COS 423: Theory of Algorithms Spring 2007

Lecture 3 — February 12, 2007

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1 Overview

In this lecture we will show demos on red-black trees, treaps, and splay trees. We will then go over the amortized time analysis for splay trees.

2 Demos

Please visit the following sites to test out the demos:

1. Red Black Trees: http://geocities.com/dmh2000/articles/code/red-blacktree.html
2. Treaps: http://www.ibr.cs.tu-bs.de/courses/ss98/audii/applets/BST/Treap-Example.html
3. Splay Trees: http://www.ibr.cs.tu-bs.de/courses/ss98/audii/applets/BST/SplayTree-Example.html

3 Splay Trees

For more info please visit http://en.wikipedia.org/wiki/Splay_tree. Splay trees are self adjusting binary search trees. Every access modifies the tree by moving the accessed item up to the root. The tree is modified by a sequence of zig-zag, zig-zig, and zig operations. These modifications guarentee basic operations such as insertion, lookup and removal to be performed in $O(log(n))$ amortized time. That is, a sequence of m operations on an n node splay tree takes $O(mlog(n))$ time.

Amortized analysis gives the average performance of each operation in the worst case. In a sequence of operations on a data structure often the worst case can not occur in each operation. There are two ways of thinking about amortized time. One is a bankers analysis and the other is (in terms of physics) potential energy. In the banking scenario each node of the tree has a savings account containing a certain amount of money. When a node x is created, we overcharge the add operation that creates x and deposits the extra credits to x's account. These credits will be used later to pay for restructuring operations on the tree. You can kinda of think of this as having a fixed amount A of restructuring operations when calling m accesses on a splay tree with n nodes. Restructuring some elements may be more expensive then others, but the total cost will be A.

http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15451-f06/www/lectures/lect0919.pdf
http://www.cs.princeton.edu/ wayne/cs423/lectures/amortized-4up.pdf