Lecture 18: More Dataflow Analysis

COS 320

Compiling Techniques

Princeton University
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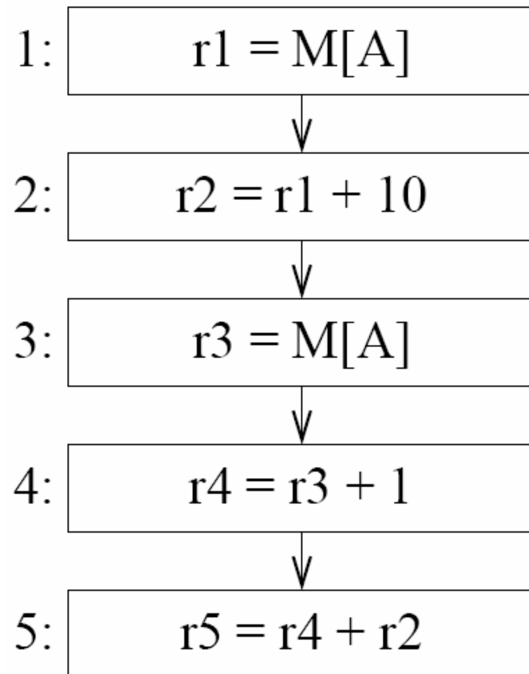
Common Subexpression Elimination

If `x op y` is computed multiple times, *common subexpression elimination* (CSE) attempts to eliminate some of the duplicate computations.



Need to track expression propagation → available expression analysis

Definitions

- Expression `x op y` is *available* at CFG node n if, on every path from CFG entry node to n, `x op y` is computed at least once, and neither `x` nor `y` are defined since last occurrence of `x op y` on path.
- Can compute set of expressions available at each statement using system of dataflow equations.
- Statement `r1 = M[r2]`:
  - *generates* expression `M[r2]`.
  - *kills* all expressions containing `r1`.
- Statement `r1 = r2 + r3`:
  - *generates* expression `r2 + r3`.
  - *kills* all expressions containing `r1`.

Iterative Dataflow Analysis Framework

- Specify:
  - Two *set definitions* - $A[n]$ and $B[n]$
  - A *transfer function* - $f\left(A, B, IN/OUT\right)$
  - A *confluence operator* - $\vee$.
  - A *direction* - FORWARD or REVERSE.
- For forward analyses:

$$IN[n] = \vee_{p \in PRED[n]} OUT[p]$$
$$OUT[n] = f\left(A, B\right)$$

- For reverse analyses:

$$OUT[n] = \vee_{s \in SUCC[n]} IN[s]$$
$$IN[n] = f\left(A, B\right)$$

Available Expression Analysis

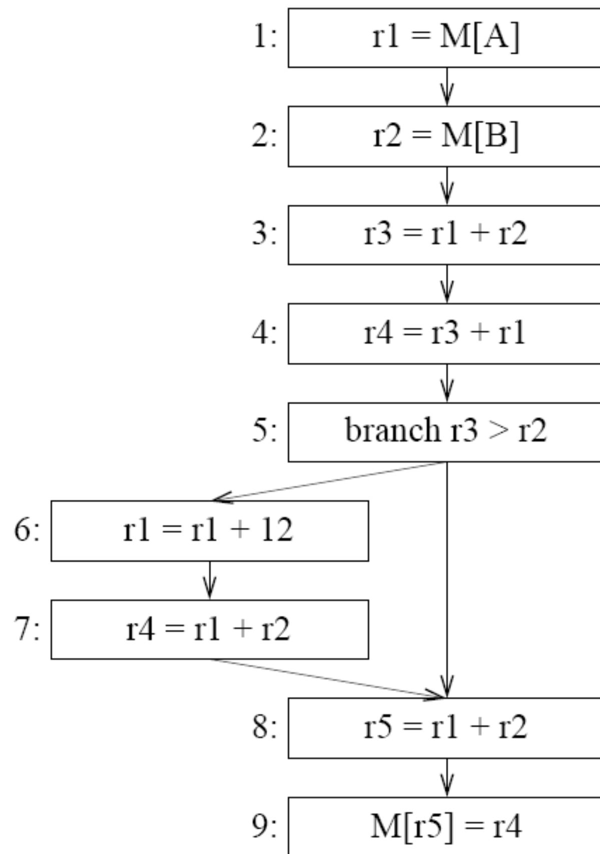
Available Expression Analysis:

- $exp(t)$ - set of all expressions containing t.
- Set definition ($A[n]$): $GEN[n]$ - the set of all expressions generated by n.
- Set definition ($B[n]$): $KILL[n]$ - the set of all expressions that n kills - $exp(n)$.
- Transfer function ($f(A, B, IN/OUT)$): $GEN[n] \cup (IN[n] - KILL[n])$
- Confluence operator ($\vee$): $\cap$
  - Use of $\cup$, required initialization of IN and OUT sets to $\emptyset$.
  - Use of $\cap$, requires initialization of IN and OUT sets to U (except for IN of entry node).
- Direction: FORWARD

$$IN[n] = \cap_{p \in PRED[n]} OUT[p]$$

$$OUT[n] = GEN[n] \cup (IN[n] - KILL[n])$$

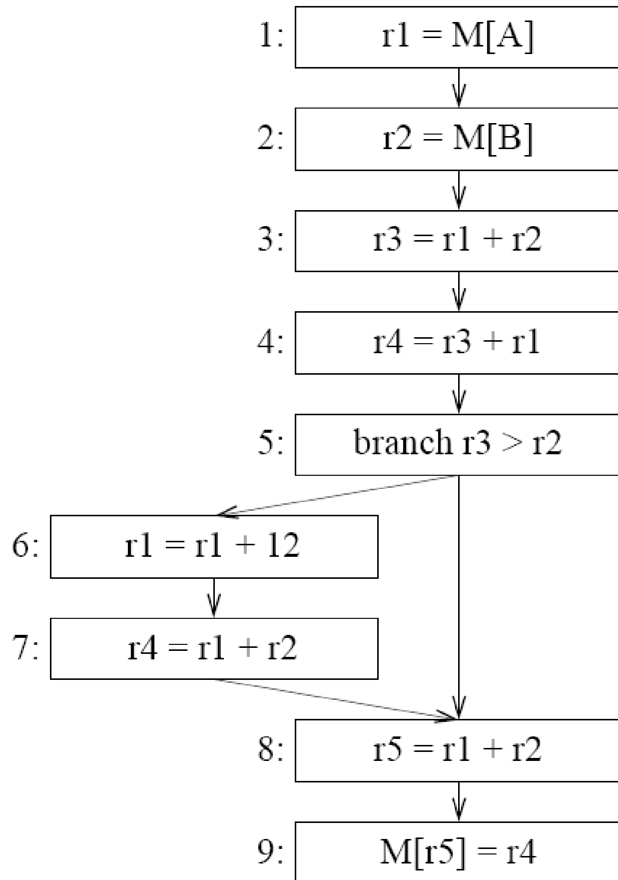
Example



Node	*GEN*	*KILL*	IN	OUT
1	M[A]	r1+r2, r1+12, r3+r1	-	*U*
2	M[B]	r1+r2	*U*	*U*
3	r1+r2	r3+r1	*U*	*U*
4	r3+r1		*U*	*U*
5			*U*	*U*
6		r1+r2, r3+r1, r1+12	*U*	*U*
7	r1+r2		*U*	*U*
8	r1+r2	M[r5]	*U*	*U*
9		M[A], M[B], M[r5]	*U*	*U*

Node	*GEN*	*KILL*	IN	OUT
1	1	378, 6, 4	-	*U*
2	2	378	*U*	*U*
3	378	4	*U*	*U*
4	4		*U*	*U*
5			*U*	*U*
6		378, 4, 6	*U*	*U*
7	378		*U*	*U*
8	378	9	*U*	*U*
9		1, 2, 9	*U*	*U*

Example



Node	GEN	$KILL$	IN	OUT
1	1	378, 4, 6	-	U
2	2	378	U	U
3	378	4	U	U
4	4		U	U
5			U	U
6		378, 4, 6	U	U
7	378		U	U
8	378	9	U	U
9		1, 2, 9	U	U

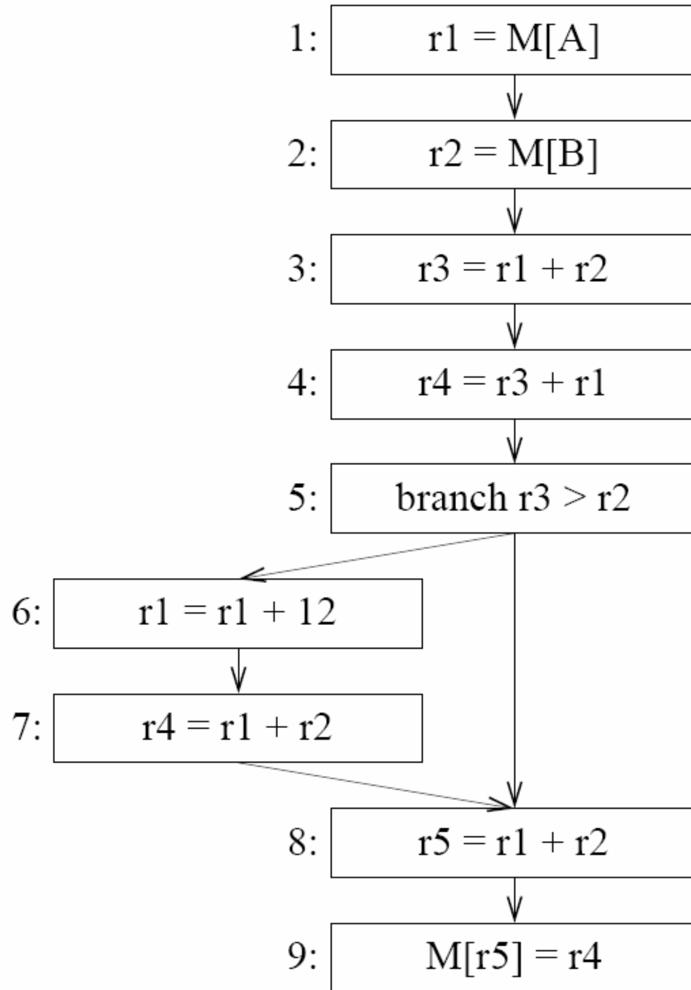
Node	IN	OUT	IN	OUT
1				
2				
3				
4				
5				
6				
7				
8				
9				

Common Subexpression Elimination (CSE)

Given statement s: `t = x op y`:
If expression `x op y` is available at beginning of node s then:

1. starting from node s, traverse CFG edges backwards to find last occurrence of `x op y` on each path from entry node to s.
2. create new temporary `w`.
3. for each statement s': `v = x op y` found in (1), replace s' by:
   ```
   w = x op y
   v = w
   ```
4. replace statement s by: `t = w`

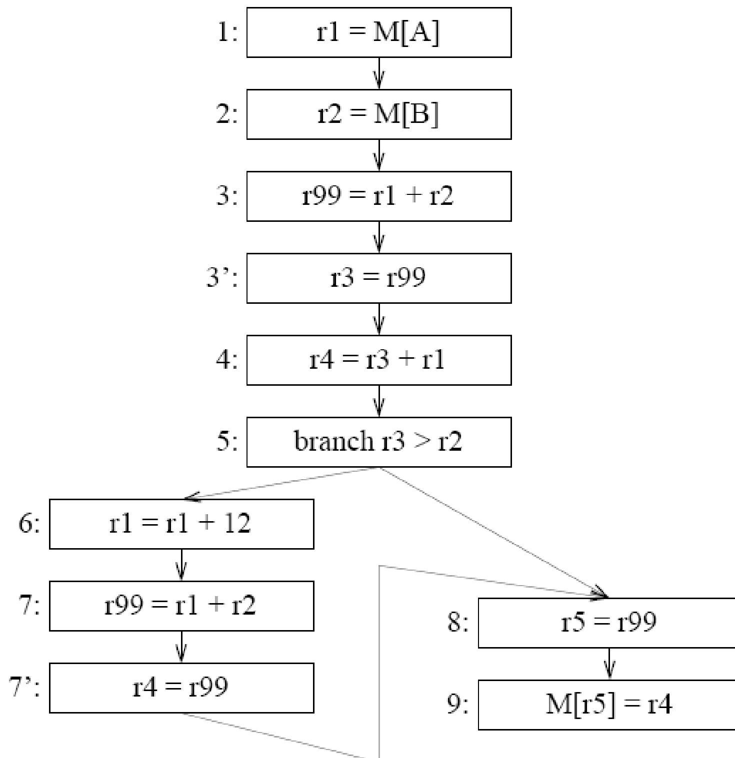
CSE Example



`r1 + r2` **in node 8 is a common subexpression.**

Copy Propagation

- Given statement d: `a = z` (`a` and `z` are both register temps) $\rightarrow$ d is a copy statement.
- Given statement u: `t = a op b`.
- If d reaches u, no other definition of `a` reaches u, and no definition of `z` exists on any path from d to u, then replace u by: `t = z op b`.



Sets

- Sets have been used in all the dataflow and control flow analyses presented.
- There are at least 3 representations which can be used:
  - Bit-Arrays:
    - Each *potential* member is stored in a bit of some array.
    - Insertion, Member is $O(1)$.
    - Assuming set size of N and word size of W - Union (OR) and Intersection (AND) is $O(N/W)$.
  - Sorted Lists/Trees:
    - Each member is stored in a list element.
    - Insertion, Member, Union, Intersection is $O(size)$. (Insertion, Member is $O(\log_2 size)$ in trees.)
    - Better for sparse sets than bit-arrays.
  - Hybrids: - Trees with bit-arrays
    - Use Tree to hold elements containing bit-arrays.
    - Union, Intersection is $O(size/W)$. Insertion, Member is $O(\log_2 size/W)$.

Basic Block Level Analysis

- To improve performance of dataflow, process at basic block level.
  - Represent the entire basic block by a single *super-instruction* which has any number of destinations and sources.
  - Run dataflow at basic block level.
  - Expand result to the instruction level.
- Example:

```
p:   r1 = r2 + r3      ->    r1, r2 = r2, r3
n:   r2 = r1
```

Basic Block Level Analysis

- Example:

```
p:   r1 = r2 + r3     ->    r1, r2 = r2, r3
n:   r2 = r1
```

- For reaching definitions:

$$OUT[n] = GEN[n] \cup (IN[n] - KILL[n])$$

But $IN[n] = OUT[p]$:

$$OUT[n] = GEN[n] \cup ((GEN[p] \cup (IN[p] - KILL[p])) - KILL[n])$$

Which (clearly) yields:

$$OUT[n] = GEN[n] \cup (GEN[p] - KILL[n]) \cup (IN[p] - (KILL[p] \cup KILL[n]))$$

So:

$$GEN[pn] = GEN[n] \cup (GEN[p] - KILL[n])$$
$$KILL[pn] = KILL[p] \cup KILL[n]$$

- Can we do this at the loop or general region level?

Reducible Flow Graphs Revisited

Definition

- A flow graph is reducible iff each edge exists in exactly one class:
  1. Forward edges (forms an acyclic graph where every node is reachable from start node)
  2. Back edges (head dominates tail)

Algorithm:

1. Remove all backedges
2. Check for cycles:
   - Cycles: Irreducible.
   - No Cycles: Reducible.

Think:

- All loop entry arcs point to header.

Reducible Flow Graphs – Structured Programs

Motivation:

- Structured programs are always reducible programs.
- Reducible programs are not always structured programs.
- Exploit the structured or reducible property in dataflow analysis.

Structures:

- Lists of instructions
- Conditionals/Hammocks
- While Loops (no breaks)

Method:

- Represent structures by a single *super-instruction* which has any number of destinations and sources.
- Run dataflow at structure level.
- Expand result to the instruction level.

Structured Program Analysis

- Lists of instructions - Basic Blocks!

$$GEN[pn] = GEN[n] \cup (GEN[p] - KILL[n])$$

$$KILL[pn] = KILL[p] \cup KILL[n]$$

- Conditionals/Hammocks

$$GEN[lr] = GEN[l] \cup GEN[r]$$

$$KILL[lr] = KILL[l] \cap KILL[r]$$

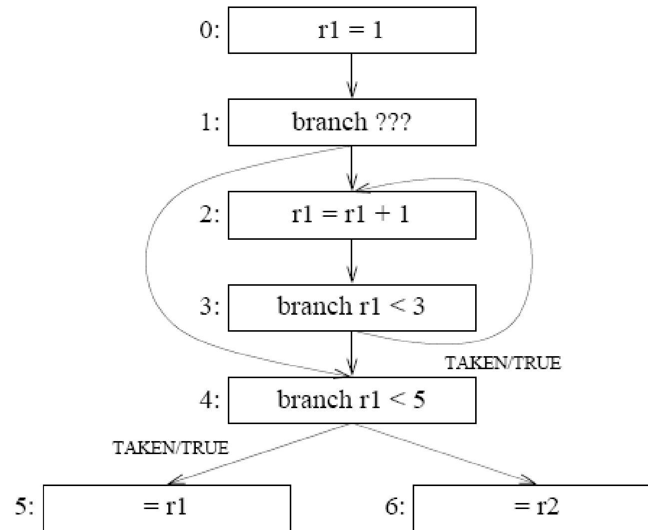
- While Loops

$$GEN[loop] = GEN[l]$$

$$KILL[loop] = KILL[l]$$

Try this on an irreducible flow graph...

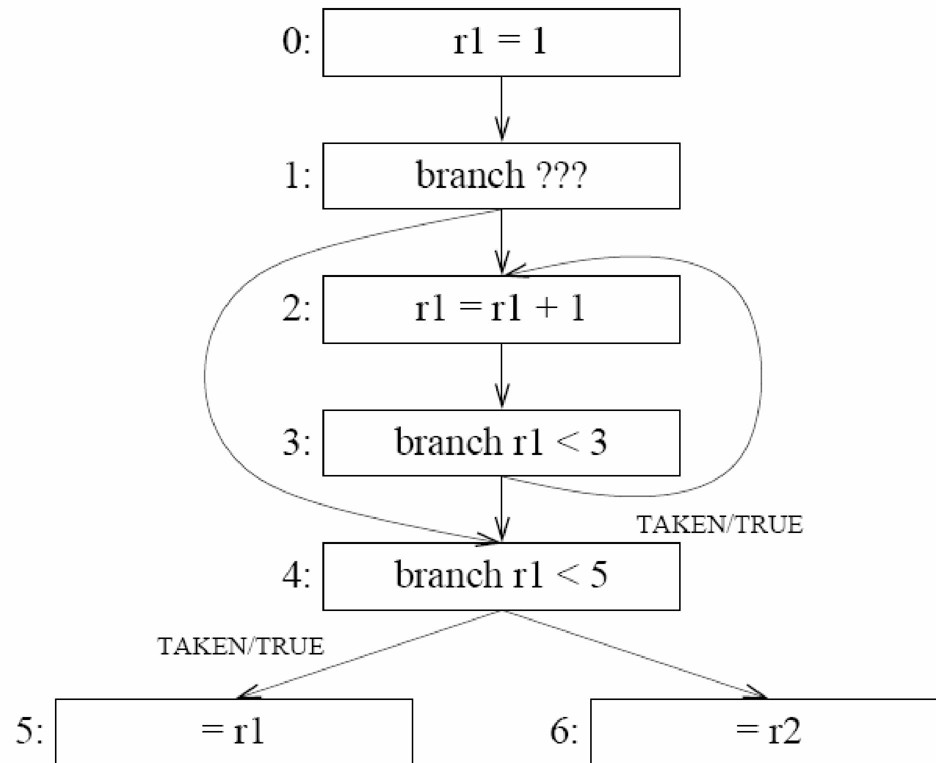
Conservative Approximations



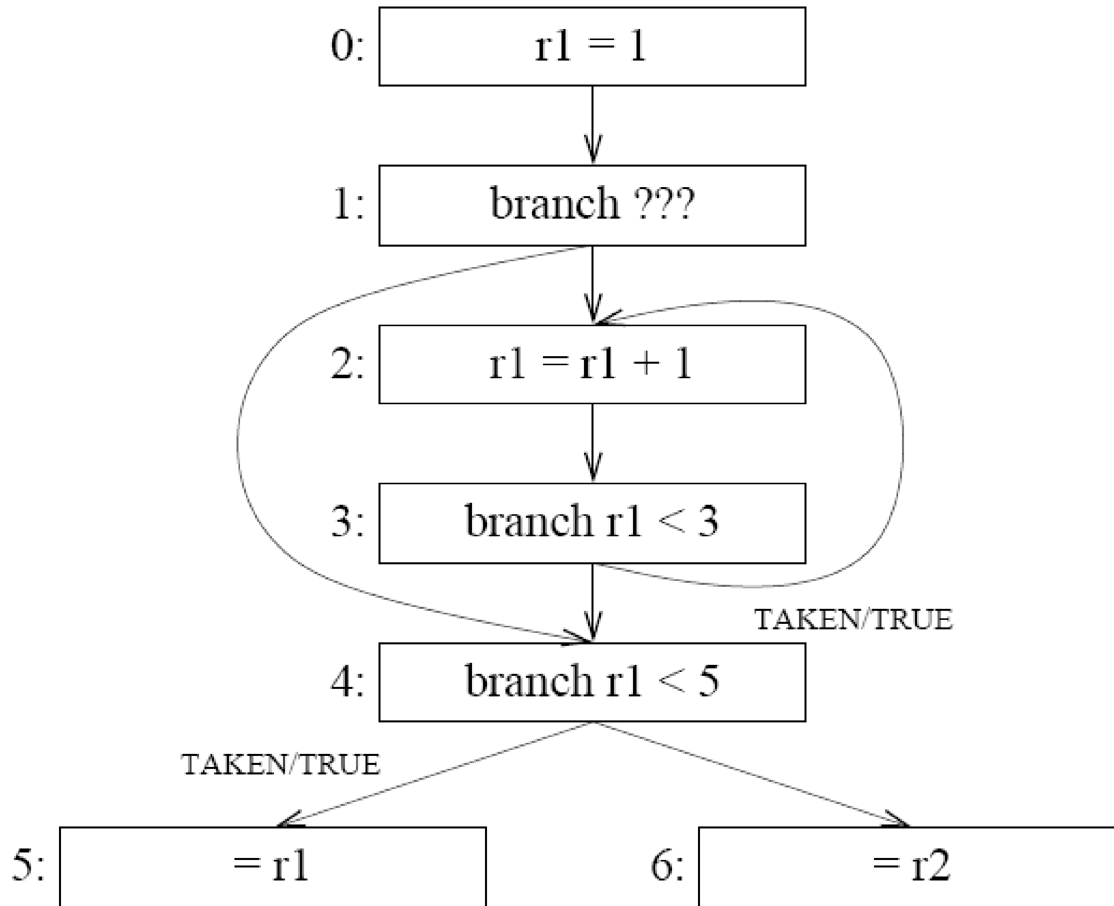
- Register r2 looks live by our live variable analysis, but is *not*.
- In general, today's compilers do not determine exactly how execution will proceed.
- Inaccuracy must lead to *conservative approximations*.
  - Optimizations must only be applied when proven safe.
  - Conservatism may not always compute best results.
- *MCI in ML* uses the terms *statically live* and *dynamically live*.

Conservative Approximations Example

Register Allocation:



New Dataflow Analysis



Limitation of Dataflow Analysis