



Data Structures and Algorithms

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The material for this lecture is drawn, in part, from
The Practice of Programming (Kernighan & Pike) Chapter 2

1



Motivating Quotation

“Every program depends on algorithms and data structures, but few programs depend on the invention of brand new ones.”

-- Kernighan & Pike

Corollary: work *smarter*, not *harder*

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Goals of this Lecture



- Help you learn (or refresh your memory) about:
 - Commonly used data structures and algorithms
- Shallow motivation
 - Provide examples of typical pointer-related C code
- Deeper motivation
 - Common data structures and algorithms serve as “high level building blocks”
 - A power programmer:
 - Rarely creates large programs from scratch
 - Creates large programs using high level building blocks whenever possible

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A Common Task



- Maintain a table of key/value pairs
 - Each key is a string; each value is an int
 - Unknown number of key-value pairs
 - For simplicity, allow duplicate keys (client responsibility)
 - In Assignment #3, must check for duplicate keys!
- Examples
 - (student name, grade)
 - (“john smith”, 84), (“jane doe”, 93), (“bill clinton”, 81)
 - (baseball player, number)
 - (“Ruth”, 3), (“Gehrig”, 4), (“Mantle”, 7)
 - (variable name, value)
 - (“maxLength”, 2000), (“i”, 7), (“j”, -10)

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Data Structures and Algorithms



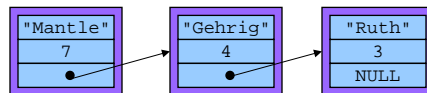
- **Data structures:** two ways to store the data
 - **Linked list** of key/value pairs
 - **Hash table** of key/value pairs
 - **Expanding array** of key/value pairs (see Appendix)
- **Algorithms:** various ways to manipulate the data
 - **Create:** Create the data structure
 - **Add:** Add a key/value pair
 - **Search:** Search for a key/value pair, by key
 - **Free:** Free the data structure

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Data Structure #1: Linked List



- **Data structure:** Nodes; each node contains a key/value pair and a pointer to the next node



- **Algorithms:**
 - **Create:** Allocate “dummy” node to point to first real node
 - **Add:** Create a new node, and insert at front of list
 - **Search:** Linear search through the list
 - **Free:** Free nodes while traversing; free dummy node

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Linked List: Data Structure



```
struct Node {
    const char *key;
    int value;
    struct Node *next;
};

struct Table {
    struct Node *first;
};
```

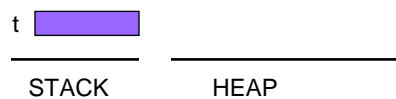
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Linked List: Create (1)



```
struct Table *Table_create(void) {
    struct Table *t;
    t = (struct Table*)
        malloc(sizeof(struct Table));
    t->first = NULL;
    return t;
}
```

```
struct Table *t;
...
t = Table_create();
...
```



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Linked List: Create (2)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)  
        malloc(sizeof(struct Table));  
    t->first = NULL;  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



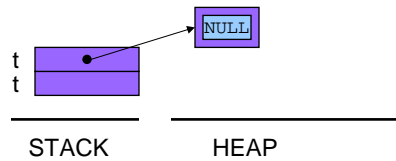
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Linked List: Create (3)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)  
        malloc(sizeof(struct Table));  
    t->first = NULL;  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



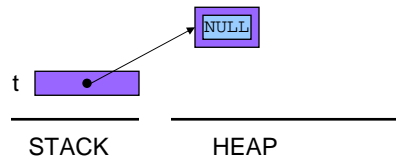
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Linked List: Create (4)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)  
        malloc(sizeof(struct Table));  
    t->first = NULL;  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



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Linked List: Add (1)



```
void Table_add(struct Table *t,  
    const char *key, int value) {  
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));  
    p->key = key;  
    p->value = value;  
    p->next = t->first;  
    t->first = p;  
}
```

```
struct Table *t;  
...  
Table_add(t, "Ruth", 3);  
Table_add(t, "Gehrig", 4);  
Table_add(t, "Mantle", 7);  
...
```

These are pointers to strings that exist in the RODATA section



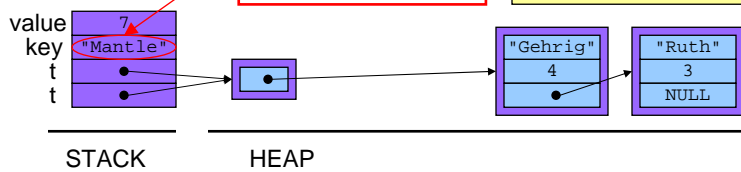
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Linked List: Add (2)



```
void Table_add(struct Table *t,
const char *key, int value) {
struct Node *p = (struct Node*)malloc(sizeof(struct Node));
p->key = key;
p->value = value;
p->next = t->first;
t->first = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



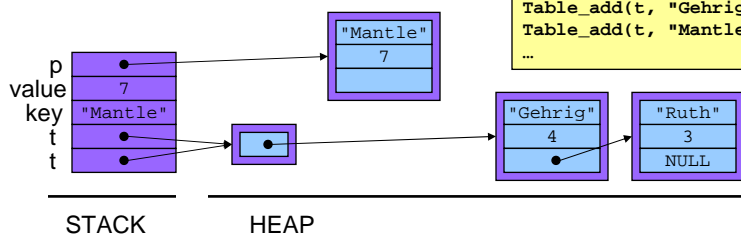
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Linked List: Add (3)



```
void Table_add(struct Table *t,
const char *key, int value) {
struct Node *p = (struct Node*)malloc(sizeof(struct Node));
p->key = key;
p->value = value;
p->next = t->first;
t->first = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```

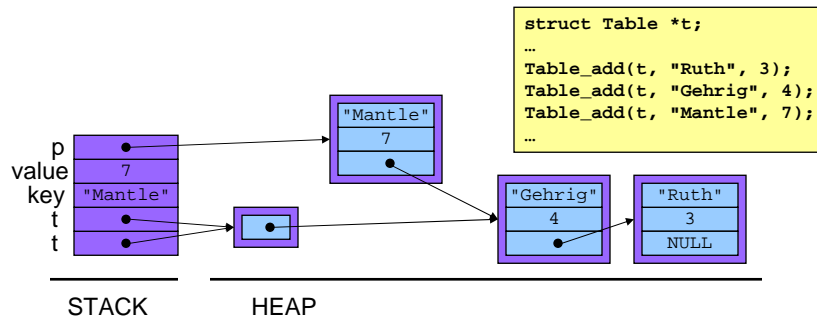


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Linked List: Add (4)



```
void Table_add(struct Table *t,
const char *key, int value) {
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));
    p->key = key;
    p->value = value;
    p->next = t->first;
    t->first = p;
}
```

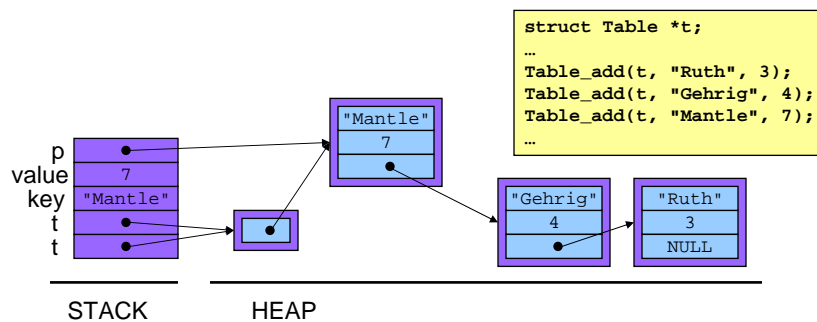


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Linked List: Add (5)



```
void Table_add(struct Table *t,
const char *key, int value) {
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));
    p->key = key;
    p->value = value;
    p->next = t->first;
    t->first = p;
}
```



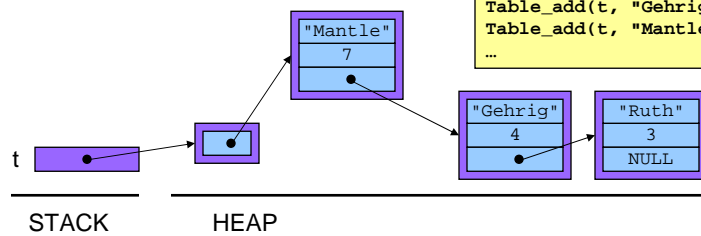
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Linked List: Add (6)



```
void Table_add(struct Table *t,
const char *key, int value) {
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));
    p->key = key;
    p->value = value;
    p->next = t->first;
    t->first = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



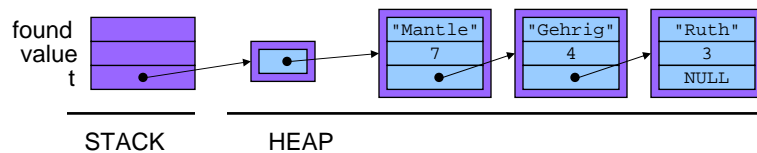
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Linked List: Search (1)



```
int Table_search(struct Table *t,
const char *key, int *value) {
    struct Node *p;
    for (p = t->first; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



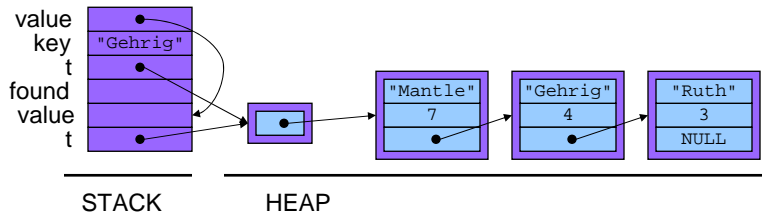
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Linked List: Search (2)



```
int Table_search(struct Table *t,
                const char *key, int *value) {
    struct Node *p;
    for (p = t->first; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



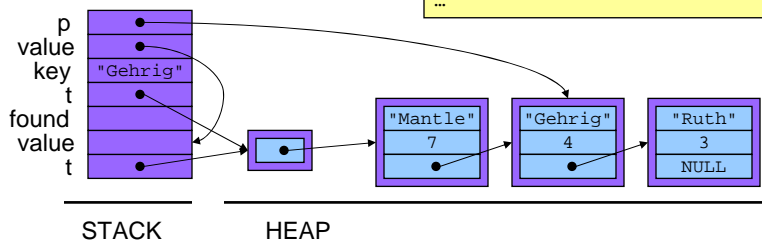
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Linked List: Search (3)



```
int Table_search(struct Table *t,
                const char *key, int *value) {
    struct Node *p;
    for (p = t->first; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



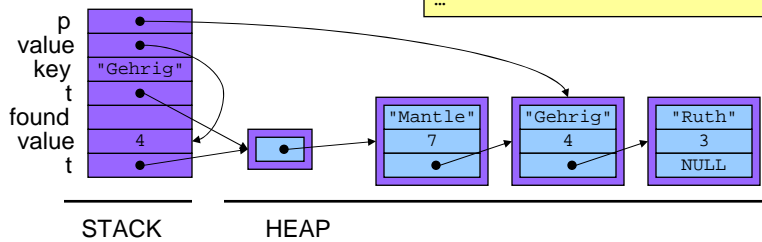
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Linked List: Search (4)



```
int Table_search(struct Table *t,
const char *key, int *value) {
struct Node *p;
for (p = t->first; p != NULL; p = p->next)
if (strcmp(p->key, key) == 0) {
    *value = p->value;
return 1;
}
return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



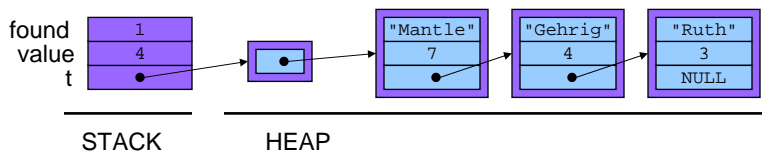
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Linked List: Search (5)



```
int Table_search(struct Table *t,
const char *key, int *value) {
struct Node *p;
for (p = t->first; p != NULL; p = p->next)
if (strcmp(p->key, key) == 0) {
    *value = p->value;
return 1;
}
return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



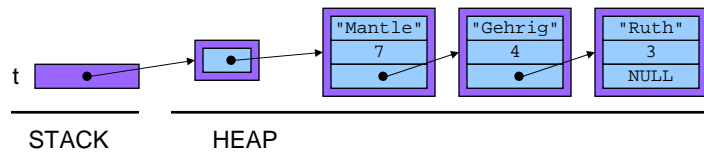
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Linked List: Free (1)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



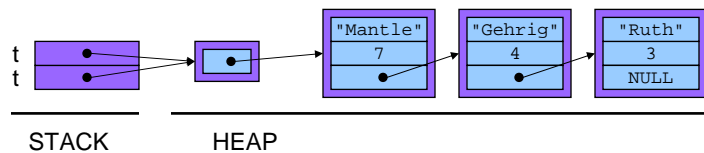
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Linked List: Free (2)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



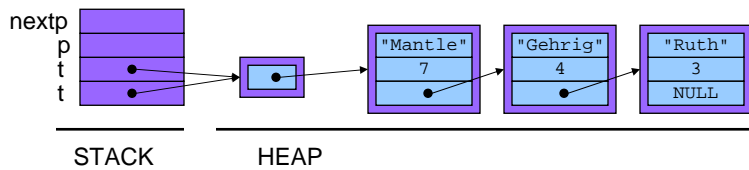
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Linked List: Free (3)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



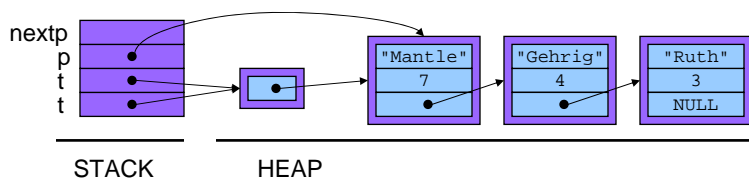
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Linked List: Free (4)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



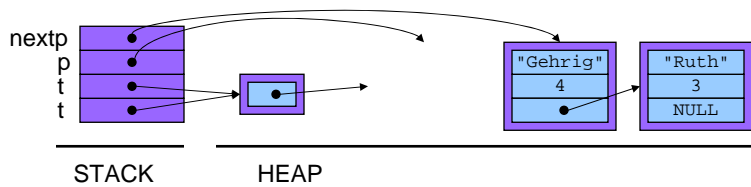
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Linked List: Free (5)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



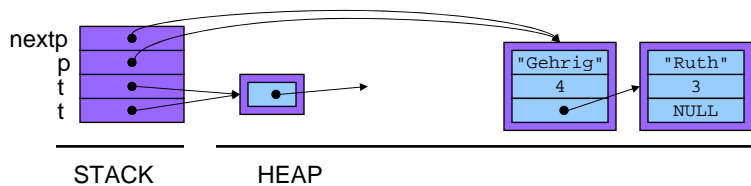
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Linked List: Free (6)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



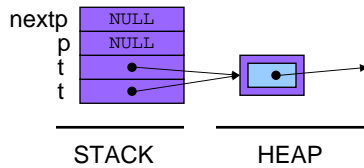
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Linked List: Free (7)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



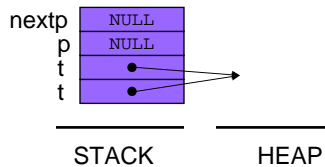
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Linked List: Free (8)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



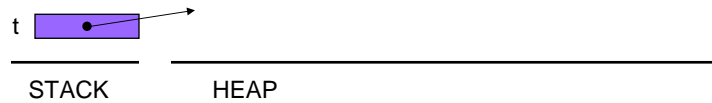
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Linked List: Free (9)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    for (p = t->first; p != NULL; p = nextp) {
        nextp = p->next;
        free(p);
    }
    free(t); /* Free the dummy node */
}
```

```
struct Table *t;
...
Table_free(t);
...
```



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Linked List Performance



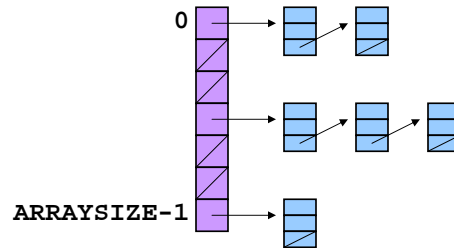
- Timing analysis of the given algorithms
 - Create: $O(1)$, fast
 - Add: $O(1)$, fast
 - Search: $O(n)$, slow
 - Free: $O(n)$, slow
- Alternative: Keep nodes in sorted order by key
 - Create: $O(1)$, fast
 - Add: $O(n)$, slow; must traverse part of list to find proper spot
 - Search: $O(n)$, still slow; must traverse part of list
 - Free: $O(n)$, slow

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Data Structure #2: Hash Table



- Fixed-size array where each element points to a linked list



```
struct Node *array[ARRAYSIZE];
```

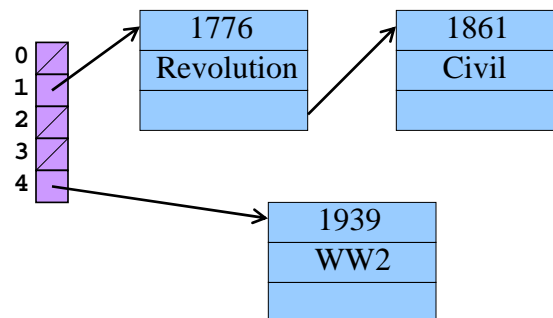
- Function maps each key to an array index
 - For example, for an integer key h
 - Hash function: $i = h \% \text{ARRAYSIZE}$ (mod function)
 - Go to array element i , i.e., the linked list `hashtab[i]`
 - Search for element, add element, remove element, etc.

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Hash Table Example



- Integer keys, array of size 5 with hash function " $h \bmod 5$ "
 - " $1776 \% 5$ " is 1
 - " $1861 \% 5$ " is 1
 - " $1939 \% 5$ " is 4

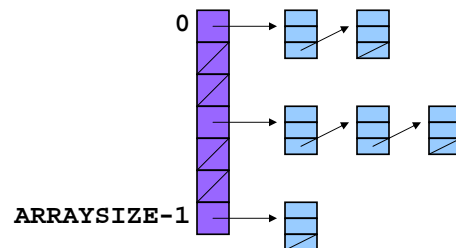


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How Large an Array?



- Large enough that average “bucket” size is 1
 - Short buckets mean fast search
 - Long buckets mean slow search
- Small enough to be memory efficient
 - Not an excessive number of elements
 - Fortunately, each array element is just storing a pointer
- This is OK:

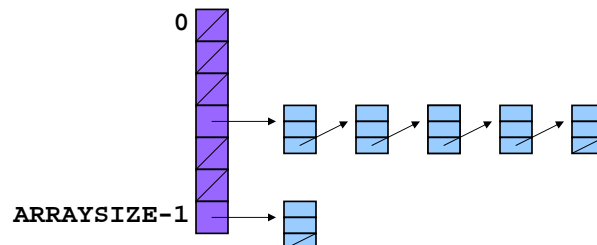


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What Kind of Hash Function?



- Good at distributing elements across the array
 - Distribute results over the range 0, 1, ..., ARRAYSIZE-1
 - Distribute results *evenly* to avoid very long buckets
- This is not so good:



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Hashing String Keys to Integers



- Simple schemes don't distribute the keys evenly enough
 - Number of characters, mod ARRAYSIZE
 - Sum the ASCII values of all characters, mod ARRAYSIZE
 - ...
- Here's a reasonably good hash function
 - Weighted sum of characters x_i in the string
 - $(\sum a^i x_i) \bmod \text{ARRAYSIZE}$
 - Best if a and ARRAYSIZE are relatively prime
 - E.g., $a = 65599$, ARRAYSIZE = 1024

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Implementing Hash Function



- Potentially expensive to compute a^i for each value of i
 - Computing a^i for each value of i
 - Instead, do $((x[0] * 65599 + x[1]) * 65599 + x[2]) * 65599 + x[3]) * \dots$

```
unsigned int hash(const char *x) {  
    int i;  
    unsigned int h = 0U;  
    for (i=0; x[i]!='\0'; i++)  
        h = h * 65599 + (unsigned char)x[i];  
    return h % 1024;  
}
```

Can be more clever than this for powers of two!
(Described in Appendix)

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Hash Table Example



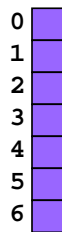
Example: `ARRAYSIZE = 7`

Lookup (and enter, if not present) these strings: the, cat, in, the, hat

Hash table initially empty.

First word: the. $\text{hash}(\text{"the"}) = 965156977$. $965156977 \% 7 = 1$.

Search the linked list `table[1]` for the string "the"; not found.



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Hash Table Example (cont.)



Example: `ARRAYSIZE = 7`

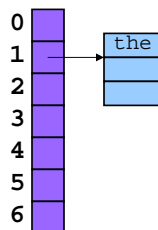
Lookup (and enter, if not present) these strings: the, cat, in, the, hat

Hash table initially empty.

First word: "the". $\text{hash}(\text{"the"}) = 965156977$. $965156977 \% 7 = 1$.

Search the linked list `table[1]` for the string "the"; not found

Now: `table[1] = makelink(key, value, table[1])`



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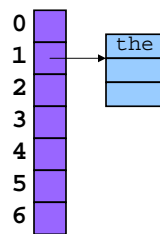
Hash Table Example (cont.)



Second word: "cat". $\text{hash}(\text{"cat"}) = 3895848756$. $3895848756 \% 7 = 2$.

Search the linked list `table[2]` for the string "cat"; not found

Now: `table[2] = makelink(key, value, table[2])`



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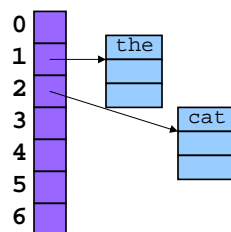
Hash Table Example (cont.)



Third word: "in". $\text{hash}(\text{"in"}) = 6888005$. $6888005 \% 7 = 5$.

Search the linked list `table[5]` for the string "in"; not found

Now: `table[5] = makelink(key, value, table[5])`



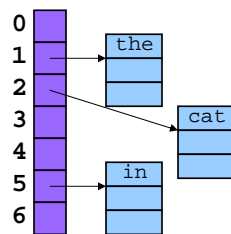
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Hash Table Example (cont.)



Fourth word: "the". $\text{hash}(\text{"the"}) = 965156977$. $965156977 \% 7 = 1$.

Search the linked list `table[1]` for the string "the"; found it!



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Hash Table Example (cont.)

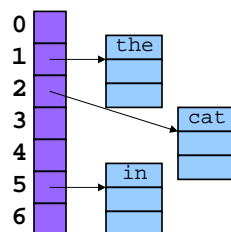


Fourth word: "hat". $\text{hash}(\text{"hat"}) = 865559739$. $865559739 \% 7 = 2$.

Search the linked list `table[2]` for the string "hat"; not found.

Now, insert "hat" into the linked list `table[2]`.

At beginning or end? Doesn't matter.

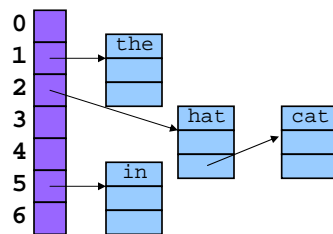


44

Hash Table Example (cont.)



Inserting at the front is easier, so add "hat" at the front



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Hash Table: Data Structure



```
enum {BUCKET_COUNT = 1024};

struct Node {
    const char *key;
    int value;
    struct Node *next;
};

struct Table {
    struct Node *array[BUCKET_COUNT];
};
```

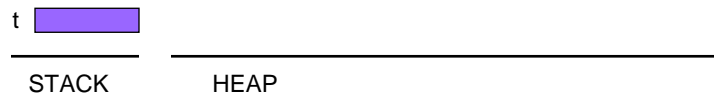
46

Hash Table: Create (1)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)calloc(1, sizeof(struct Table));  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



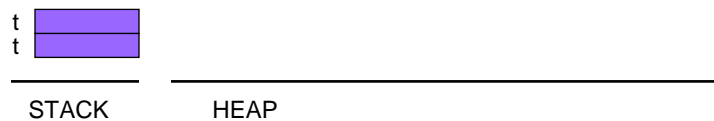
47

Hash Table: Create (2)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)calloc(1, sizeof(struct Table));  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



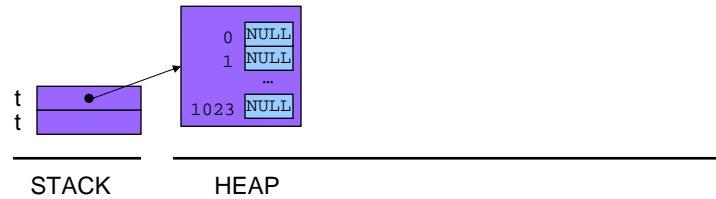
48

Hash Table: Create (3)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)calloc(1, sizeof(struct Table));  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



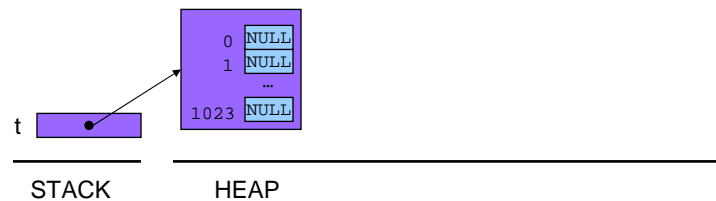
49

Hash Table: Create (4)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)calloc(1, sizeof(struct Table));  
    return t;  
}
```

```
struct Table *t;  
...  
t = Table_create();  
...
```



50

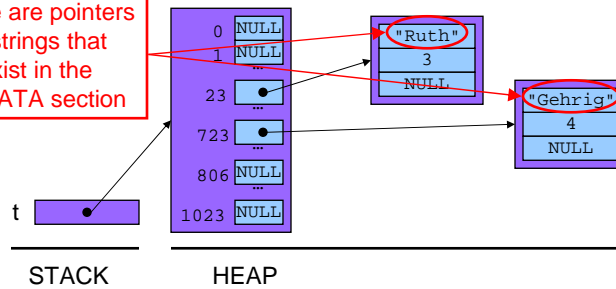
Hash Table: Add (1)



```
void Table_add(struct Table *t,
  const char *key, int value) {
  struct Node *p = (struct Node*)malloc(sizeof(struct Node));
  int h = hash(key);
  p->key = key;
  p->value = value;
  p->next = t->array[h];
  t->array[h] = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```

These are pointers to strings that exist in the RODATA section



51

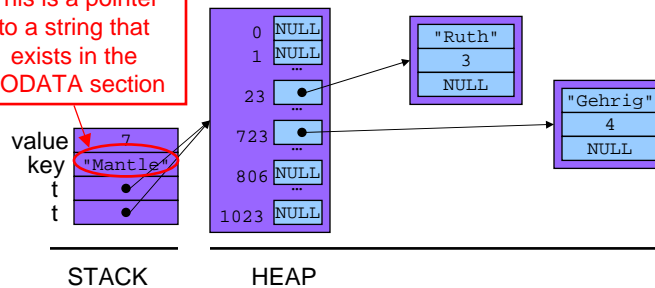
Hash Table: Add (2)



```
void Table_add(struct Table *t,
  const char *key, int value) {
  struct Node *p = (struct Node*)malloc(sizeof(struct Node));
  int h = hash(key);
  p->key = key;
  p->value = value;
  p->next = t->array[h];
  t->array[h] = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```

This is a pointer to a string that exists in the RODATA section



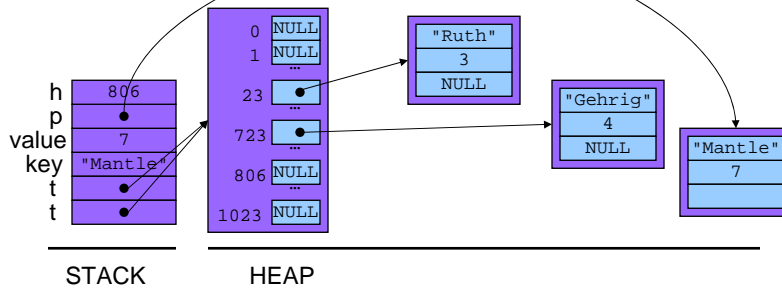
52

Hash Table: Add (3)



```
void Table_add(struct Table *t,
const char *key, int value) {
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));
    int h = hash(key);
    p->key = key;
    p->value = value;
    p->next = t->array[h];
    t->array[h] = p;
}

struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



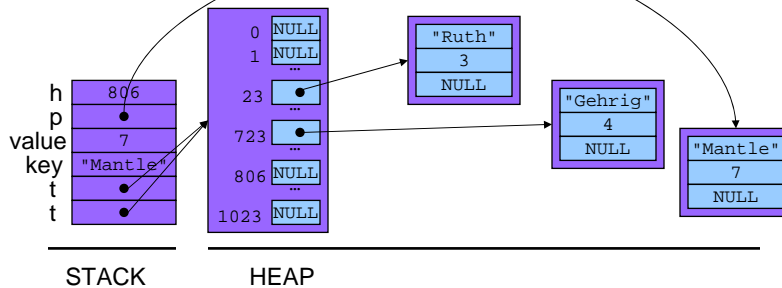
53

Hash Table: Add (4)



```
void Table_add(struct Table *t,
const char *key, int value) {
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));
    int h = hash(key);
    p->key = key;
    p->value = value;
    p->next = t->array[h];
    t->array[h] = p;
}

struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



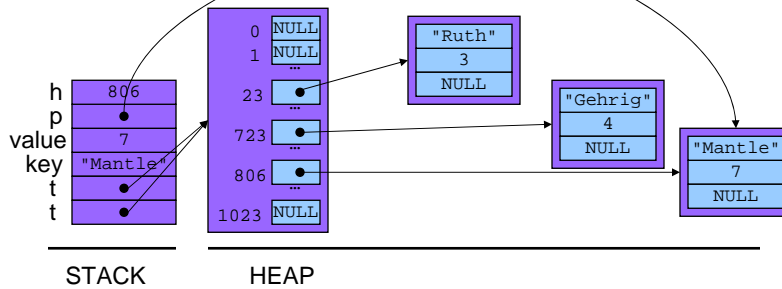
54

Hash Table: Add (5)



```
void Table_add(struct Table *t,
  const char *key, int value) {
  struct Node *p = (struct Node*)malloc(sizeof(struct Node));
  int h = hash(key);
  p->key = key;
  p->value = value;
  p->next = t->array[h];
  t->array[h] = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



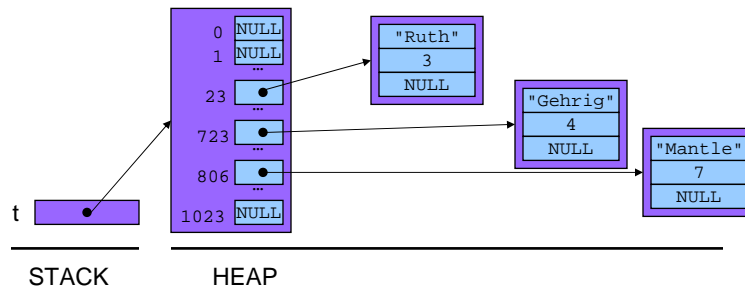
55

Hash Table: Add (6)



```
void Table_add(struct Table *t,
  const char *key, int value) {
  struct Node *p = (struct Node*)malloc(sizeof(struct Node));
  int h = hash(key);
  p->key = key;
  p->value = value;
  p->next = t->array[h];
  t->array[h] = p;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



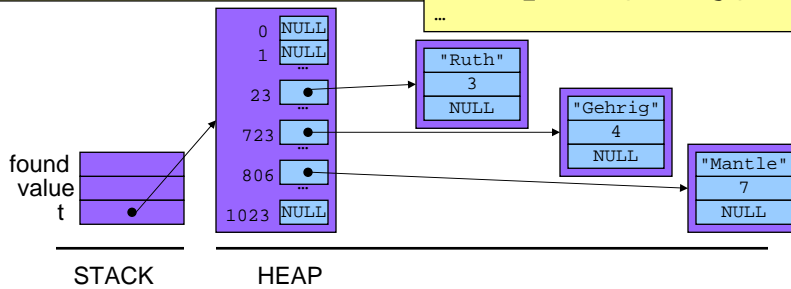
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Hash Table: Search (1)



```
int Table_search(struct Table *t,
const char *key, int *value) {
struct Node *p;
int h = hash(key);
for (p = t->array[h]; p != NULL; p = p->next)
if (strcmp(p->key, key) == 0) {
*value = p->value;
return 1;
}
return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
Table_search(t, "Gehrig", &value);
...
```



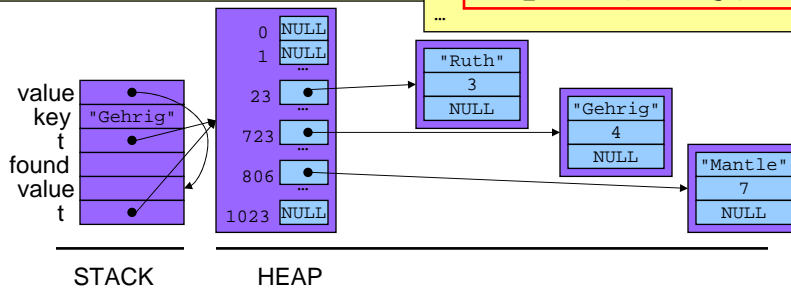
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Hash Table: Search (2)



```
int Table_search(struct Table *t,
const char *key, int *value) {
struct Node *p;
int h = hash(key);
for (p = t->array[h]; p != NULL; p = p->next)
if (strcmp(p->key, key) == 0) {
*value = p->value;
return 1;
}
return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
Table_search(t, "Gehrig", &value);
...
```



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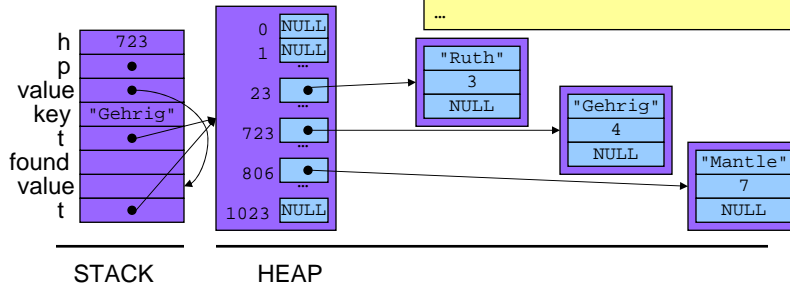
Hash Table: Search (3)



```

int Table_search(struct Table *t,
const char *key, int *value) {
    struct Node *p;
    int h = hash(key);
    for (p = t->array[h]; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}

struct Table *t;
int value;
int found;
...
found = Table_search(t, "Gehrig", &value);
...
    
```



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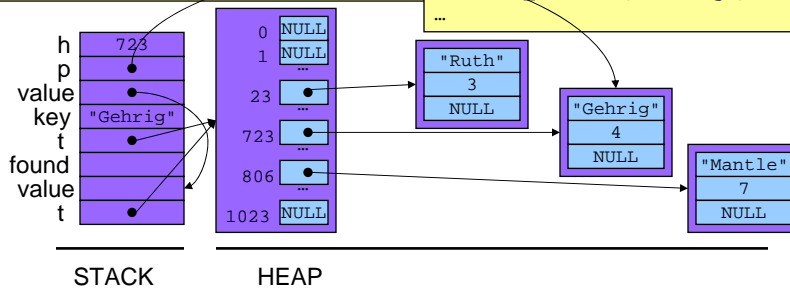
Hash Table: Search (4)



```

int Table_search(struct Table *t,
const char *key, int *value) {
    struct Node *p;
    int h = hash(key);
    for (p = t->array[h]; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}

struct Table *t;
int value;
int found;
...
found = Table_search(t, "Gehrig", &value);
...
    
```



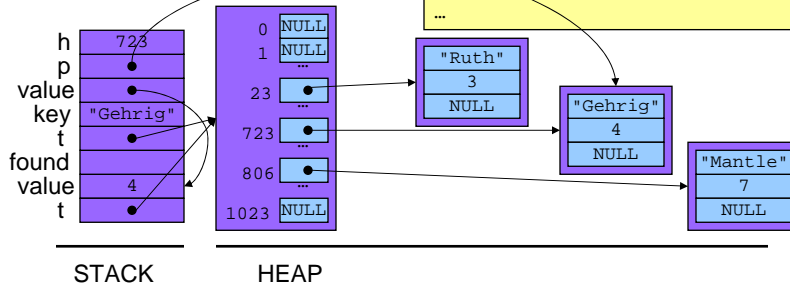
60

Hash Table: Search (5)



```
int Table_search(struct Table *t,
const char *key, int *value) {
    struct Node *p;
    int h = hash(key);
    for (p = t->array[h]; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



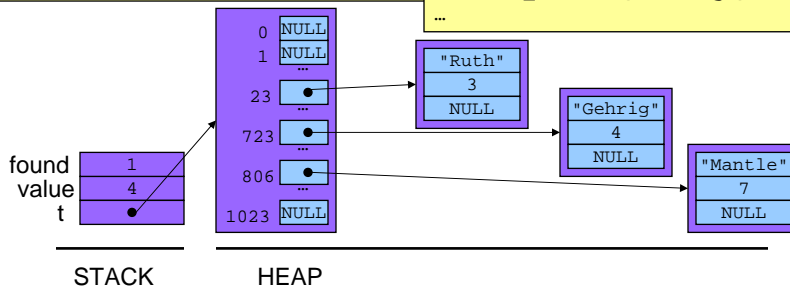
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Hash Table: Search (6)



```
int Table_search(struct Table *t,
const char *key, int *value) {
    struct Node *p;
    int h = hash(key);
    for (p = t->array[h]; p != NULL; p = p->next)
        if (strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



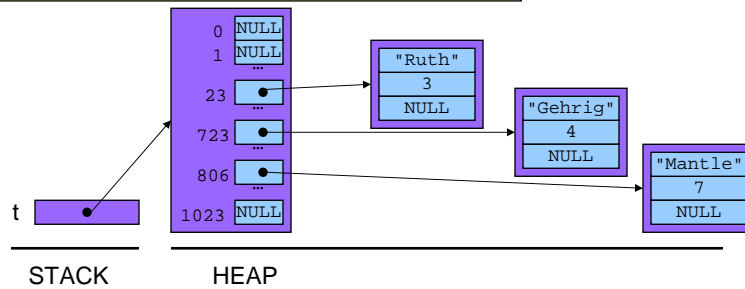
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Hash Table: Free (1)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    int b;
    for (b = 0; b < BUCKET_COUNT; b++)
        for (p = t->array[b]; p != NULL; p = nextp) {
            nextp = p->next;
            free(p);
        }
    free(t);
}
```

```
struct Table *t;
...
Table_free(t);
...
```



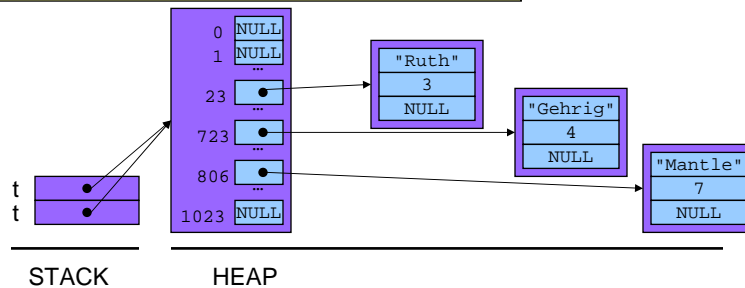
63

Hash Table: Free (2)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    int b;
    for (b = 0; b < BUCKET_COUNT; b++)
        for (p = t->array[b]; p != NULL; p = nextp) {
            nextp = p->next;
            free(p);
        }
    free(t);
}
```

```
struct Table *t;
...
Table_free(t);
...
```



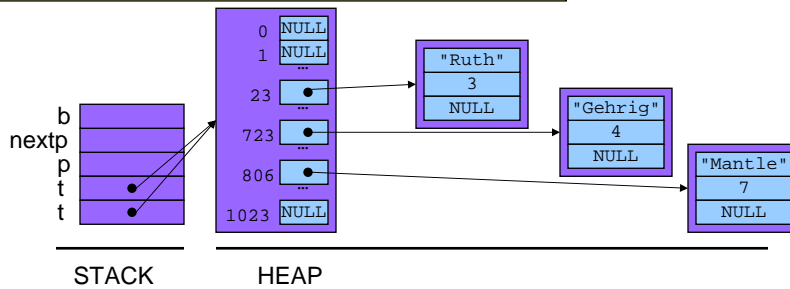
64

Hash Table: Free (3)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    int b;
    for (b = 0; b < BUCKET_COUNT; b++)
        for (p = t->array[b]; p != NULL; p = nextp) {
            nextp = p->next;
            free(p);
        }
    free(t);
}
```

```
struct Table *t;
...
Table_free(t);
...
```



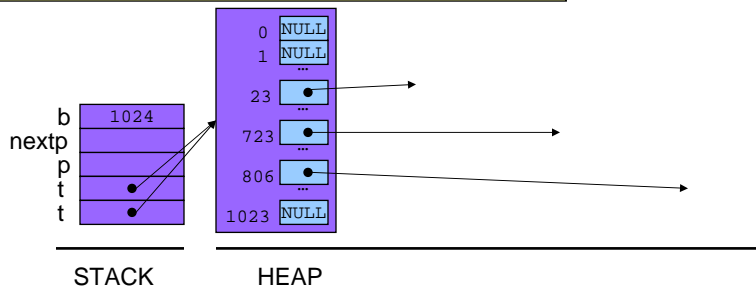
65

Hash Table: Free (4)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    int b;
    for (b = 0; b < BUCKET_COUNT; b++)
        for (p = t->array[b]; p != NULL; p = nextp) {
            nextp = p->next;
            free(p);
        }
    free(t);
}
```

```
struct Table *t;
...
Table_free(t);
...
```



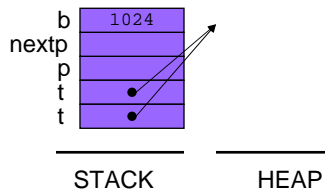
66

Hash Table: Free (5)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    int b;
    for (b = 0; b < BUCKET_COUNT; b++)
        for (p = t->array[b]; p != NULL; p = nextp) {
            nextp = p->next;
            free(p);
        }
    free(t);
}
```

```
struct Table *t;
...
Table_free(t);
...
```



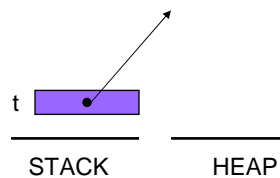
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Hash Table: Free (6)



```
void Table_free(struct Table *t) {
    struct Node *p;
    struct Node *nextp;
    int b;
    for (b = 0; b < BUCKET_COUNT; b++)
        for (p = t->array[b]; p != NULL; p = nextp) {
            nextp = p->next;
            free(p);
        }
    free(t);
}
```

```
struct Table *t;
...
Table_free(t);
...
```



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Hash Table Performance



- Create: $O(1)$, fast
- Add: $O(1)$, fast
- Search: $O(1)$, fast – if and only if bucket sizes are small
- Free: $O(n)$, slow

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Key Ownership



- Note: `Table_add()` functions contain this code:

```
void Table_add(struct Table *t, const char *key, int value) {  
    ...  
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));  
    p->key = key;  
    ...  
}
```

- Caller passes `key`, which is a pointer to memory where a string resides
- `Table_add()` function simply stores within the table the address where the string resides

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Key Ownership (cont.)



- Problem: Consider this calling code:

```
struct Table t;  
char k[100] = "Ruth";  
...  
Table_add(t, k, 3);  
strcpy(k, "Gehrig");  
...
```

- Via Table_add(), table contains memory address k
- Client changes string at memory address k
- Thus client changes key within table

- Trouble in hash table
 - Existing node's key has been changed from "Ruth" to "Gehrig"
 - Existing node now is in wrong bucket!!!
 - Hash table has been corrupted!!!
- Could be trouble in other data structures too

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Key Ownership (cont.)



- Solution: Table_add() saves **copy** of given key

```
void Table_add(struct Table *t, const char *key, int value) {  
    ...  
    struct Node *p = (struct Node*)malloc(sizeof(struct Node));  
    p->key = (const char*)malloc(strlen(key) + 1);  
    strcpy(p->key, key);  
    ...  
}
```

Allow room for '\0'

- If client changes string at memory address k, data structure is not affected
- Then the data structure "owns" the copy, that is:
 - The data structure is responsible for freeing the memory in which the copy resides
 - The Table_free() function must free the copy

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Summary



- Common data structures and associated algorithms
 - **Linked list**
 - Unsorted => fast insert, slow search
 - Sorted => slow insert, slow search
 - **Hash table**
 - Fast insert, fast search – iff hash function works well
 - Invaluable for storing key/value pairs
 - Very common
- **Related issues**
 - Hashing algorithms
 - Memory ownership
- **Two appendices**
 - Appendix #1: tricks for faster hash tables
 - Appendix #2: example of a third data structure

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Appendix 1



- “Stupid programmer tricks” related to hash tables...

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Revisiting Hash Functions



- Potentially expensive to compute “mod c”
 - Involves division by c and keeping the remainder
 - Easier when c is a power of 2 (e.g., $16 = 2^4$)
- An alternative (by example)

- $53 = 32 + 16 + 4 + 1$

••• 32 16 8 4 2 1

0	0	1	1	0	1	0	1
---	---	---	---	---	---	---	---

- $53 \% 16$ is 5, the last four bits of the number

••• 32 16 8 4 2 1

0	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---

- Would like an easy way to isolate the last four bits...

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Recall: Bitwise Operators in C



- Bitwise AND (&)
- Bitwise OR (|)

&	0	1
0	0	0
1	0	1

	0	1
0	0	1
1	1	1

- Mod on the cheap!
 - E.g., $h = 53 \& 15;$

53

0	0	1	1	0	1	0	1
---	---	---	---	---	---	---	---

& 15

0	0	0	0	1	1	1	1
---	---	---	---	---	---	---	---

5

0	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---

- One's complement (~)
 - Turns 0 to 1, and 1 to 0
 - E.g., set last three bits to 0
 - $x = x \& \sim 7;$

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A Faster Hash Function



```
unsigned int hash(const char *x) {
    int i;
    unsigned int h = 0U;
    for (i=0; x[i]!='\0'; i++)
        h = h * 65599 + (unsigned char)x[i];
    return h % 1024;
}
```

Previous
version



```
unsigned int hash(const char *x) {
    int i;
    unsigned int h = 0U;
    for (i=0; x[i]!='\0'; i++)
        h = h * 65599 + (unsigned char)x[i];
    return h & 1023;
}
```

Faster

- Beware: Don't write "h & 1024"

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Speeding Up Key Comparisons



- Speeding up key comparisons
 - For any non-trivial value comparison function
 - Trick: store full hash result in structure

```
int Table_search(struct Table *t,
    const char *key, int *value) {
    struct Node *p;
    int h = hash(key); /* No % in hash function */
    for (p = t->array[h%1024]; p != NULL; p = p->next)
        if ((p->hash == h) && strcmp(p->key, key) == 0) {
            *value = p->value;
            return 1;
        }
    return 0;
}
```

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Appendix 2: Another Data Structure

- Expanding array...

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Expanding Array

- The general idea...
- **Data structure:** An array that expands as necessary
- **Create algorithm:** Allocate an array of key/value pairs; initially the array has few elements
- **Add algorithm:** If out of room, double the size of the array; copy the given key/value pair into the first unused element
 - Note: For efficiency, expand the array *geometrically* instead of *linearly*
- **Search algorithm:** Simple linear search
- **Free algorithm:** Free the array

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Expanding Array: Data Structure



```
enum {INITIAL_SIZE = 2};
enum {GROWTH_FACTOR = 2};

struct Pair {
    const char *key;
    int value;
};

struct Table {
    int pairCount;      /* Number of pairs in table */
    int arraySize;     /* Physical size of array */
    struct Pair *array; /* Address of array */
};
```

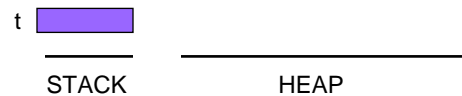
81

Expanding Array: Create (1)



```
struct Table *Table_create(void) {
    struct Table *t;
    t = (struct Table*)
        malloc(sizeof(struct Table));
    t->pairCount = 0;
    t->arraySize = INITIAL_SIZE;
    t->array = (struct Pair*)
        calloc(INITIAL_SIZE,
              sizeof(struct Pair));
    return t;
}
```

```
{
    struct Table *t;
    ...
    t = Table_create();
    ...
}
```



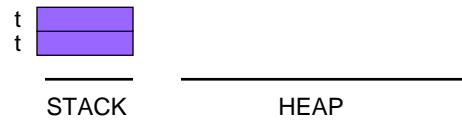
82

Expanding Array: Create (2)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)  
        malloc(sizeof(struct Table));  
    t->pairCount = 0;  
    t->arraySize = INITIAL_SIZE;  
    t->array = (struct Pair*)  
        calloc(INITIAL_SIZE,  
              sizeof(struct Pair));  
    return t;  
}
```

```
{  
    struct Table *t;  
    ...  
    t = Table_create();  
    ...  
}
```



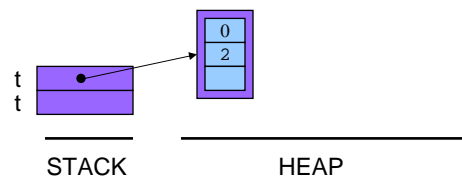
83

Expanding Array: Create (3)



```
struct Table *Table_create(void) {  
    struct Table *t;  
    t = (struct Table*)  
        malloc(sizeof(struct Table));  
    t->pairCount = 0;  
    t->arraySize = INITIAL_SIZE;  
    t->array = (struct Pair*)  
        calloc(INITIAL_SIZE,  
              sizeof(struct Pair));  
    return t;  
}
```

```
{  
    struct Table *t;  
    ...  
    t = Table_create();  
    ...  
}
```



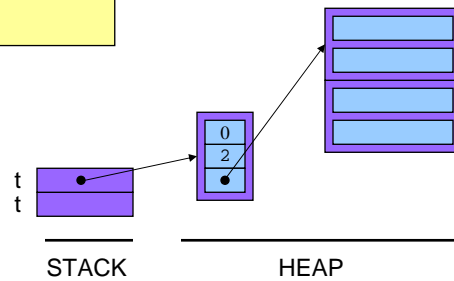
84

Expanding Array: Create (4)



```
struct Table *Table_create(void) {
    struct Table *t;
    t = (struct Table*)
        malloc(sizeof(struct Table));
    t->pairCount = 0;
    t->arraySize = INITIAL_SIZE;
    t->array = (struct Pair*)
        calloc(INITIAL_SIZE,
              sizeof(struct Pair));
    return t;
}
```

```
{
    struct Table *t;
    ...
    t = Table_create();
    ...
}
```



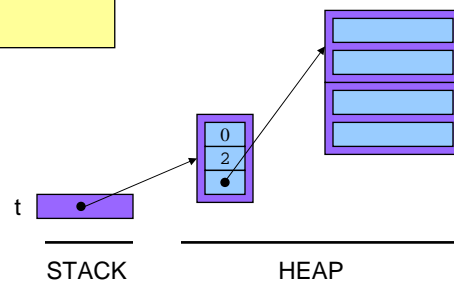
85

Expanding Array: Create (5)



```
struct Table *Table_create(void) {
    struct Table *t;
    t = (struct Table*)
        malloc(sizeof(struct Table));
    t->pairCount = 0;
    t->arraySize = INITIAL_SIZE;
    t->array = (struct Pair*)
        calloc(INITIAL_SIZE,
              sizeof(struct Pair));
    return t;
}
```

```
{
    struct Table *t;
    ...
    t = Table_create();
    ...
}
```



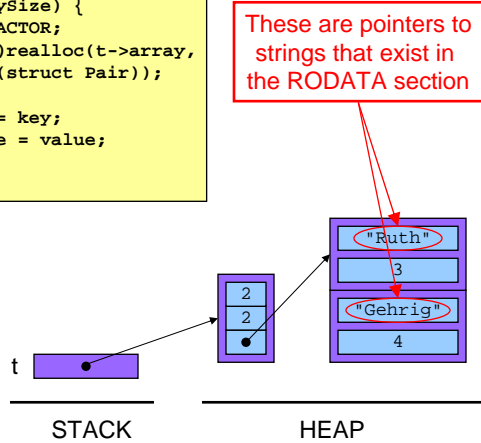
86

Expanding Array: Add (1)



```
void Table_add(struct Table *t,
               const char *key, int value) {
    /* Expand if necessary. */
    if (t->pairCount == t->arraySize) {
        t->arraySize *= GROWTH_FACTOR;
        t->array = (struct Pair*)realloc(t->array,
                                         t->arraySize * sizeof(struct Pair));
    }
    t->array[t->pairCount].key = key;
    t->array[t->pairCount].value = value;
    t->pairCount++;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



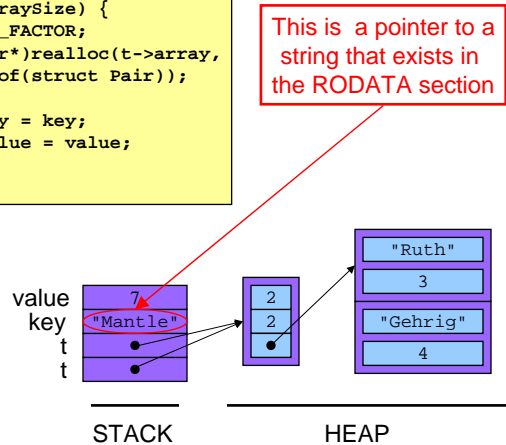
87

Expanding Array: Add (2)



```
void Table_add(struct Table *t,
               const char *key, int value) {
    /* Expand if necessary. */
    if (t->pairCount == t->arraySize) {
        t->arraySize *= GROWTH_FACTOR;
        t->array = (struct Pair*)realloc(t->array,
                                         t->arraySize * sizeof(struct Pair));
    }
    t->array[t->pairCount].key = key;
    t->array[t->pairCount].value = value;
    t->pairCount++;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



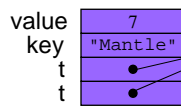
88

Expanding Array: Add (3)

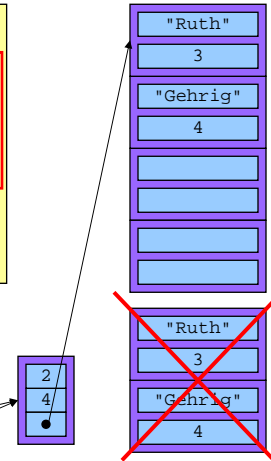


```
void Table_add(struct Table *t,
               const char *key, int value) {
    /* Expand if necessary. */
    if (t->pairCount == t->arraySize) {
        t->arraySize *= GROWTH_FACTOR;
        t->array = (struct Pair*)realloc(t->array,
                                         t->arraySize * sizeof(struct Pair));
    }
    t->array[t->pairCount].key = key;
    t->array[t->pairCount].value = value;
    t->pairCount++;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



STACK



HEAP

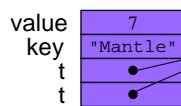
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Expanding Array: Add (4)

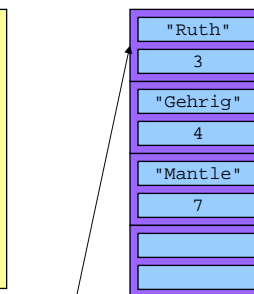


```
void Table_add(struct Table *t,
               const char *key, int value) {
    /* Expand if necessary. */
    if (t->pairCount == t->arraySize) {
        t->arraySize *= GROWTH_FACTOR;
        t->array = (struct Pair*)realloc(t->array,
                                         t->arraySize * sizeof(struct Pair));
    }
    t->array[t->pairCount].key = key;
    t->array[t->pairCount].value = value;
    t->pairCount++;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



STACK



HEAP

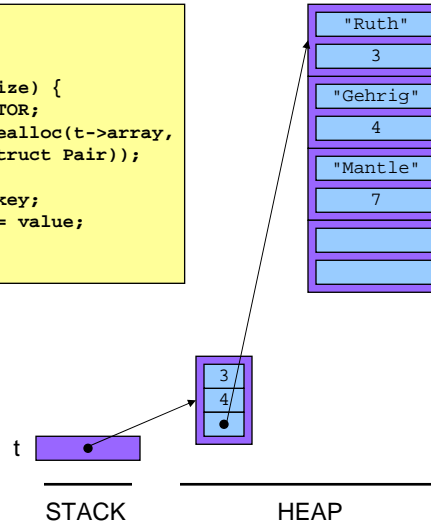
90

Expanding Array: Add (5)



```
void Table_add(struct Table *t,
const char *key, int value) {
    /* Expand if necessary. */
    if (t->pairCount == t->arraySize) {
        t->arraySize *= GROWTH_FACTOR;
        t->array = (struct Pair*)realloc(t->array,
            t->arraySize * sizeof(struct Pair));
    }
    t->array[t->pairCount].key = key;
    t->array[t->pairCount].value = value;
    t->pairCount++;
}
```

```
struct Table *t;
...
Table_add(t, "Ruth", 3);
Table_add(t, "Gehrig", 4);
Table_add(t, "Mantle", 7);
...
```



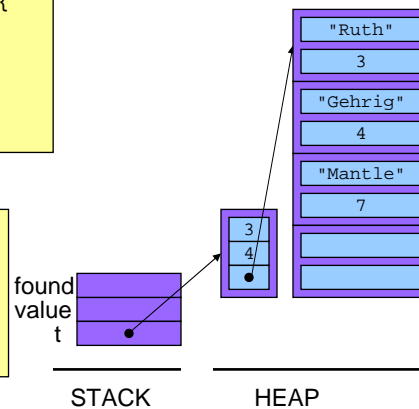
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Expanding Array: Search (1)



```
int Table_search(struct Table *t,
const char *key, int *value) {
    int i;
    for (i = 0; i < t->pairCount; i++) {
        struct Pair p = t->array[i];
        if (strcmp(p.key, key) == 0) {
            *value = p.value;
            return 1;
        }
    }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



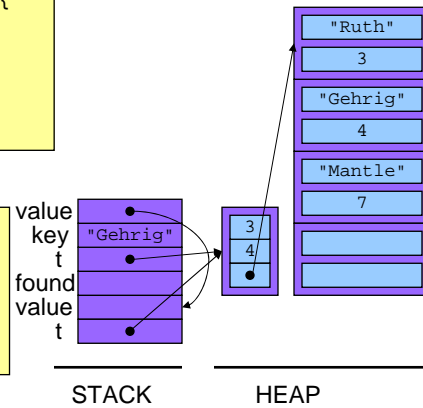
92

Expanding Array: Search (2)



```
int Table_search(struct Table *t,
                const char *key, int *value) {
    int i;
    for (i = 0; i < t->pairCount; i++) {
        struct Pair p = t->array[i];
        if (strcmp(p.key, key) == 0) {
            *value = p.value;
            return 1;
        }
    }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



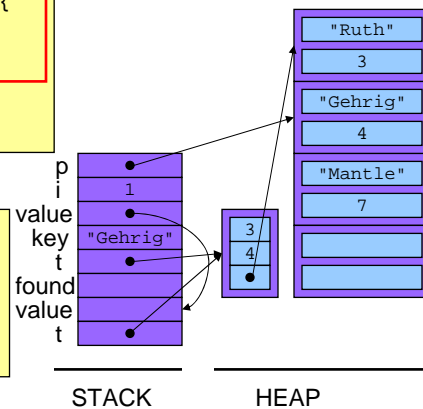
93

Expanding Array: Search (3)



```
int Table_search(struct Table *t,
                const char *key, int *value) {
    int i;
    for (i = 0; i < t->pairCount; i++) {
        struct Pair p = t->array[i];
        if (strcmp(p.key, key) == 0) {
            *value = p.value;
            return 1;
        }
    }
    return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
    Table_search(t, "Gehrig", &value);
...
```



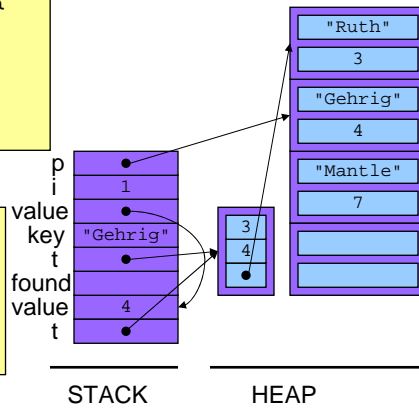
94

Expanding Array: Search (4)



```
int Table_search(struct Table *t,
  const char *key, int *value) {
  int i;
  for (i = 0; i < t->pairCount; i++) {
    struct Pair p = t->array[i];
    if (strcmp(p.key, key) == 0) {
      *value = p.value;
      return 1;
    }
  }
  return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
  Table_search(t, "Gehrig", &value);
...
```



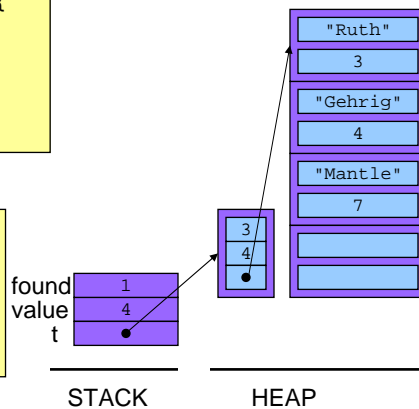
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Expanding Array: Search (5)



```
int Table_search(struct Table *t,
  const char *key, int *value) {
  int i;
  for (i = 0; i < t->pairCount; i++) {
    struct Pair p = t->array[i];
    if (strcmp(p.key, key) == 0) {
      *value = p.value;
      return 1;
    }
  }
  return 0;
}
```

```
struct Table *t;
int value;
int found;
...
found =
  Table_search(t, "Gehrig", &value);
...
```



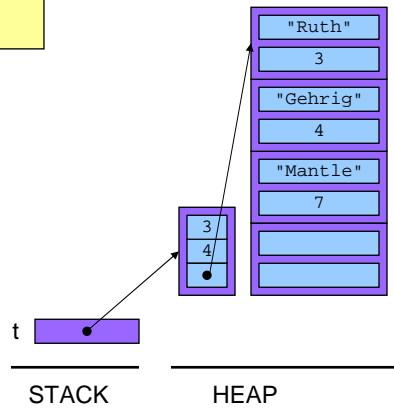
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Expanding Array: Free (1)



```
void Table_free(struct Table *t) {  
    free(t->array);  
    free(t);  
}
```

```
struct Table *t;  
...  
Table_free(t);  
...
```



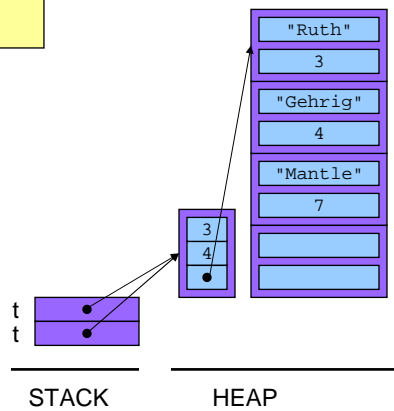
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Expanding Array: Free (2)



```
void Table_free(struct Table *t) {  
    free(t->array);  
    free(t);  
}
```

```
struct Table *t;  
...  
Table_free(t);  
...
```



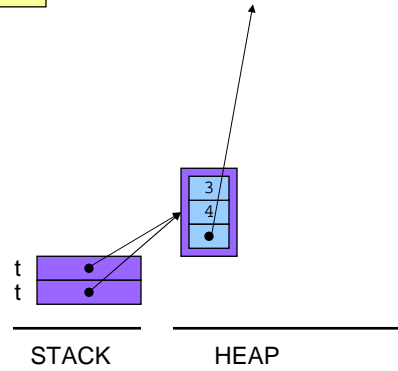
98

Expanding Array: Free (3)



```
void Table_free(struct Table *t) {  
    free(t->array);  
    free(t);  
}
```

```
struct Table *t;  
...  
Table_free(t);  
...
```



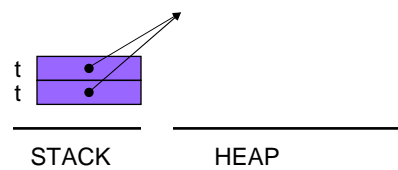
99

Expanding Array: Free (4)



```
void Table_free(struct Table *t) {  
    free(t->array);  
    free(t);  
}
```

```
struct Table *t;  
...  
Table_free(t);  
...
```



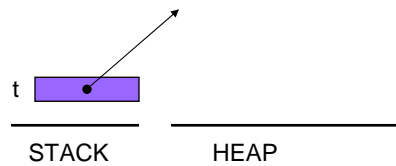
100

Expanding Array: Free (5)



```
void Table_free(struct Table *t) {  
    free(t->array);  
    free(t);  
}
```

```
struct Table *t;  
...  
Table_free(t);  
...
```



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Expanding Array Performance



- Timing analysis of given algorithms
 - Create: $O(1)$, fast
 - Add: $O(1)$, fast
 - Search: $O(n)$, slow
 - Free: $O(1)$, fast
- Alternative: Keep the array sorted by key
 - Create: $O(1)$, fast
 - Add: $O(n)$, slow; must move pairs to make room for new one
 - Search: $O(\log n)$, moderate; can use binary search
 - Free: $O(1)$, fast

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