Keyframe Animation

Adam Finkelstein & Tim Weyrich
Princeton University
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Computer Animation

• What is animation?
  ◦ Make objects change over time according to scripted actions

• What is simulation?
  ◦ Predict how objects change over time according to physical laws

Outline

➤ Keyframe animation
  • Adding inverse kinematics
  • Adding dynamics

Keyframe Animation

• Define character poses at specific time steps called “keyframes”

Keyframe Animation

• Interpolate variables describing keyframes to determine poses for character “in-between”
Example: 2-Link Structure
• Two links connected by rotational joints

\[ X = (x, y) \]

End-Effector

Example (online)
• Inbetweening:
  - Linear interpolation - usually not enough continuity
    - H&B Figure 16.16

Keyframe Animation
• Inbetweening:
  - Spline interpolation - maybe good enough
    - H&B Figure 16.11

Forward Kinematics
• Animator specifies joint angles: \( \theta_1 \) and \( \theta_2 \)
• Computer finds positions of end-effector: \( X \)

\[ X = (l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2), l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2)) \]
Keyframe Animation

- Inbetweening:
  - Cubic spline interpolation - maybe good enough
    » May not follow physical laws

Example: Walk Cycle

- Articulated figure:

- Hip joint orientation:

Example: Walk Cycle

- Knee joint orientation:

Example: Walk Cycle

- Ankle joint orientation:
Example: Ice Skating

(Mao Chen, Zaijin Guan, Zhiyan Liu, Xiaobo Que, CS426, Fall98, Princeton University)

Outline

• Keyframe animation
• Adding inverse kinematics
• Adding dynamics

Example: 2-Link Structure

• What if animator knows position of “end-effector”

Example: 2-Link Structure

Inverse Kinematics

• Animator specifies end-effector positions: $X$ 
• Computer finds joint angles: $\theta_1$ and $\theta_2$:

$$\theta_2 = \cos^{-1} \left( \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2} \right)$$

$$\theta_1 = \frac{-l_1 \sin(\theta_2)x + (l_1 + l_2 \cos(\theta_1))y}{(l_2 \sin(\theta_2)y + (l_1 + l_2 \cos(\theta_2))x}$$

Inverse Kinematics

• End-effector postions can be specified by spline curves

Inverse Kinematics

• Problem for more complex structures
  ° System of equations is usually under-defined
  ° Multiple solutions

Landmarks: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24
Inverse Kinematics

- Solution for more complex structures:
  - Find best solution (e.g., minimize energy in motion)
  - Non-linear optimization

\[ X = (x, y) \]

Example: Ball Boy

Fujito, Milliron, Ngan, & Sanocki
Princeton University

More examples: online

Pixar

Outline

- Keyframe animation
- Adding inverse kinematics
- Adding dynamics

Dynamics

- Simulation of physics insures realism of motion

Spacetime Constraints

- Animator specifies constraints:
  - What the character's physical structure is
    - e.g., articulated figure
  - What the character has to do (keyframes)
    - e.g., jump from here to there within time \( t \)
  - What other physical structures are present
    - e.g., floor to push off and land
  - How the motion should be performed
    - e.g., minimize energy
Spacetime Constraints

- Computer finds the “best” physical motion satisfying constraints
- Example: particle with jet propulsion
  - \( x(t) \) is position of particle at time \( t \)
  - \( f(t) \) is force of jet propulsion at time \( t \)
  - Particle’s equation of motion is:
    \[ m\dddot{x} - f - mg = 0 \]
  - Suppose we want to move from \( a \) to \( b \) within \( t_0 \) to \( t_1 \) with minimum jet fuel:
    \[
    \text{Minimize } \int_{t_0}^{t_1} f(t)^2 dt
    \text{ subject to } x(t_0) = a \text{ and } x(t_1) = b
    \]

Witkin & Kass ‘88

Spacetime Constraints

- Advantages:
  - Free animator from having to specify details of physically realistic motion with spline curves
  - Easy to vary motions due to new parameters and/or new constraints
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization

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Spacetime Constraints

- Adapting motion:

  - Heavier Base

  - Original Jump

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Spacetime Constraints

- Adapting motion:

  - Hurdle

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Spacetime Constraints

- Adapting motion:

  - Ski Jump

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Spacetime Constraints

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Example: Manipulation of Sims.


Summary

• Keyframe animation
  ° Poses specified at key times
  ° In-betweening to fill in the rest

• Incorporating inverse kinematics
  ° Makes keyframes easier to specify

• Incorporating dynamics
  ° Makes animation easier to adapt