
- quick union
- improvements
- applications

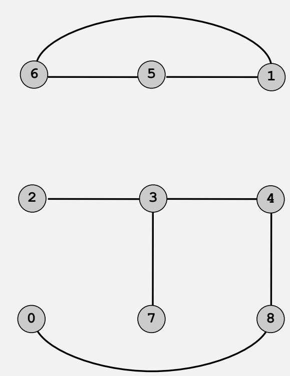
Dynamic connectivity

Given a set of objects

- Union: connect two objects.
- Find: is there a path connecting the two objects?

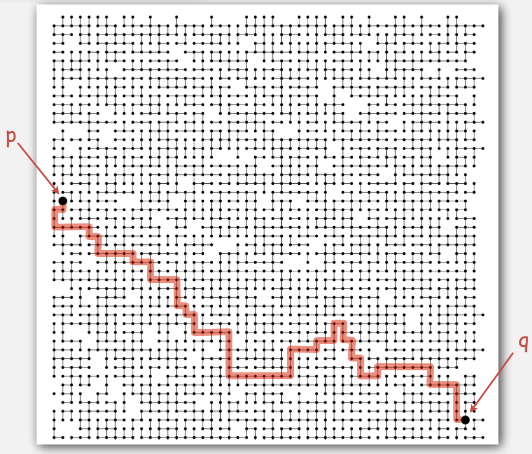
more difficult problem: find the path

```
union(3, 4)
union(8, 0)
union(2, 3)
union(5, 6)
 find(0, 2)     no
 find(2, 4)     yes
union(5, 1)
union(7, 3)
union(1, 6)
union(4, 8)
 find(0, 2)     yes
 find(2, 4)     yes
```



Network connectivity: larger example

Q. Is there a path from p to q?



A. Yes. ← but finding the path is more difficult: stay tuned (Chapter 4)

Modeling the objects

Dynamic connectivity applications involve manipulating objects of all types.

- Variable name aliases.
- Pixels in a digital photo.
- Computers in a network.
- Web pages on the Internet.
- Transistors in a computer chip.
- Metallic sites in a composite system.
- Terrorists communicating to develop a plot.

When programming, convenient to name objects 0 to N-1.

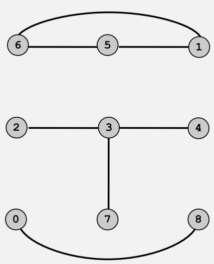
- Use integers as array index.
- Suppress details not relevant to union-find.

can use symbol table to translate from object names to integers (stay tuned)

Modeling the connections

Transitivity. If `p` is connected to `q` and `q` is connected to `r`, then `p` is connected to `r`.

Connected components. Maximal set of objects that are mutually connected.



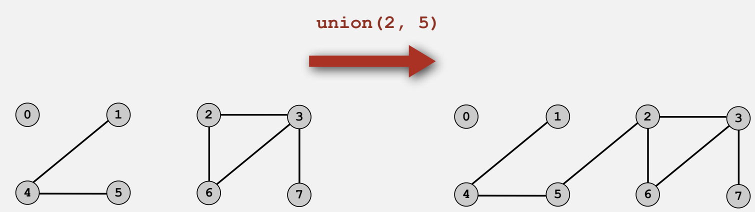
{ 1 5 6 } { 2 3 4 7 } { 0 8 }

connected components

Implementing the operations

Find query. Check if two objects are in the same set.

Union command. Replace sets containing two objects with their union.



{ 0 } { 1 4 5 } { 2 3 6 7 }

3 connected components

{ 0 } { 1 2 3 4 5 6 7 }

2 connected components

Union-find data type (API)

Goal. Design efficient data structure for union-find.

- Number of objects N can be huge.
- Number of operations M can be huge.
- Find queries and union commands may be intermixed.

public class UF		
	`UF(int N)`	*create union-find data structure with N objects and no connections*
`boolean`	`find(int p, int q)`	*are p and q in the same component?*
`void`	`union(int p, int q)`	*add connection between p and q*
`int`	`count()`	*number of components*

Dynamic-connectivity client

- Read in number of objects N from standard input.
- Repeat:
  - read in pair of integers from standard input
  - write out pair if they are not already connected

```
public static void main(String[] args)
{
   int N = StdIn.readInt();
   UF uf = new UF(N);
   while (!StdIn.isEmpty())
   {
      int p = StdIn.readInt();
      int q = StdIn.readInt();
      if (uf.find(p, q)) continue;
      uf.union(p, q);
      StdOut.println(p + " " + q);
   }
}
```

```
% more tiny.txt
10
4 3
3 8
6 5
9 4
2 1
8 9
5 0
7 2
6 1
1 0
6 7
```


- dynamic connectivity
- **quick find**
- quick union
- improvements
- applications

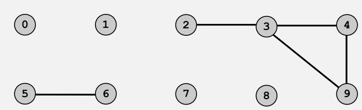
Quick-find [eager approach]

Data structure.

- Integer array `id[]` of size `N`.
- Interpretation: `p` and `q` are connected if they have the same id.

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	9	9	9	6	6	7	8	9

5 and 6 are connected
2, 3, 4, and 9 are connected



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5 and 6 are connected
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Find. Check if `p` and `q` have the same id.

`id[3] = 9; id[6] = 6`
3 and 6 not connected

Quick-find [eager approach]

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Find. Check if `p` and `q` have the same id.

`id[3] = 9; id[6] = 6`
3 and 6 not connected

Union. To merge sets containing `p` and `q`, change all entries with `id[p]` to `id[q]`.

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	6	6	6	6	6	7	8	6

union of 3 and 6
2, 3, 4, 5, 6, and 9 are connected

problem: many values can change

Quick-find example

p	q	id[0]	id[1]	id[2]	id[3]	id[4]	id[5]	id[6]	id[7]	id[8]	id[9]
4	3	0	1	2	3	4	5	6	7	8	9
		0	1	2	3	3	5	6	7	8	9
3	8	0	1	2	3	3	5	6	7	8	9
		0	1	2	8	8	5	6	7	8	9
6	5	0	1	2	8	8	5	6	7	8	9
		0	1	2	8	8	5	5	7	8	9
9	4	0	1	2	8	8	5	5	7	8	9
		0	1	2	8	8	5	5	7	8	8
2	1	0	1	2	8	8	5	5	7	8	8
		0	1	1	8	8	5	5	7	8	8
8	9	0	1	1	8	8	5	5	7	8	8
5	0	0	1	1	8	8	5	5	7	8	8
		0	1	1	8	8	0	0	7	8	8
7	2	0	1	1	8	8	0	0	7	8	8
		0	1	1	8	8	0	0	1	8	8
6	1	0	1	1	8	8	0	0	1	8	8
		1	1	1	8	8	1	1	1	8	8
1	0	1	1	1	8	8	1	1	1	8	8
6	7	1	1	1	8	8	1	1	1	8	8

`id[p]` and `id[q]` differ, so `union()` changes entries equal to `id[p]` to `id[q]` (in red)

`id[p]` and `id[q]` match, so no change

Quick-find: Java implementation

```
public class QuickFindUF
{
   private int[] id;

   public QuickFindUF(int N)
   {
      id = new int[N];
      for (int i = 0; i < N; i++)
         id[i] = i;
   }

   public boolean find(int p, int q)
   {  return id[p] == id[q];  }

   public void union(int p, int q)
   {
      int pid = id[p];
      int qid = id[q];
      for (int i = 0; i < id.length; i++)
         if (id[i] == pid) id[i] = qid;
   }
}
```

set id of each object to itself (N array accesses)

check whether `p` and `q` are in the same component (2 array accesses)

change all entries with `id[p]` to `id[q]` (linear number of array accesses)

Quick-find is too slow

Cost model. Number of array accesses (for read or write).

algorithm	init	union	find
quick-find	N	N	1

Quick-find defect.

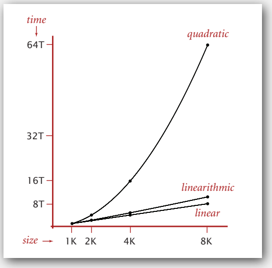
- Union too expensive.
- Trees are flat, but too expensive to keep them flat.
- Ex. Takes N^2 array accesses to process sequence of N union commands on N objects.

Quadratic algorithms do not scale

Rough standard (for now).

- 10^9 operations per second.
- 10^9 words of main memory.
- Touch all words in approximately 1 second.

a truism (roughly) since 1950 !



Ex. Huge problem for quick-find.

- 10^9 union commands on 10^9 objects.
- Quick-find takes more than 10^{18} operations.
- 30+ years of computer time!

Paradoxically, quadratic algorithms get worse with newer equipment.

- New computer may be 10x as fast.
- But, has 10x as much memory so problem may be 10x bigger.
- With quadratic algorithm, takes 10x as long!

- dynamic connectivity
- quick find
- **quick union**
- improvements
- applications

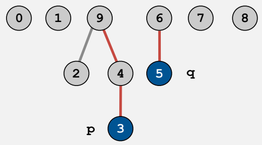
Quick-union [lazy approach]

Data structure.

- Integer array `id[]` of size `N`.
- Interpretation: `id[i]` is parent of `i`.
- Root of `i` is `id[id[id[...id[i]...]]]`.

keep going until it doesn't change

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	9	4	9	6	6	7	8	9



3's root is 9; 5's root is 6

Quick-union [lazy approach]

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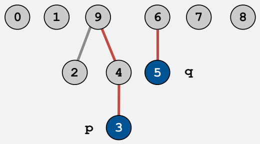

3's root is 9; 5's root is 6
3 and 5 are not connected

Find. Check if `p` and `q` have the same root.

Quick-union [lazy approach]

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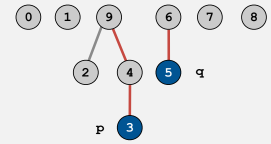


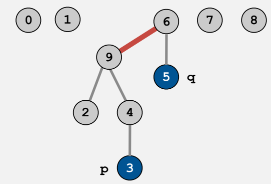
3's root is 9; 5's root is 6
3 and 5 are not connected

Find. Check if `p` and `q` have the same root.

Union. To merge sets containing `p` and `q`, set the id of `p`'s root to the id of `q`'s root.

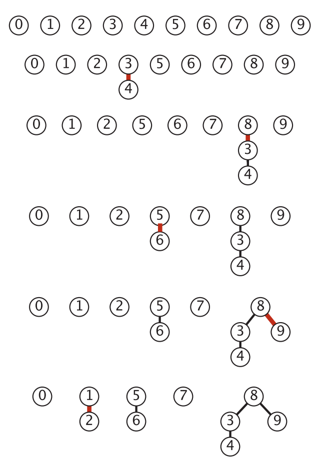
i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	9	4	9	6	6	7	8	6

only one value changes



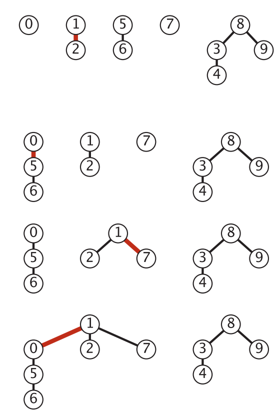
Quick-union example

p	q	id[0]	id[1]	id[2]	id[3]	id[4]	id[5]	id[6]	id[7]	id[8]	id[9]
4	3	0	1	2	3	4	5	6	7	8	9
		0	1	2	3	3	5	6	7	8	9
3	8	0	1	2	3	3	5	6	7	8	9
		0	1	2	8	3	5	6	7	8	9
6	5	0	1	2	8	3	5	6	7	8	9
		0	1	2	8	3	5	5	7	8	9
9	4	0	1	2	8	3	5	5	7	8	9
		0	1	2	8	3	5	5	7	8	8
2	1	0	1	2	8	3	5	5	7	8	8
		0	1	1	8	3	5	5	7	8	8



Quick-union example

p	q	id[0]	id[1]	id[2]	id[3]	id[4]	id[5]	id[6]	id[7]	id[8]	id[9]
8	9	0	1	1	8	3	5	5	7	8	8
5	0	0	1	1	8	3	5	5	7	8	8
		0	1	1	8	3	0	5	7	8	8
7	2	0	1	1	8	3	0	5	7	8	8
		0	1	1	8	3	0	5	1	8	8
6	1	0	1	1	8	3	0	5	1	8	8
		1	1	1	8	3	0	5	1	8	8
1	0	1	1	1	8	3	0	5	1	8	8
6	7	1	1	1	8	3	0	5	1	8	8



Quick-union: Java implementation

```
public class QuickUnionUF
{
   private int[] id;

   public QuickUnionUF(int N)
   {
      id = new int[N];
      for (int i = 0; i < N; i++) id[i] = i;
   }

   private int root(int i)
   {
      while (i != id[i]) i = id[i];
      return i;
   }

   public boolean find(int p, int q)
   {
      return root(p) == root(q);
   }

   public void union(int p, int q)
   {
      int i = root(p), j = root(q);
      id[i] = j;
   }
}
```

set id of each object to itself (N array accesses)

chase parent pointers until reach root (depth of i array accesses)

check if p and q have same root (depth of p and q array accesses)

change root of p to point to root of q (depth of p and q array accesses)

Quick-union is also too slow

Cost model. Number of array accesses (for read or write).

algorithm	init	union	find
quick-find	N	N	1
quick-union	N	N †	N

← worst case

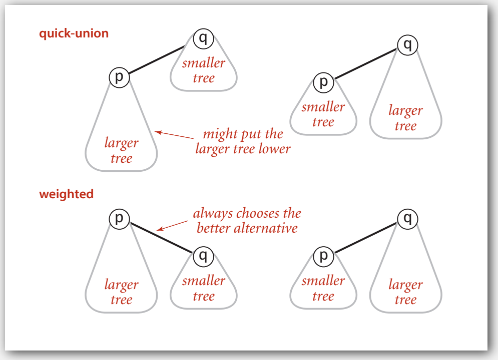
† includes cost of finding root

Quick-find defect.
- Union too expensive (N array accesses).
- Trees are flat, but too expensive to keep them flat.

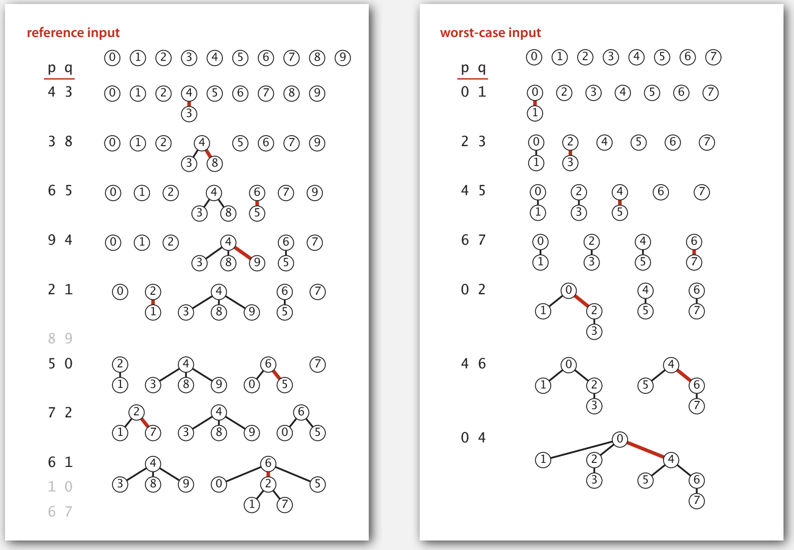
Quick-union defect.
- Trees can get tall.
- Find too expensive (could be N array accesses).

Improvement 1: weighting

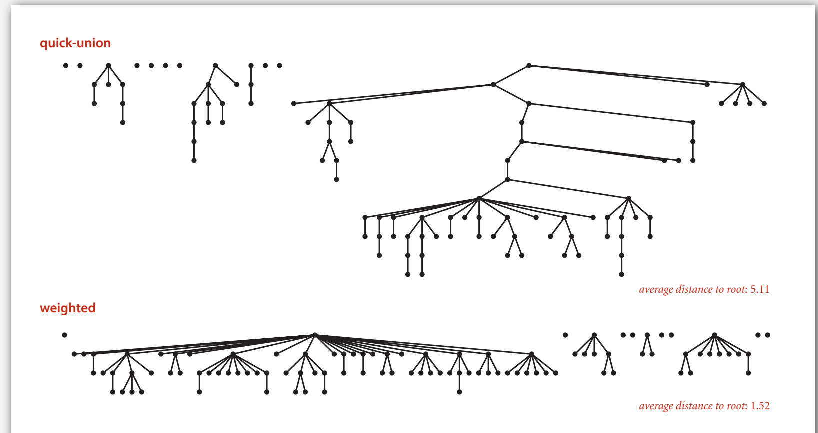
Weighted quick-union.
- Modify quick-union to avoid tall trees.
- Keep track of size of each tree (number of objects).
- Balance by linking small tree below large one.



Weighted quick-union examples



Quick-union and weighted quick-union example



Quick-union and weighted quick-union (100 sites, 88 `union()` operations)

Weighted quick-union: Java implementation

Data structure. Same as quick-union, but maintain extra array `sz[i]` to count number of objects in the tree rooted at `i`.

Find. Identical to quick-union.

```
return root(p) == root(q);
```

Union. Modify quick-union to:

- Merge smaller tree into larger tree.
- Update the `sz[]` array.

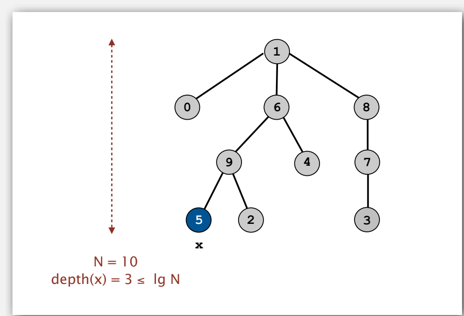
```
int i = root(p);
int j = root(q);
if  (sz[i] < sz[j]) { id[i] = j; sz[j] += sz[i]; }
else                { id[j] = i; sz[i] += sz[j]; }
```


Weighted quick-union analysis

Running time.

- Find: takes time proportional to depth of p and q.
- Union: takes constant time, given roots.

Proposition. Depth of any node x is at most $\lg N$.



Weighted quick-union analysis

Running time.

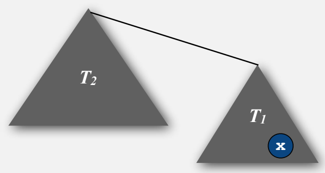
- Find: takes time proportional to depth of p and q.
- Union: takes constant time, given roots.

Proposition. Depth of any node x is at most $\lg N$.

Pf. When does depth of x increase?

Increases by 1 when tree T_1 containing x is merged into another tree T_2.

- The size of the tree containing x at least doubles since $|T_2| \geq |T_1|$.
- Size of tree containing x can double at most $\lg N$ times. Why?



Weighted quick-union analysis

Running time.

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- Union: takes constant time, given roots.

Proposition. Depth of any node x is at most $\lg N$.

algorithm	init	union	find
quick-find	N	N	1
quick-union	N	N †	N
weighted QU	N	lg N †	lg N

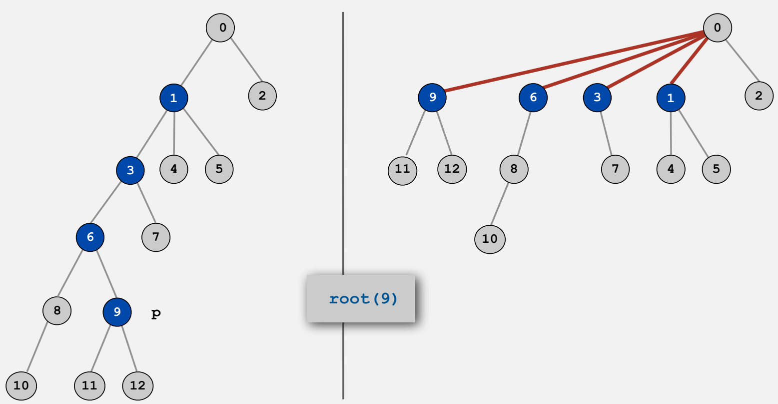
† includes cost of finding root

Q. Stop at guaranteed acceptable performance?

A. No, easy to improve further.

Improvement 2: path compression

Quick union with path compression. Just after computing the root of p, set the id of each examined node to point to that root.



Path compression: Java implementation

Standard implementation: add second loop to `find()` to set the `id[]` of each examined node to the root.

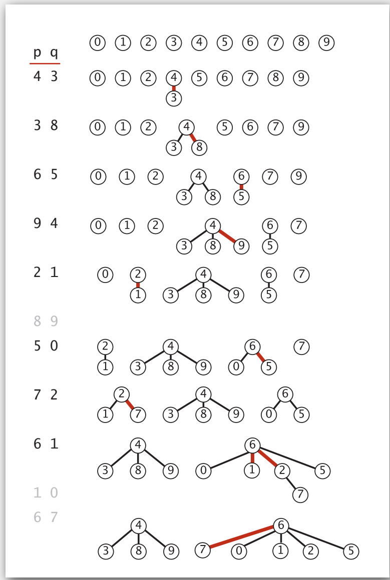
Simpler one-pass variant: halve the path length by making every other node in path point to its grandparent.

```
public int root(int i)
{
   while (i != id[i])
   {
      id[i] = id[id[i]];
      i = id[i];
   }
   return i;
}
```

only one extra line of code !

In practice. No reason not to! Keeps tree almost completely flat.

Weighted quick-union with path compression example



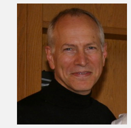
1 linked to 6 because of path compression

7 linked to 6 because of path compression

Weighted quick-union with path compression: amortized analysis

Proposition. Starting from an empty data structure, any sequence of M union-find operations on N objects makes at most proportional to $N + M \lg^* N$ array accesses.

- Proof is very difficult.
- Can be improved to $N + M\,\alpha(M, N)$. ← see COS 423
- But the algorithm is still simple!



Bob Tarjan
(Turing Award '86)

Linear-time algorithm for M union-find ops on N objects?

- Cost within constant factor of reading in the data.
- In theory, WQUPC is not quite linear.
- In practice, WQUPC is linear.

because $\lg^* N$ is a constant in this universe

N	lg* N
1	0
2	1
4	2
16	3
65536	4
2^{65536}	5

lg* function

Amazing fact. No linear-time algorithm exists.

in "cell-probe" model of computation

Summary

Bottom line. WQUPC makes it possible to solve problems that could not otherwise be addressed.

algorithm	worst-case time
quick-find	M N
quick-union	M N
weighted QU	N + M log N
QU + path compression	N + M log N
weighted QU + path compression	N + M lg* N

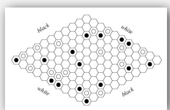
M union-find operations on a set of N objects

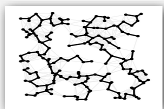
Ex. [10^9 unions and finds with 10^9 objects]

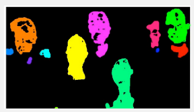
- WQUPC reduces time from 30 years to 6 seconds.
- Supercomputer won't help much; good algorithm enables solution.

Union-find applications

- Percolation.
- Games (Go, Hex).
- ✓ Network connectivity.
- Least common ancestor.
- Equivalence of finite state automata.
- Hoshen-Kopelman algorithm in physics.
- Hinley-Milner polymorphic type inference.
- Kruskal's minimum spanning tree algorithm.
- Compiling equivalence statements in Fortran.
- Morphological attribute openings and closings.
- Matlab's `bwlabel()` function in image processing.



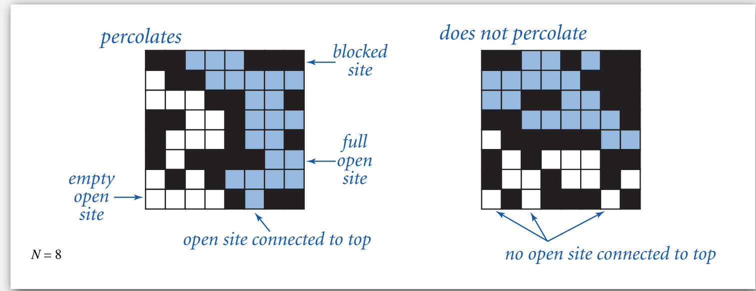




Percolation

A model for many physical systems:

- N-by-N grid of sites.
- Each site is open with probability p (or blocked with probability $1 - p$).
- System percolates if top and bottom are connected by open sites.



Percolation

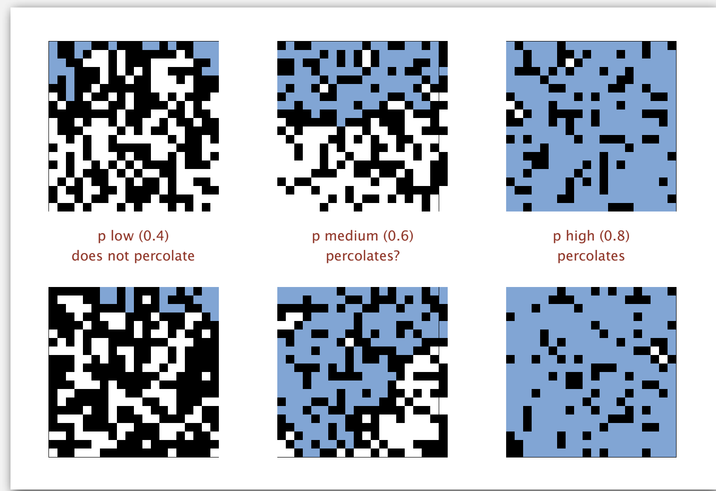
A model for many physical systems:

- N-by-N grid of sites.
- Each site is open with probability p (or blocked with probability $1 - p$).
- System percolates if top and bottom are connected by open sites.

model	system	vacant site	occupied site	percolates
electricity	material	conductor	insulated	conducts
fluid flow	material	empty	blocked	porous
social interaction	population	person	empty	communicates

Likelihood of percolation

Depends on site vacancy probability p.



p low (0.4)
does not percolate

p medium (0.6)
percolates?

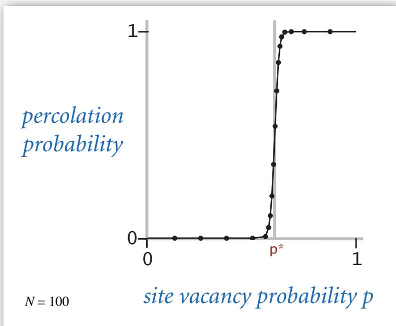
p high (0.8)
percolates

Percolation phase transition

When N is large, theory guarantees a sharp threshold p^*.

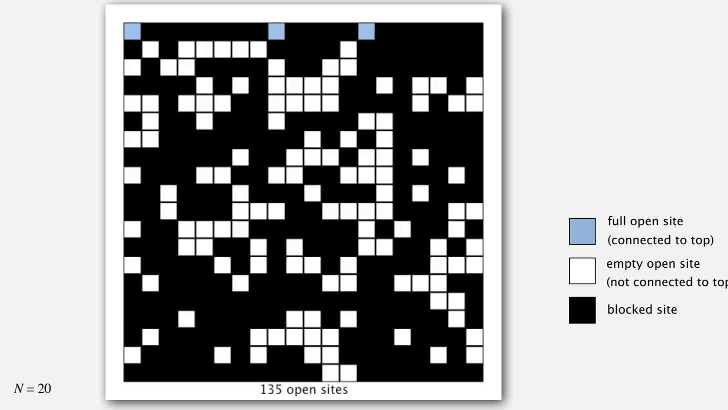
- $p > p^*$: almost certainly percolates.
- $p < p^*$: almost certainly does not percolate.

Q. What is the value of p^* ?



Monte Carlo simulation

- Initialize N-by-N whole grid to be blocked.
- Declare random sites open until top connected to bottom.
- Vacancy percentage estimates p^*.

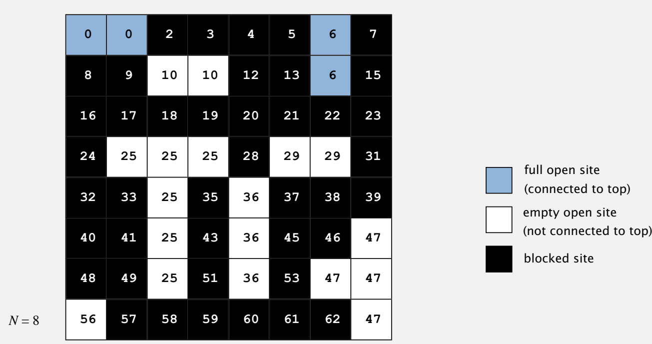


UF solution to find percolation threshold

How to check whether system percolates?

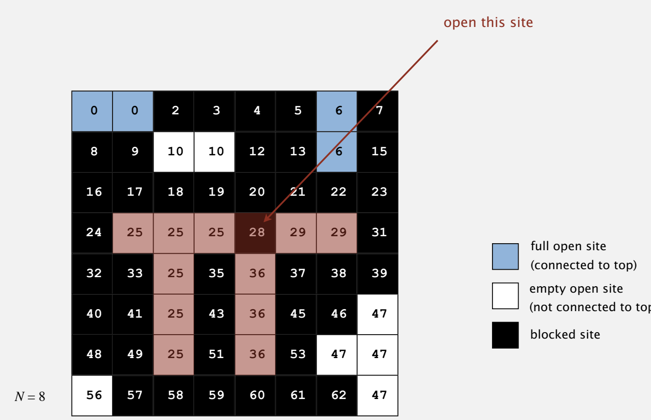
- Create an object for each site.
- Sites are in same set if connected by open sites.
- Percolates if any site in top row is in same set as any site in bottom row.

brute force algorithm: check all N^2 pairs



UF solution to find percolation threshold

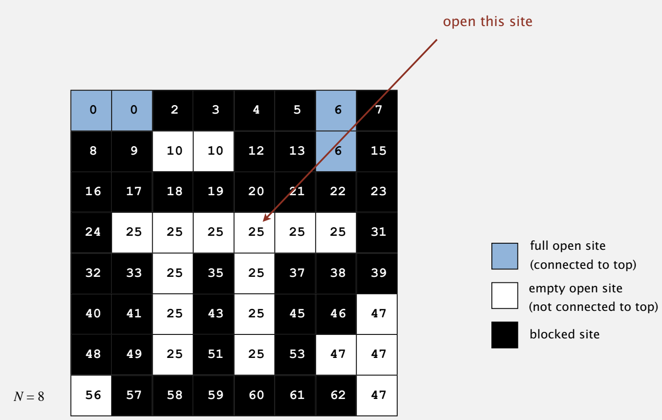
Q. How to declare a new site open?



UF solution to find percolation threshold

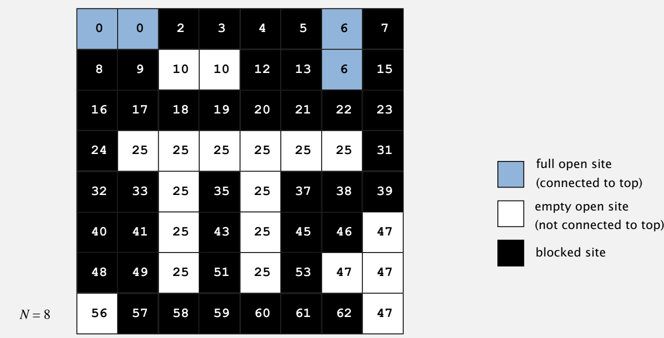
Q. How to declare a new site open?

A. Take union of new site and all adjacent open sites.



UF solution: a critical optimization

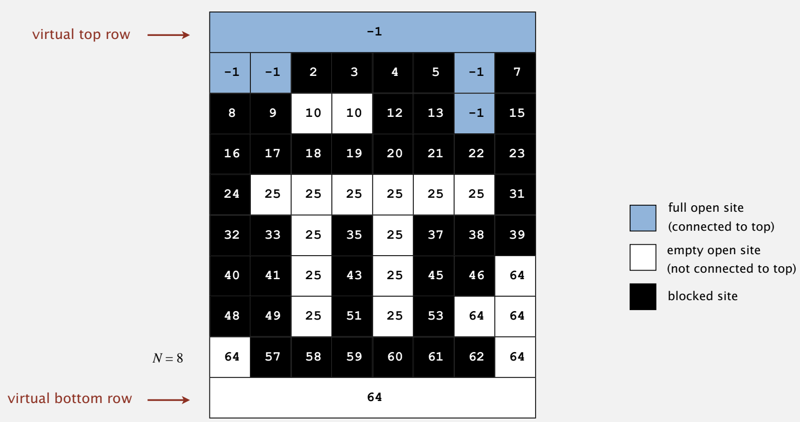
Q. How to avoid checking all pairs of top and bottom sites?



UF solution: a critical optimization

Q. How to avoid checking all pairs of top and bottom sites?

A. Create a virtual top and bottom objects;
system percolates when virtual top and bottom objects are in same set.

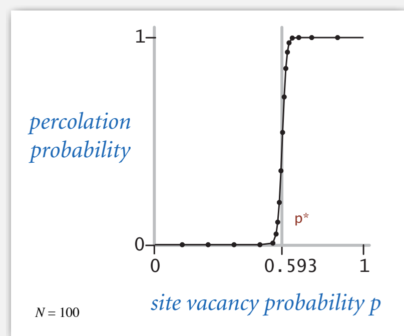


Percolation threshold

Q. What is percolation threshold p^* ?

A. About 0.592746 for large square lattices.

percolation constant known only via simulation



Fast algorithm enables accurate answer to scientific question.

Subtext of today's lecture (and this course)

Steps to developing a usable algorithm.

- Model the problem.
- Find an algorithm to solve it.
- Fast enough? Fits in memory?
- If not, figure out why.
- Find a way to address the problem.
- Iterate until satisfied.

The scientific method.

Mathematical analysis.