Computer Animation

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Syllabus

I. Image processing
II. Modeling
III. Rendering
IV. Animation

Simulation

• Dynamics
  ◦ Considers underlying forces
  ◦ Compute motion from initial conditions and physics

• Kinematics
  ◦ Considers only motion
  ◦ Determined by positions, velocities, accelerations

Dynamics

Passive—no muscles or motors

- model
- particle systems
- leaves
- water spray
- clothing
- state
- numerical integrator
- initial conditions
- user

Active—internal source of energy

- running human
- trotting dog
- swimming fish
- state
- numerical integrator
- forces and torques
- model
- desired behavior
- user

Passive Dynamics

• No muscles or motors
  ◦ Smoke
  ◦ Water
  ◦ Cloth
  ◦ Fire
  ◦ Fireworks
  ◦ Dice

Pixar

University of Illinois

Hodgins

McAllister
Passive Dynamics

• Physical laws
  ◦ Newton's laws
  ◦ Hook's law
  ◦ Etc.

• Physical phenomena
  ◦ Gravity
  ◦ Momentum
  ◦ Friction
  ◦ Collisions
  ◦ Elasticity
  ◦ Fracture

Particle Systems

• A particle is a point mass
  ◦ Mass
  ◦ Position
  ◦ Velocity
  ◦ Forces
  ◦ Color
  ◦ Lifetime

• Use lots of particles to model complex phenomena
  ◦ Keep array of particles
  ◦ Newton's laws

Particle Systems

• For each frame:
  ◦ Create new particles and assign attributes
  ◦ Delete any expired particles
  ◦ Update particles based on attributes and physics
  ◦ Render particles

Creating Particles

• Where to create particles?
  ◦ Predefined source
  ◦ Surface of shape
  ◦ Where particle density is low
  ◦ etc.

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Deleting Particles

• When to delete particles?
  ◦ Predefined sink
  ◦ Surface of shape
  ◦ Where density is high
  ◦ Life span
  ◦ Random
**Rendering Particles**

- Rendering styles
  - Points
  - Polygons
  - Shapes
  - Trails
  - etc.

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**Particle Systems**

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**Equations of Motion**

- Newton’s Law for a point mass
  - \( f = ma \)

- Computing particle motion requires solving second-order differential equation
  \[
  \ddot{x} = \frac{f(x, \dot{x}, t)}{m}
  \]

- Add variable \( v \) to form coupled first-order differential equations
  \[
  \begin{align*}
  \dot{x} &= v \\
  \dot{v} &= \frac{f}{m}
  \end{align*}
  \]
Solving the Equations of Motion

- Initial value problem
  - Know $p(0)$, $v(0)$, $a(0)$
  - Can compute force at any time and position
  - Compute $p(t)$ by forward integration

- Euler integration
  - $p(t+\Delta t) = p(t) + \Delta t \, v(t)$
  - $v(t+\Delta t) = v(t) + \Delta t \, f(x, t)/m$

- Problem:
  - Accuracy decreases as $\Delta t$ gets bigger

- Midpoint method (2nd order Runge-Kutta)
  - Compute an Euler step
  - Evaluate $f$ at the midpoint
  - Take an Euler step using midpoint force
    - $v(t+\Delta t) = v(t) + \Delta t \, f( p(t) + 0.5 \times \Delta t \, v(t), t)$

- Adapting step size
  - Compute $p_a$ by taking one step of size $h$
  - Compute $p_b$ by taking 2 steps of size $h/2$
  - Error = $| p_a - p_b |$
  - Multiply step size by factor ($constant/error$)

Particle System Forces

- Force fields
  - Gravity, wind, pressure

- Viscosity/damping
  - Liquids, drag

- Collisions
  - Environment
  - Other particles

- Other particles
  - Springs between neighboring particles (mesh)
  - Useful for cloth
Particle System Forces

Example: Gravity

Example: Fire

Example: Bouncing Off Wall

Example: Bouncing Off Wall
Example: Bouncing Off Wall

Collision Response

$$V' = V_T - k_s V_N$$

Before

After

Example: Bouncing Off Wall

Contact Force

$$F' = F_T$$

The wall pushes back, cancelling the normal component of $F$.

(An example of a constraint force.)

Example: Bouncing Off Particles

Example: More Bouncing

Example: Cloth

- Spring-mass mesh
- Hooke’s law

$$f = -k_s (|d| - s) \frac{d}{|d|}$$

$q$ = force

$k_s$ = spring constant

$d = p - q$

$s =$ resting length

Example: Cloth

Breen

Hodgins
Example: Cloth

• Spring-mass mesh

Example: Cloth

• Hooke’s law
  \[ f = -k_s(|d| - s) \frac{d}{|d|} \]

  - f = force
  - \( k_s \) = spring constant
  - d = p - q
  - s = resting length

• Damping term
  \[ f = -\left( k_s(|d| - s) + k_d \frac{d^2}{|d|^3} \right) \frac{d}{|d|} \]
  \[ \dot{d} = p - q \]

Example: Cloth

Summary

• Particle systems
  - Lots of particles
  - Simple physics

• Interesting behaviors
  - Waterfalls
  - Smoke
  - Cloth
  - Flocks

• Solving motion equations
  - Simplest method is Euler integration
  - Better to use adaptive step sizes