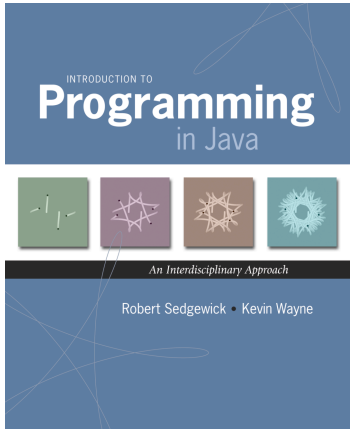
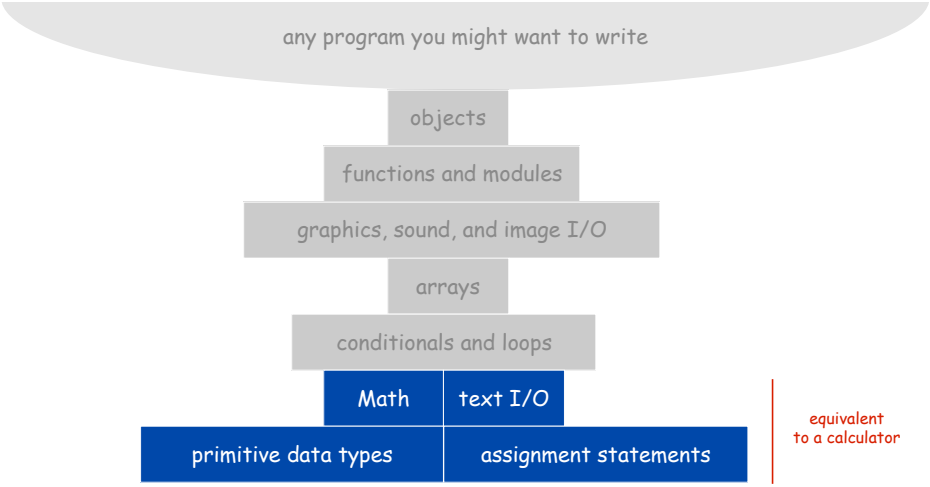
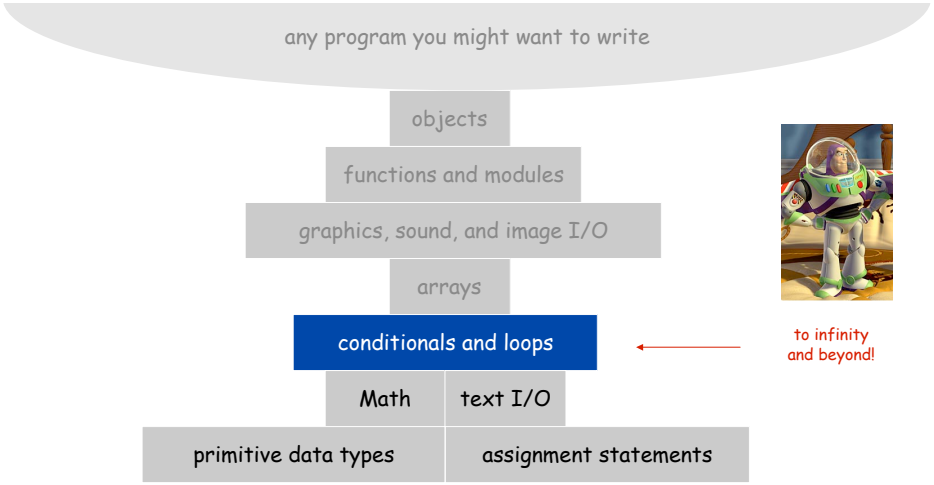
1.3 Conditionals and Loops

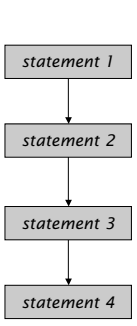




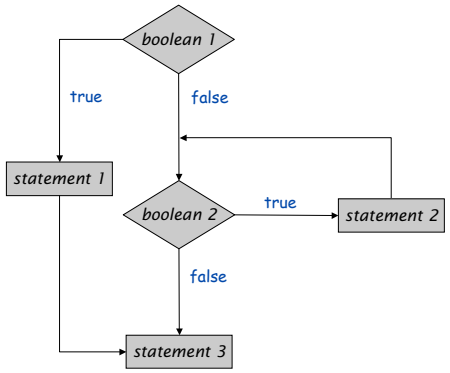


Control flow.

- Sequence of statements that are actually executed in a program.
- Conditionals and loops: enable us to choreograph control flow.



straight-line control flow



control flow with conditionals and loops

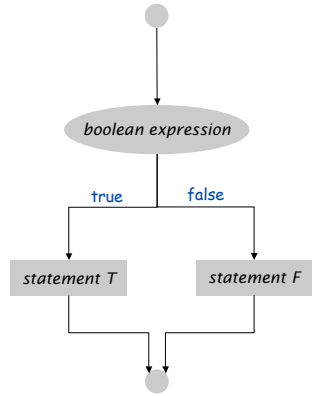
Conditionals

The `if` statement. A common branching structure.

- Evaluate a `boolean` expression.
- If `true`, execute some statements.
- If `false`, execute other statements.

```
if (boolean expression) {
   statement T;
}
else {
   statement F;
}
```

can be any sequence of statements



The `if` statement. A common branching structure.

- Evaluate a `boolean` expression.
- If `true`, execute some statements.
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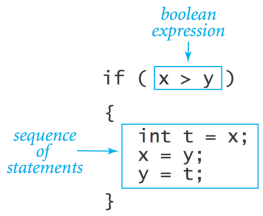
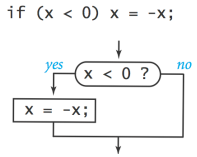
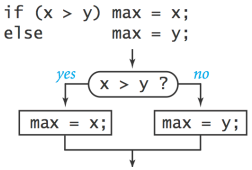






Ex. Take different action depending on value of variable.

```
public class Flip {
   public static void main(String[] args) {
      if (Math.random() < 0.5) System.out.println("Heads");
      else                     System.out.println("Tails");
   }
}
```

```
% java Flip
Heads

% java Flip
Heads

% java Flip
Tails

% java Flip
Heads
```

If Statement Examples

absolute value	`if (x < 0) x = -x;`

put x and y into sorted order	`if (x > y)`
	`{`
	`  int t = x;`
	`  y = x;`
	`  x = t;`
	`}`

maximum of x and y	`if (x > y) max = x;` `else  max = y;`
error check for division operation	`if (den == 0) System.out.println("Division by zero");` `else  System.out.println("Quotient = " + num/den);`

error check for quadratic formula	`double discriminant = b*b - 4.0*c;`
	`if (discriminant < 0.0)`
	`{`
	`  System.out.println("No real roots");`
	`}`
	`else`
	`{`
	`  System.out.println((-b + Math.sqrt(discriminant))/2.0);`
	`  System.out.println((-b - Math.sqrt(discriminant))/2.0);`
	`}`

The While Loop

While Loop

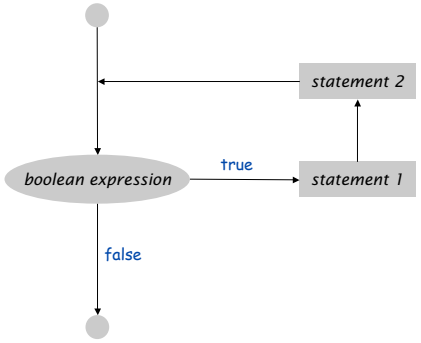
The `while` loop. A common repetition structure.

- Evaluate a `boolean` expression.
- If `true`, execute some statements.
- Repeat.

loop continuation condition

```
while (boolean expression) {
   statement 1;
   statement 2;
}
```

loop body



While Loop: Powers of Two

Ex. Print powers of 2 that are ≤ 2^N.

- Increment `i` from `0` to `N`.
- Double `v` each time.

```
int i = 0;
int v = 1;
while (i <= N) {
   System.out.println(i + " " + v);
   i = i + 1;
   v = 2 * v;
}
```

i	v	i <= N
0	1	true
1	2	true
2	4	true
3	8	true
4	16	true
5	32	true
6	64	true
7	128	false

```
0 1
1 2
2 4
3 8
4 16
5 32
6 64
```

N = 6

Click for demo

Powers of Two

```
public class PowersOfTwo {
   public static void main(String[] args) {

      // last power of two to print
      int N = Integer.parseInt(args[0]);

      int i = 0;  // loop control counter
      int v = 1;  // current power of two
      while (i <= N) {
         System.out.println(i + " " + v);
         i = i + 1;
         v = 2 * v;
      }
   }
}
```

print i and ith power of two

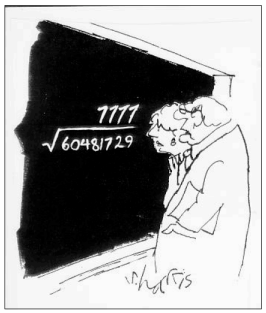
```
% java PowersOfTwo 4
0 1
1 2
2 4
3 8

% java PowersOfTwo 6
0 1
1 2
2 4
3 8
4 16
5 32
6 64
```


While Loop Challenge

Q. Anything wrong with the following code for printing powers of 2?

```
int i = 0;
int v = 1;
while (i <= N)
 System.out.println(i + " " + v);
   i = i + 1;
   v = 2 * v;
```


A Wonderful Square Root



"A wonderful square root. Let's hope it can be used for the good of mankind."


```
% java Sqrt 60481729
7777.0
```


While Loops: Square Root

Q. How might we implement `Math.sqrt()` ?

A. To compute the square root of c:

- Initialize $t_0 = c$.
- Repeat until $t_i = c / t_i$, up to desired precision: set t_{i+1} to be the average of t_i and c / t_i.

$$
\begin{aligned}
t_0 &= 2.0 \\
t_1 &= \tfrac{1}{2}(t_0 + \tfrac{2}{t_0}) = 1.5 \\
t_2 &= \tfrac{1}{2}(t_1 + \tfrac{2}{t_1}) = 1.416666666666665 \\
t_3 &= \tfrac{1}{2}(t_2 + \tfrac{2}{t_2}) = 1.4142156862745097 \\
t_4 &= \tfrac{1}{2}(t_3 + \tfrac{2}{t_3}) = 1.4142135623746899 \\
t_5 &= \tfrac{1}{2}(t_4 + \tfrac{2}{t_4}) = 1.414213562373095
\end{aligned}
$$

computing the square root of 2

While Loops: Square Root

Q. How might we implement `Math.sqrt()` ?

A. To compute the square root of c:

- Initialize t_0 = c.
- Repeat until t_i = c / t_i, up to desired precision: set t_{i+1} to be the average of t_i and c / t_i.

```
public class Sqrt {
   public static void main(String[] args) {
      double epsilon = 1e-15;
      double c = Double.parseDouble(args[0]);
      double t = c;
      while (Math.abs(t - c/t) > t*epsilon) {
         t = (c/t + t) / 2.0;
      }
      System.out.println(t);
   }
}
```

relative error tolerance

```
% java Sqrt 2.0
1.414213562373095
```

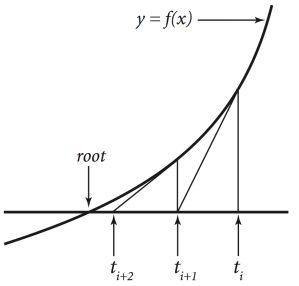
15 decimal digits of accuracy in 5 iterations

Newton-Raphson Method

Square root method explained.

- Goal: find root of any function f(x).
- Start with estimate t_0.
- Draw line tangent to curve at x= t_i.
- Set t_{i+1} to be x-coordinate where line hits x-axis.
- Repeat until desired precision.

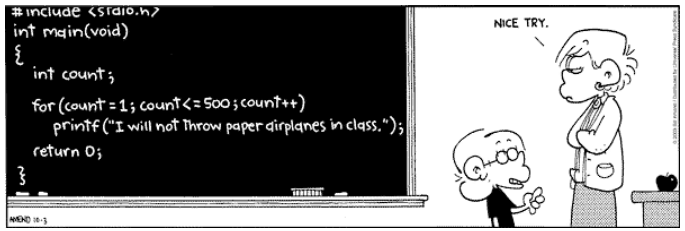
$f(x) = x^2 - c$ to compute $\sqrt{c}$



$$t_{i+1} = t_i - \frac{f(t_i)}{f'(t_i)}$$

Caveat. f(x) must be smooth; t_0 must be good estimate.

The For Loop



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For Loops

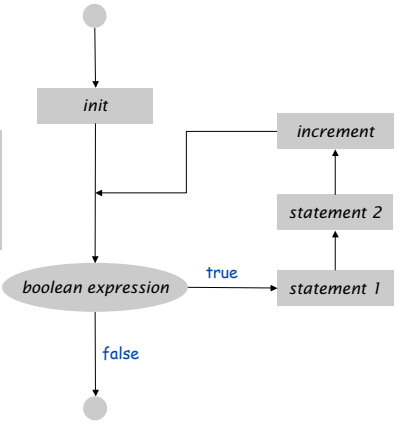
The `for` loop. Another common repetition structure.

- Execute initialization statement.
- Evaluate a `boolean` expression.
- If `true`, execute some statements.
- And then the increment statement.
- Repeat.

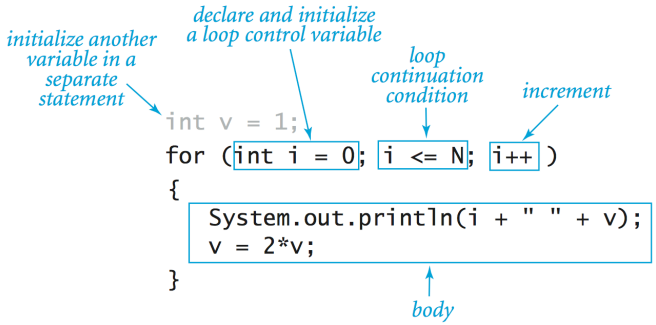
loop continuation condition

```
for (init; boolean expression; increment) {
   statement 1;
   statement 2;
}
```

body



Anatomy of a For Loop



Q. What does it print?

A.

For Loops: Subdivisions of a Ruler

Create subdivision of a ruler.

- Initialize `ruler` to `" "`.
- For each value `i` from `1` to `N`:
  sandwich two copies of `ruler` on either side of `i`.

```
public class RulerN {
   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      String ruler = " ";
      for (int i = 1; i <= N; i++) {
         ruler = ruler + i + ruler;
      }
      System.out.println(ruler);
   }
}
```

i	ruler
	" "
1	" 1 "
2	" 1 2 1 "
3	" 1 2 1 3 1 2 1 "

For Loops: Subdivisions of a Ruler

```
% java RulerN 1
 1

% java RulerN 2
 1 2 1

% java RulerN 3
 1 2 1 3 1 2 1

% java RulerN 4
 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

% java RulerN 5
 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 5 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

% java RulerN 100
Exception in thread "main"
java.lang.OutOfMemoryError
```

Observation. Loops can produce a huge amount of output!

Loop Examples

print largest power of two less than or equal to N	`int v = 1;`
	`while (v <= N/2)`
	`  v = 2*v;`
	`System.out.println(v);`

compute a finite sum $(1 + 2 + \ldots + N)$	`int sum = 0;`
	`for (int i = 1; i <= N; i++)`
	`  sum += i;`
	`System.out.println(sum);`

compute a finite product $(N! = 1 \times 2 \times \ldots \times N)$	`int product = 1;`
	`for (int i = 1; i <= N; i++)`
	`  product *= i;`
	`System.out.println(product);`

print a table of function values	`for (int i = 0; i <= N; i++)` `  System.out.println(i + " " + 2*Math.PI*i/N);`

print the ruler function (see Program 1.2.1)	`String ruler = " ";`
	`for (int i = 1; i <= N; i++)`
	`  ruler = ruler + i + ruler;`
	`System.out.println(ruler);`

Nesting

Conditionals enable you to do one of 2^n sequences of operations with n lines.

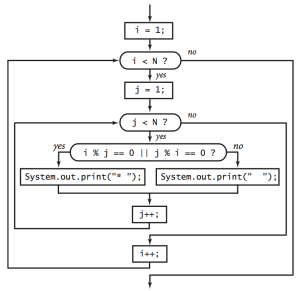
```
if (a0 > 0) System.out.print(0);
if (a1 > 0) System.out.print(1);
if (a2 > 0) System.out.print(2);
if (a3 > 0) System.out.print(3);
if (a4 > 0) System.out.print(4);
if (a5 > 0) System.out.print(5);
if (a6 > 0) System.out.print(6);
if (a7 > 0) System.out.print(7);
if (a8 > 0) System.out.print(8);
if (a9 > 0) System.out.print(9);
```

2^{10} = 1024 possible results, depending on input

Loops enable you to do an operation n times using only 2 lines of code.

```
double sum = 0.0;
for (int i = 1; i <= 1024; i++)
   sum = sum + 1.0 / i;
```

computes 1/1 + 1/2 + ... + 1/1024



More sophisticated programs.

- Nest conditionals within conditionals.
- Nest loops within loops.
- Nest conditionals within loops within loops.

Ex. Pay a certain tax rate depending on income level.

Income	Rate
0 - 47,450	22%
47,450 - 114,650	25%
114,650 - 174,700	28%
174,700 - 311,950	33%
311,950 -	35%

5 mutually exclusive alternatives

```
double rate;
if      (income <  47450) rate = 0.22;
else if (income < 114650) rate = 0.25;
else if (income < 174700) rate = 0.28;
else if (income < 311950) rate = 0.33;
else                      rate = 0.35;
```

graduated income tax calculation


```
if      (income <  47450) rate = 0.22;
else if (income < 114650) rate = 0.25;
else if (income < 174700) rate = 0.28;
else if (income < 311950) rate = 0.33;
else if (income < 311950) rate = 0.35;
```

is shorthand for

```
if (income <  47450) rate = 0.22;
else {
   if (income < 114650) rate = 0.25;
   else {
      if (income < 174700) rate = 0.28;
      else {
         if (income < 311950) rate = 0.33;
         else if (income < 311950) rate = 0.35;
      }
   }
}
```

Be careful when nesting if-else statements. (See Q+A on p. 75.)

Nested If Statement Challenge

Q. Anything wrong with the following for income tax calculation?

Income	Rate
0 - 47,450	22%
47,450 - 114,650	25%
114,650 - 174,700	28%
174,700 - 311,950	33%
311,950 -	35%

```
double rate = 0.35;
if (income <  47450) rate = 0.22;
if (income < 114650) rate = 0.25;
if (income < 174700) rate = 0.28;
if (income < 311950) rate = 0.33;
```

wrong graduated income tax calculation

Monte Carlo Simulation

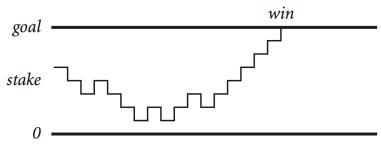
Gambler's Ruin

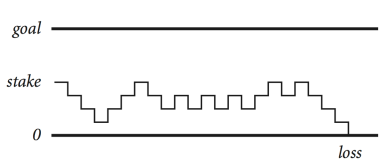
Gambler's ruin. Gambler starts with $stake and places $1 fair bets until going broke or reaching $goal.

- What are the chances of winning?
- How many bets will it take?

One approach. Monte Carlo simulation.

- Flip digital coins and see what happens.
- Repeat and compute statistics.





Gambler's Ruin

```
public class Gambler {
   public static void main(String[] args) {
      int stake  = Integer.parseInt(args[0]);
      int goal   = Integer.parseInt(args[1]);
      int T      = Integer.parseInt(args[2]);
      int wins   = 0;
      // repeat experiment N times
      for (int t = 0; t < T; t++) {
         // do one gambler's ruin experiment
         int cash = stake;
         while (cash > 0 && cash < goal) {
            // flip coin and update
            if (Math.random() < 0.5) cash++;
            else                     cash--;
         }
         if (cash == goal) wins++;
      }
      System.out.println(wins + " wins of " + T);
   }
}
```

Digression: Simulation and Analysis

stake goal T

```
% java Gambler 5 25 1000
191 wins of 1000

% java Gambler 5 25 1000
203 wins of 1000

% java Gambler 500 2500 1000
197 wins of 1000
```

after a substantial wait....

Fact. [see ORF 309] Probability of winning = stake ÷ goal.
Fact. [see ORF 309] Expected number of bets = stake × desired gain.
Ex. 20% chance of turning $500 into $2500, but expect to make one million $1 bets.

500/2500 = 20%
500 * (2500 - 500) = 1 million

Remark. Both facts can be proved mathematically; for more complex scenarios, computer simulation is often the best plan of attack.

Control Flow Summary

Control flow.
- Sequence of statements that are actually executed in a program.
- Conditionals and loops: enables us to choreograph the control flow.

Control Flow	Description	Examples
straight-line programs	all statements are executed in the order given	
conditionals	certain statements are executed depending on the values of certain variables	`if` `if-else`
loops	certain statements are executed repeatedly until certain conditions are met	`while` `for` `do-while`

Program Development

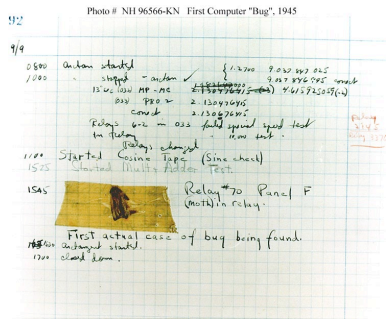

Ada Lovelace


Admiral Grace Murray Hopper

95% of Program Development

Program development. Creating a program and putting it to good use.
Def. A bug is a mistake in a computer program.

Programming is primarily a process of finding and fixing bugs.



Good news. Can use computer to test program.
Bad news. Cannot use computer to automatically find all bugs.

95% of Program Development

Debugging. Cyclic process of editing, compiling, and fixing errors.

- Always a logical explanation.
- What would the machine do?
- Explain it to the teddy bear.

You will make many mistakes as you write programs. It's normal.

> *"As soon as we started programming, we found out to our surprise that it wasn't as easy to get programs right as we had thought. I can remember the exact instant when I realized that a large part of my life from then on was going to be spent in finding mistakes in my own programs." — Maurice Wilkes*

> *" If I had eight hours to chop down a tree, I would spend six hours sharpening an axe." — Abraham Lincoln*

Debugging Example

Factor. Given an integer N > 1, compute its prime factorization.

$3{,}757{,}208 = 2^3 \times 7 \times 13^2 \times 397$

$98 = 2 \times 7^2$

$17 = 17$

$11{,}111{,}111{,}111{,}111{,}111 = 2{,}071{,}723 \times 5{,}363{,}222{,}357$

Application. Break RSA cryptosystem (factor 200-digit numbers).

Debugging Example

Factor. Given an integer N > 1, compute its prime factorization.

Brute-force algorithm. For each putative factor i = 2, 3, 4, …, check if N is a multiple of i, and if so, divide it out.

3757208/8

i	N	output	i	N	output	i	N	output
2	3757208	2 2 2	9	67093		16	397	
3	469651		10	67093		17	397	
4	469651		11	67093		18	397	
5	469651		12	67093		19	397	
6	469651		13	67093	13 13	20	397	
7	469651	7	14	397				397
8	67093		15	397				

Debugging: 95% of Program Development

Programming. A process of finding and fixing mistakes.

- Compiler error messages help locate syntax errors.
- Run program to find semantic and performance errors.

check if i is a factor

as long as i is a factor, divide it out

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0])
      for (i = 0; i < N; i++) {
         while (N % i == 0)
            System.out.print(i + " ")
            N = N / i

      }
   }
}
```

this program has many bugs!

Debugging: Syntax Errors

Syntax error. Illegal Java program.

- Compiler error messages help locate problem.
- Goal: no errors and a file named `Factors.class`.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0])
      for (i = 0; i < N; i++) {
         while (N % i == 0)
            System.out.print(i + " ")
            N = N / i

      }
   }
}
```

```
% javac Factors.java
Factors.java:6: ';' expected
      for (i = 2; i < N; i++)
      ^
1 error
```

the first error

Debugging: Syntax Errors

Syntax error. Illegal Java program.

- Compiler error messages help locate problem.
- Goal: no errors and a file named `Factors.class`.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 0; i < N; i++) {
         while (N % i == 0)
            System.out.print(i + " ");
            N = N / i;
      }
   }
}
```

need to declare variable i

need terminating semicolons

syntax (compile-time) errors

Debugging: Semantic Errors

Semantic error. Legal but wrong Java program.

- Run program to identify problem.
- Add print statements if needed to produce trace.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 0; i < N; i++) {
         while (N % i == 0)
            System.out.print(i + " ");
            N = N / i;
      }
   }
}
```

```
% javac Factors.java
% java Factors
Exception in thread "main"
java.lang.ArrayIndexOutOfBoundsException: 0
        at Factors.main(Factors.java:5)
```

oops, no argument

Debugging: Semantic Errors

Semantic error. Legal but wrong Java program.

- Run program to identify problem.
- Add print statements if needed to produce trace.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 0; i < N; i++) {
         while (N % i == 0)
            System.out.print(i + " ");
            N = N / i;
      }
   }
}
```

need to start at 2 because 0 and 1 cannot be factors

```
% javac Factors.java
% java Factors 98
Exception in thread "main"
java.lang.ArithmeticExeption: / by zero
        at Factors.main(Factors.java:8)
```

Debugging: Semantic Errors

Semantic error. Legal but wrong Java program.

- Run program to identify problem.
- Add print statements if needed to produce trace.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 2; i < N; i++) {
         while (N % i == 0)
            System.out.print(i + " ");
            N = N / i;
      }
   }
}
```

indents do not imply braces

```
% javac Factors.java
% java Factors 98
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 ...
```

infinite loop!

Debugging: The Beat Goes On

Success. Program factors 98 = 2 × 7².

- But that doesn't mean it works for all inputs.
- Add trace to find and fix (minor) problems.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 2; i < N; i++) {
         while (N % i == 0) {
            System.out.print(i + " ");
            N = N / i;
         }
      }
   }
}
```

```
% java Factors 98
2 7 %
```

need newline

```
% java Factors 5
```

??? no output

```
% java Factors 6
2 %
```

??? missing the 3

Debugging: The Beat Goes On

Success. Program factors 98 = 2 × 7².

- But that doesn't mean it works for all inputs.
- Add trace to find and fix (minor) problems.

```
% java Factors 5
TRACE 2 5
TRACE 3 5
TRACE 4 5

% java Factors 6
2
TRACE 2 3
```

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 2; i < N; i++) {
         while (N % i == 0) {
            System.out.println(i + " ");
            N = N / i;
         }
         System.out.println("TRACE: " + i + " " + N);
      }
   }
}
```

Aha! Print out N after for loop (if it is not 1)

Debugging: Success?

Success. Program seems to work.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 2; i < N; i++) {
         while (N % i == 0) {
            System.out.print(i + " ");
            N = N / i;
         }
      }
      if (N > 1) System.out.println(N);
      else       System.out.println();
   }
}
```

"corner case"

```
% java Factors 5
5

% java Factors 6
2 3

% java Factors 98
2 7 7

% java Factors 3757208
2 2 2 7 13 13 397
```

Debugging: Performance Error

Performance error. Correct program, but too slow.

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 2; i < N; i++) {
         while (N % i == 0) {
            System.out.print(i + " ");
            N = N / i;
         }
      }
      if (N > 1) System.out.println(N);
      else       System.out.println();
   }
}
```

```
% java Factors 11111111
11 73 11 137

% java Factors 11111111111
21649 51329

% java Factors 11111111111111
11 239 4649 909091

% java Factors 1111111111111111
2071723
```

very long wait (with a surprise ending)

Debugging: Performance Error

Performance error. Correct program, but too slow.

Solution. Improve or change underlying algorithm.

fixes performance error: if N has a factor, it has one less than or equal to its square root

```
public class Factors {
   public static void main(String[] args) {
      long N = Long.parseLong(args[0]);
      for (int i = 2; i <= N/i; i++) {
         while (N % i == 0) {
            System.out.print(i + " ");
            N = N / i;
         }
      }
      if (N > 1) System.out.println(N);
      else       System.out.println();
   }
}
```

```
% java Factors 11111111
11 73 11 137

% java Factors 11111111111
21649 51329

% java Factors 11111111111111
11 239 4649 909091

% java Factors 1111111111111111
2071723 5363222357
```


Program Development: Analysis

Q. How large an integer can I factor?

```
% java Factors 3757208
2 2 2 7 13 13 397

% java Factors 9201111169755555703
9201111169755555703
```

after a few minutes of computing....

largest factor

digits	(i <= N)	(i*i <= N)
3	instant	instant
6	0.15 seconds	instant
9	77 seconds	instant
12	21 hours †	0.16 seconds
15	2.4 years †	2.7 seconds
18	2.4 millennia †	92 seconds

† estimated

Note. Can't break RSA this way (experts are still trying).

Debugging

Programming. A process of finding and fixing mistakes.

1. Create the program.
2. Compile it.
   Compiler says: That's not a legal program.
   Back to step 1 to fix syntax errors.
3. Execute it.
   Result is bizarrely (or subtly) wrong.
   Back to step 1 to fix semantic errors.
4. Enjoy the satisfaction of a working program!
5. Too slow? Back to step 1 to try a different algorithm.