Hierarchical clustering

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- Visualizing this tree provides a useful summary of the data

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- Recall that k-means or k-medoids requires
  - A number of clusters k
  - An initial assignment of data to clusters
  - A distance measure between data $d(x_n, x_m)$
- Hierarchical clustering only requires a measure of similarity between *groups* of data points.

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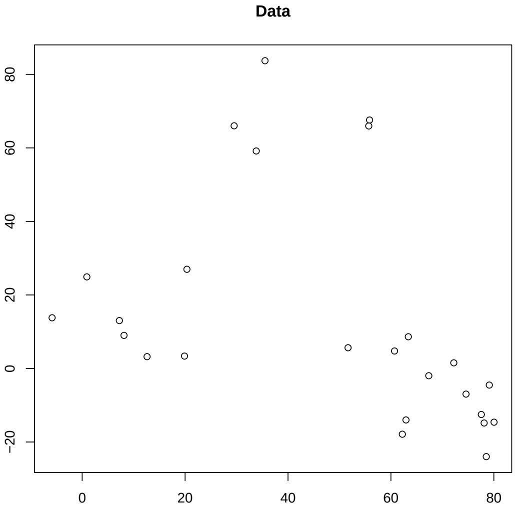
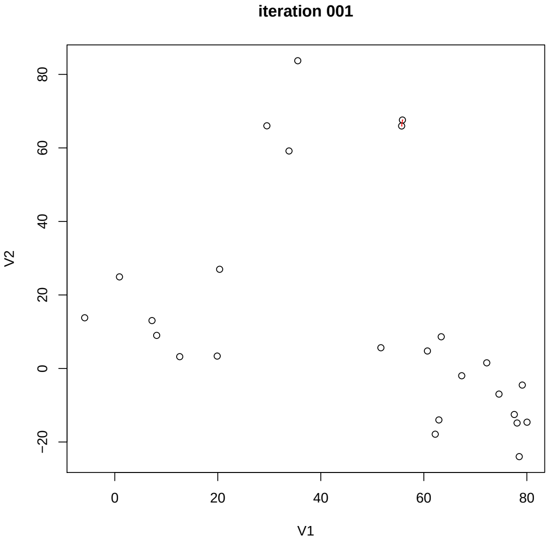
  2. Repeat: iteratively merge the two closest groups
  3. Until: all the data are merged into a single cluster

Example



Example



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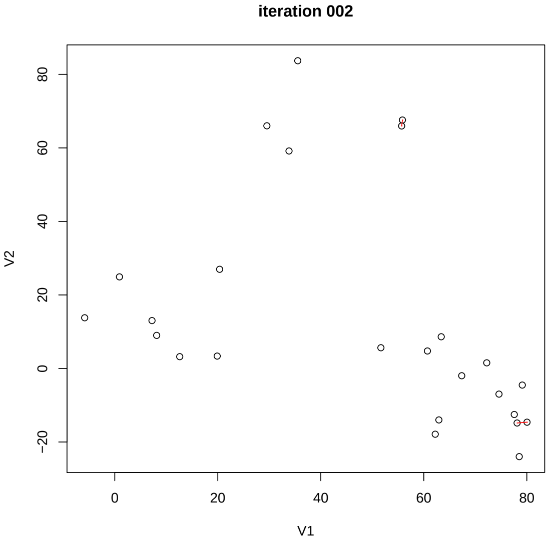

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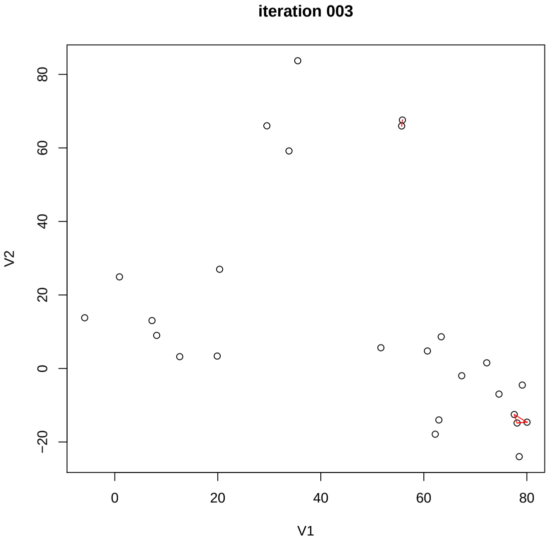

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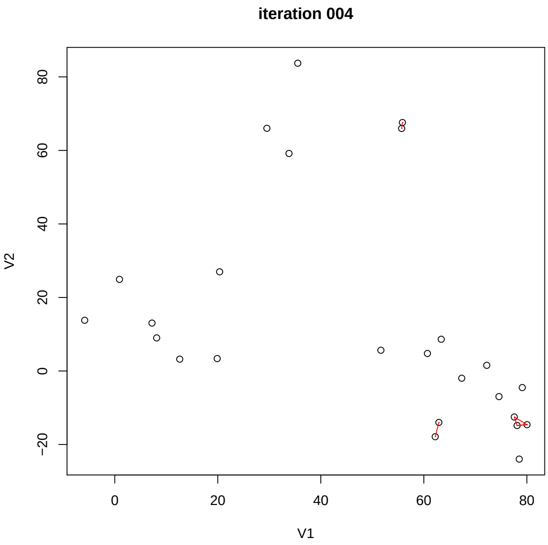

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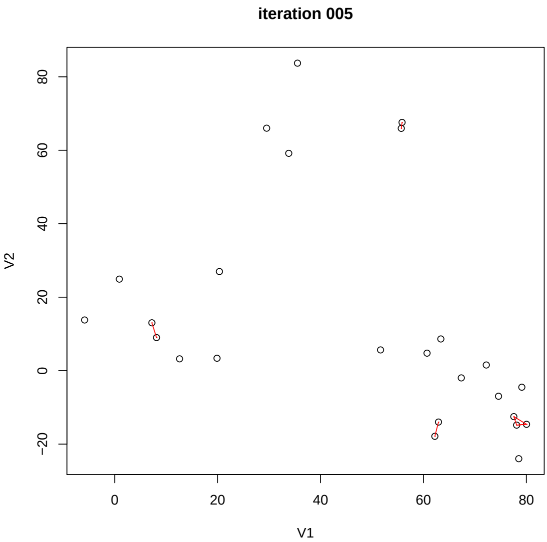

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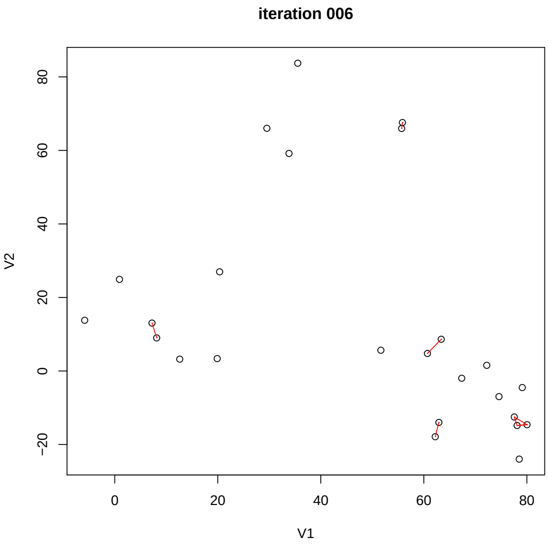

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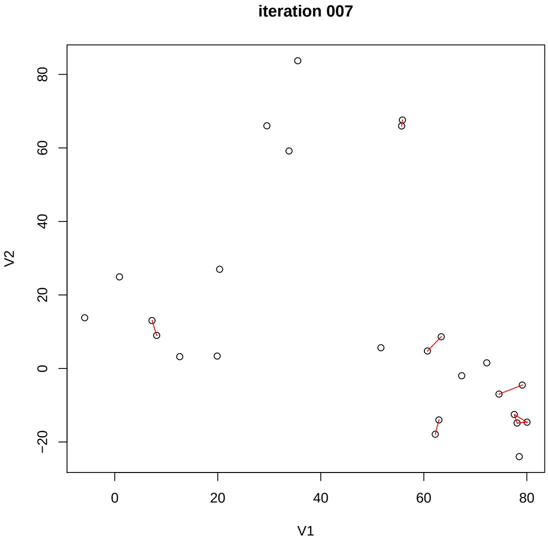

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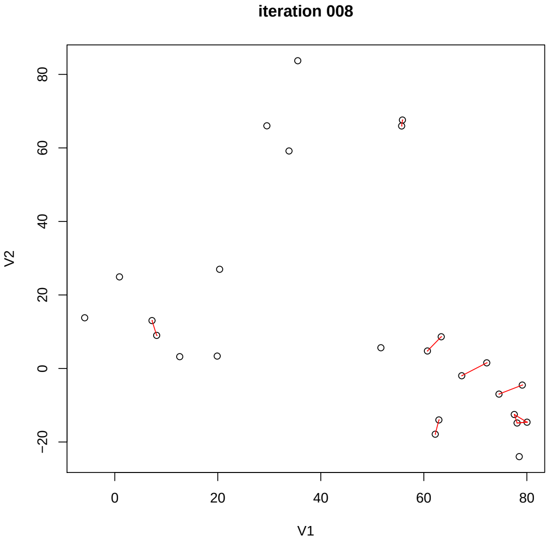

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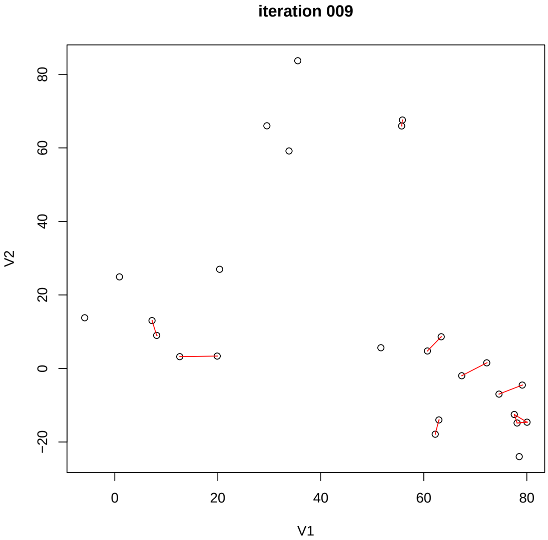

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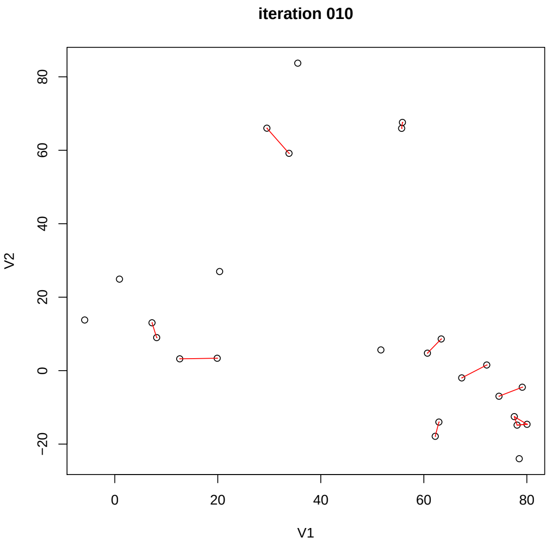

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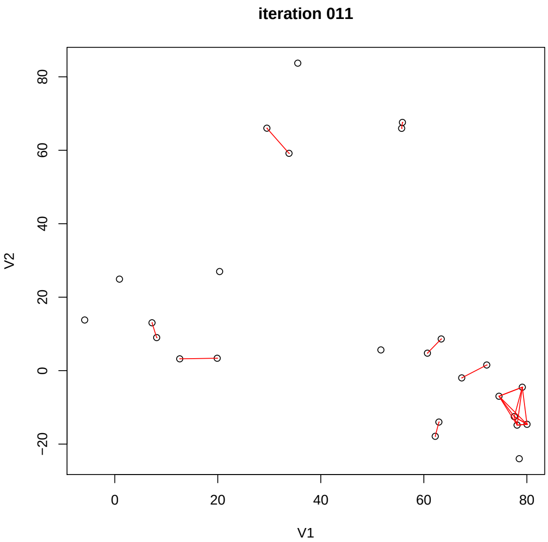

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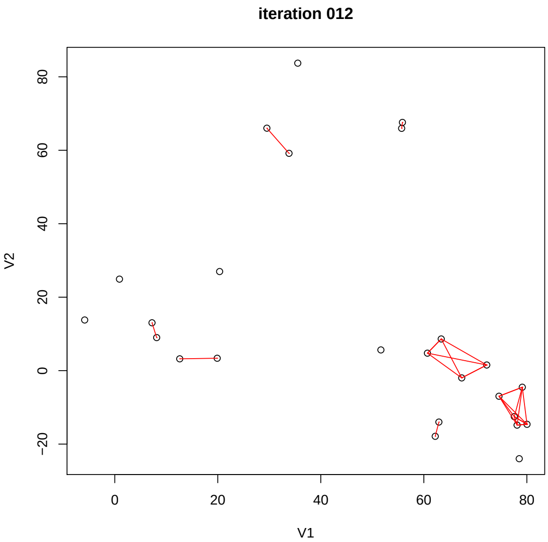

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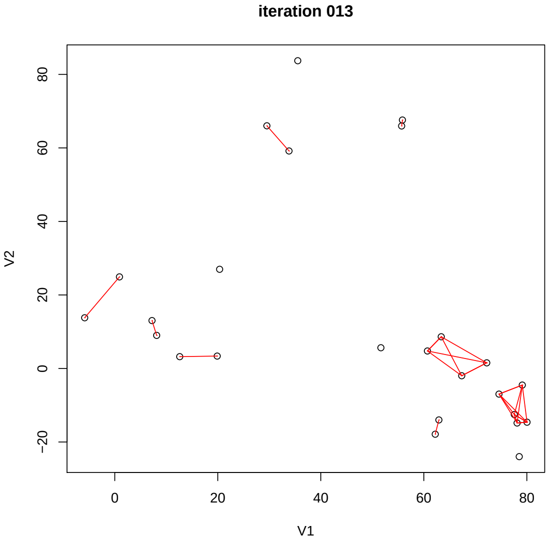

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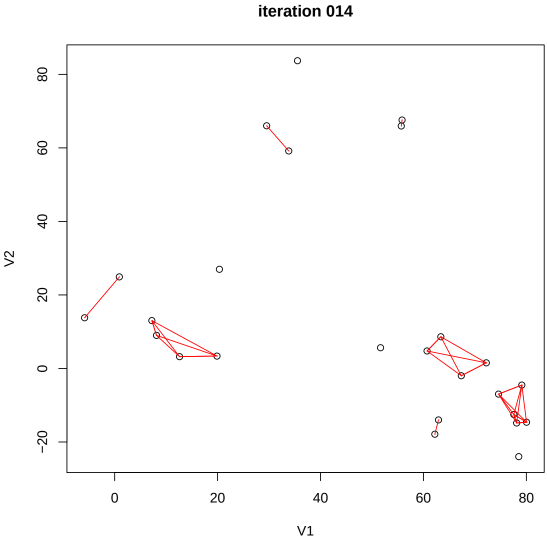

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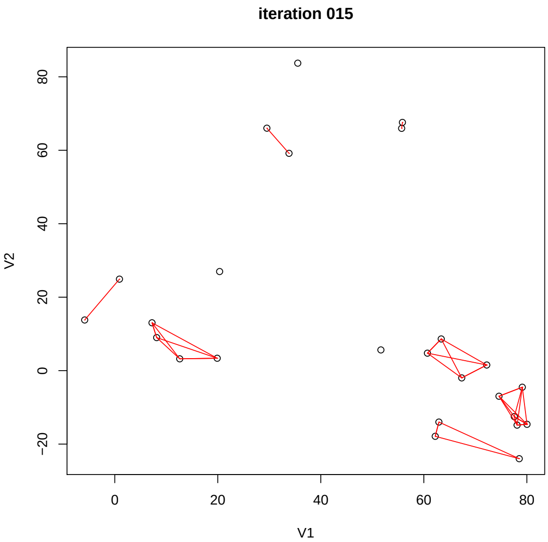

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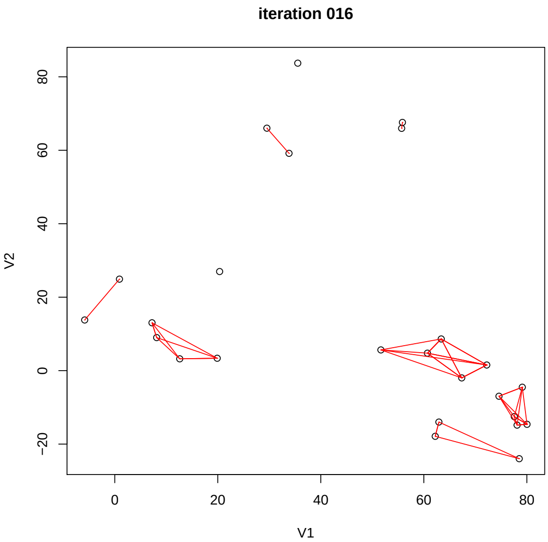

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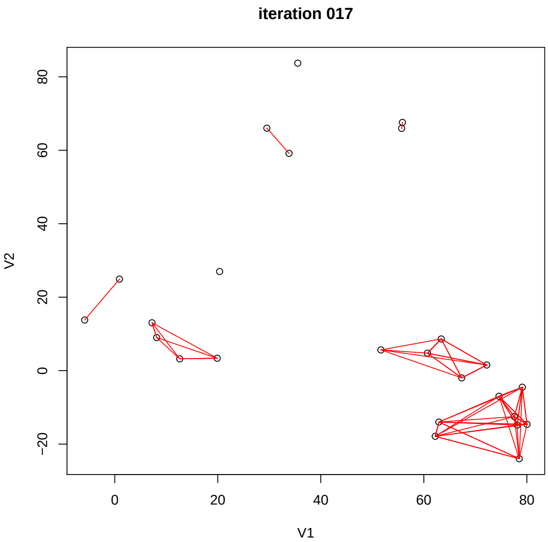

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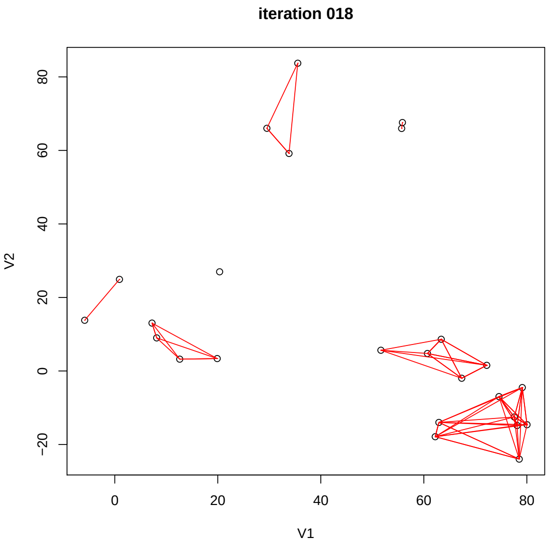

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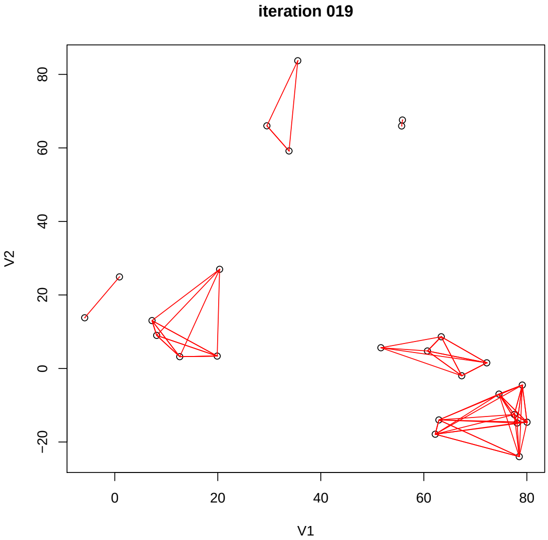

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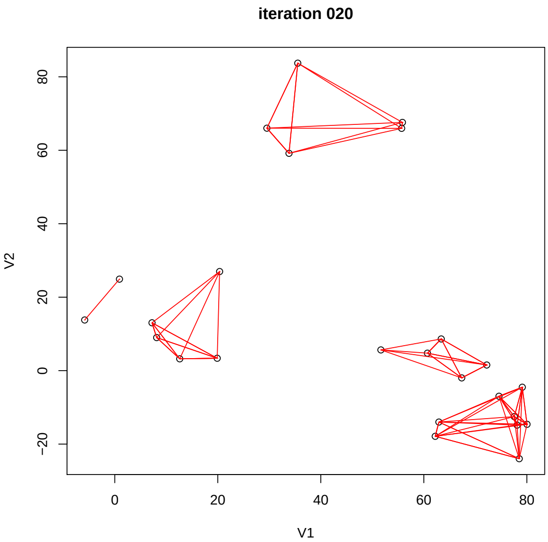

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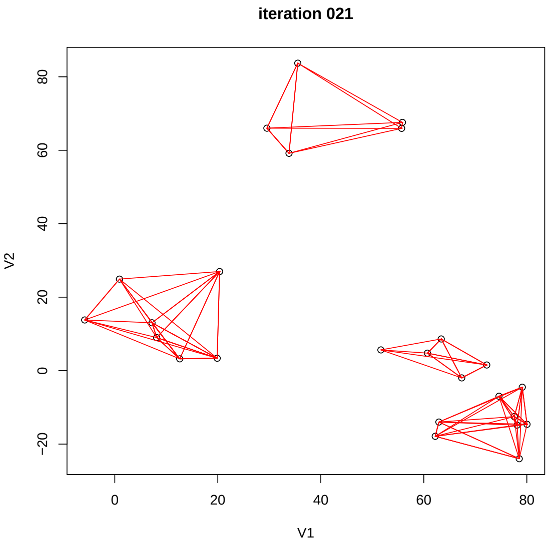

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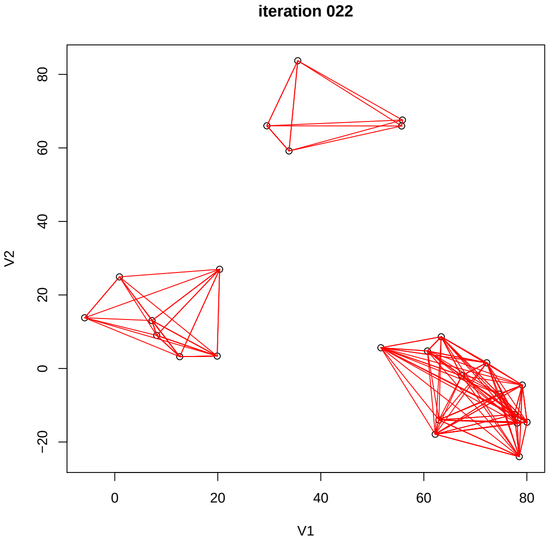

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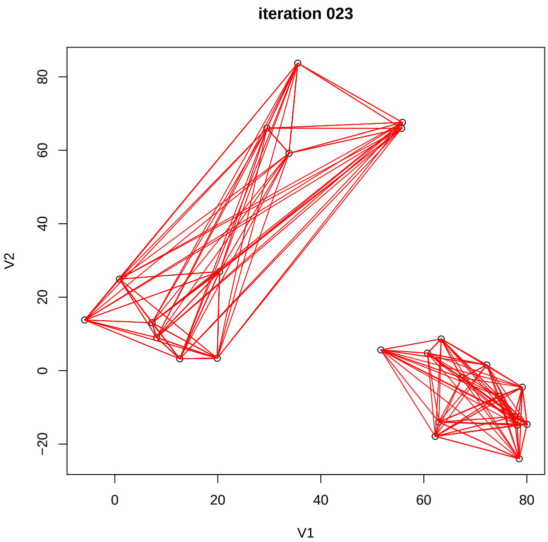

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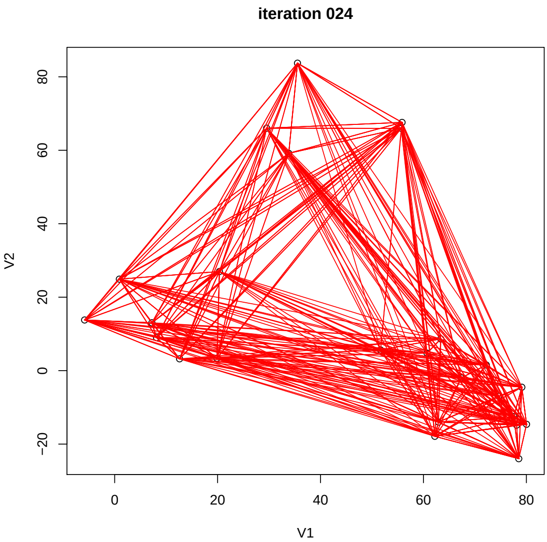

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- It is up to the user to choose a "natural" clustering from this sequence

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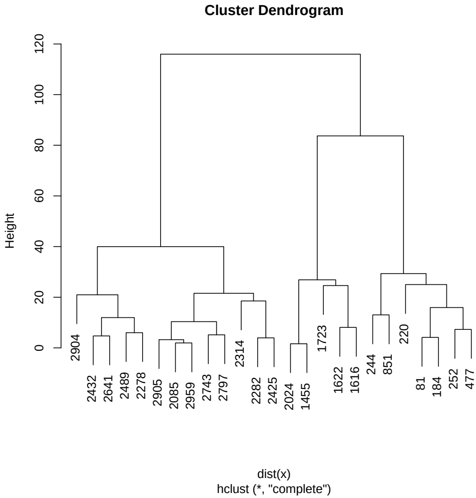
- *Dendrogram*: Plot each merge at the (negative) similarity between the two merged groups
- Provides an interpretable visualization of the algorithm and data
- Useful summarization tool, part of why hierarchical clustering is popular

Dendrogram of example data



Groups that merge at high values relative to the merger values of their subgroups are candidates for natural clusters. (Tibshirani et al., 2001)

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  - *Group average*: the average similarity between groups

$$d_{GA} = \frac{1}{N_G N_H} \sum_{i \in G} \sum_{j \in H} d_{i,j}$$

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- Complete linkage has the opposite problem. It might not merge close groups because of outlier members that are far apart.
- Group average represents a natural compromise, but depends on the scale of the similarities. Applying a monotone transformation to the similarities can change the results.

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- Different decisions about group similarities can lead to vastly different dendrograms.
- The algorithm *imposes* a hierarchical structure on the data, even data for which such structure is not appropriate.

Examples

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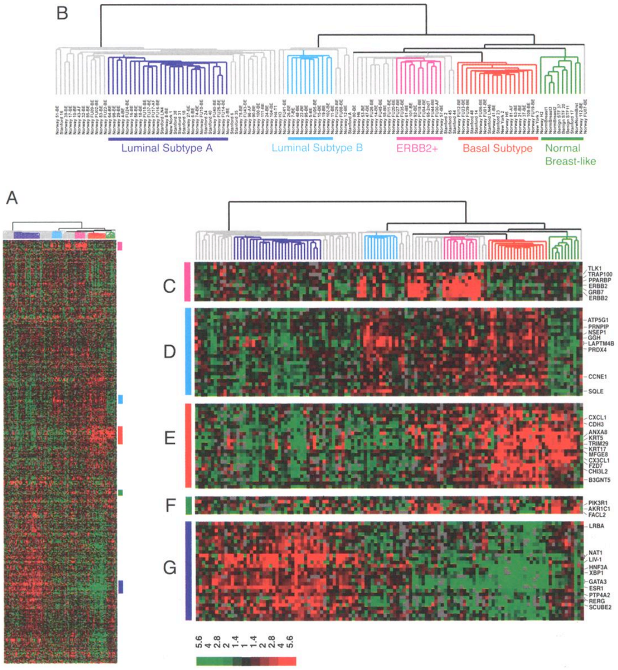
- Hierarchical clustering of gene expression data lead to new theories
- Later, theories tested in the lab.


B
Luminal Subtype A
Luminal Subtype B
ERBB2+
Basal Subtype
Normal
Breast-like
A
C
D
E
F
G
TLK1
TRAP100
PPARBP
ERBB2
GRB7
ERBB2
ATP5G1
PRNPIP
NSEP1
GGH
LAPTM4B
PRDX4
CCNE1
SQLE
CXCL1
CDH3
ANXA8
KRT5
TRIM29
KRT17
MFGE8
CX3CL1
FZD7
CHI3L2
B3GNT5
PIK3R1
AKR1C1
FACL2
LRBA
NAT1
LIV-1
HNF3A
XBP1
GATA3
ESR1
PTP4A2
RERG
SCUBE2
5.6 4 2.8 2 1.4 1 1.4 2 2.8 4 5.6

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- These authors cluster them using different methods, including hierarchical clustering
- They discuss the different clusters. (They are art critics.)

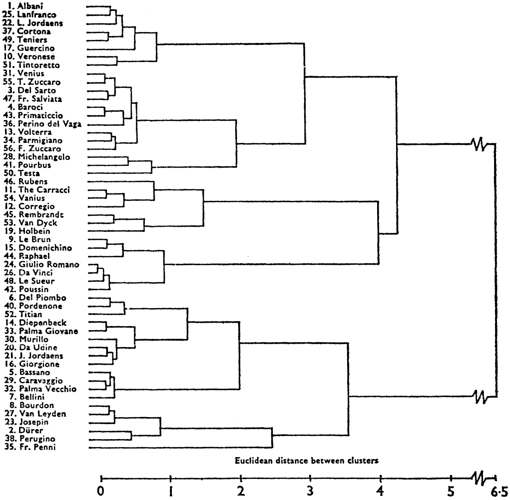

FIG. 1.

Good: They are cautious. “The value of this analysis...will depend on any interesting speculation it may provoke.”

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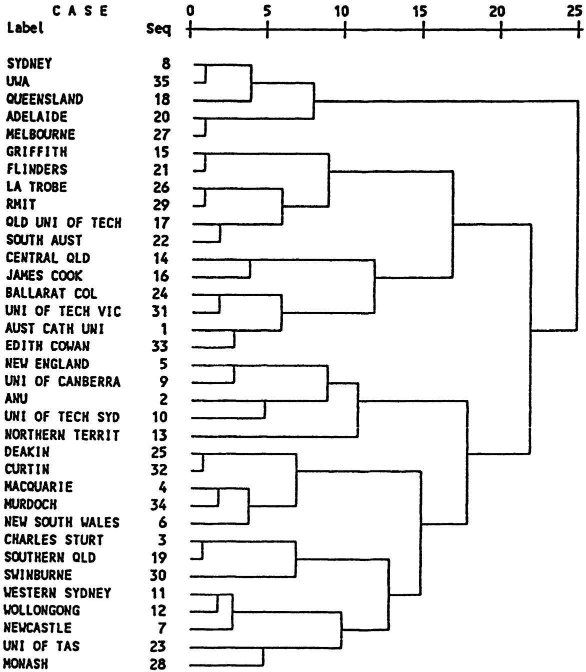
    - funding
    - evaluations



C A S E
Label
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0
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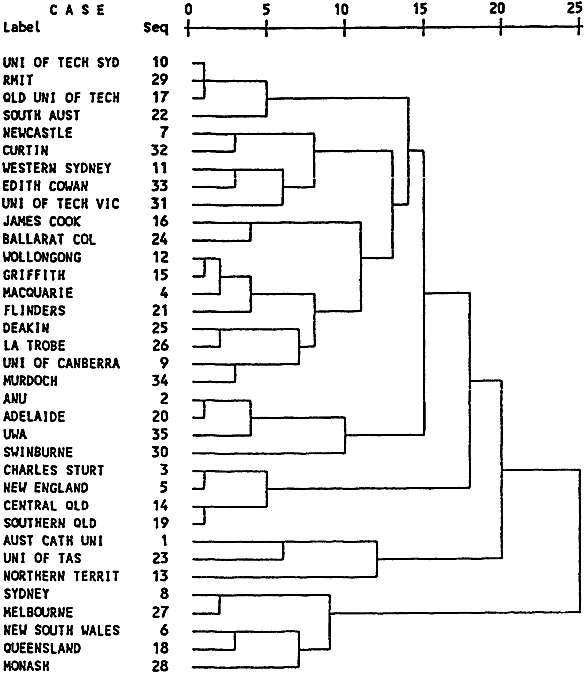
SYDNEY 8
UWA 35
QUEENSLAND 18
ADELAIDE 20
MELBOURNE 27
GRIFFITH 15
FLINDERS 21
LA TROBE 26
RMIT 29
QLD UNI OF TECH 17
SOUTH AUST 22
CENTRAL QLD 14
JAMES COOK 16
BALLARAT COL 24
UNI OF TECH VIC 31
AUST CATH UNI 1
EDITH COWAN 33
NEW ENGLAND 5
UNI OF CANBERRA 9
ANU 2
UNI OF TECH SYD 10
NORTHERN TERRIT 13
DEAKIN 25
CURTIN 32
MACQUARIE 4
MURDOCH 34
NEW SOUTH WALES 6
CHARLES STURT 3
SOUTHERN QLD 19
SWINBURNE 30
WESTERN SYDNEY 11
WOLLONGONG 12
NEWCASTLE 7
UNI OF TAS 23
MONASH 28


CASE
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RMIT 29
QLD UNI OF TECH 17
SOUTH AUST 22
NEWCASTLE 7
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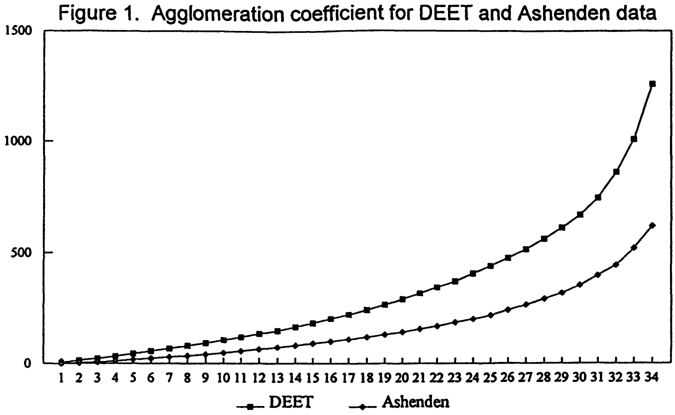
WESTERN SYDNEY 11
EDITH COWAN 33
UNI OF TECH VIC 31
JAMES COOK 16
BALLARAT COL 24
WOLLONGONG 12
GRIFFITH 15
MACQUARIE 4
FLINDERS 21
DEAKIN 25
LA TROBE 26
UNI OF CANBERRA 9
MURDOCH 34
ANU 2
ADELAIDE 20
UWA 35
SWINBURNE 30
CHARLES STURT 3
NEW ENGLAND 5
CENTRAL QLD 14
SOUTHERN QLD 19
AUST CATH UNI 1
UNI OF TAS 23
NORTHERN TERRIT 13
SYDNEY 8
MELBOURNE 27
NEW SOUTH WALES 6
QUEENSLAND 18
MONASH 28



- Split values: They notice that there's no kink and conclude that there is no cluster structure in Austrailian universities.
- **Good**: Cautious interpretation of clustering, analysis of clustering based on multiple subsets of the features.
- **Bad**: Their conclusions—we can't cluster Australian universities—ignores all the algorithmic choices that were made.

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- Data: weekly rates of return for stocks in twelve countries
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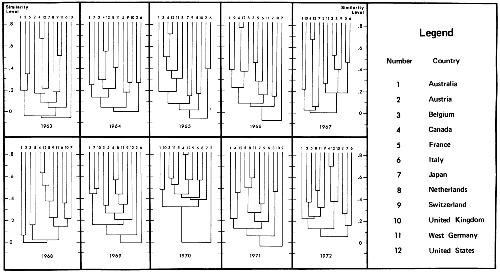
- Interpret the structure and examine stability over different time periods

Examples

FIGURE II

ONE-YEAR DENDROGRAMS
1963-1972



Good: Cautious. "This study is only descriptive...A logical subsequent research area is to explain observed structural properties and the causes of structural change."